Wavelets in Physics

This book surveys the application of the recently developed technique of the wavelet transform to a wide range of physical fields, including astrophysics, turbulence, meteorology, plasma physics, atomic and solid state physics, multifractals occurring in physics, biophysics (in medicine and physiology) and mathematical physics. The wavelet transorm can analyse scale-dependent characteristics of a signal (or image) locally, unlike the Fourier transform, and more flexibly than the windowed Fourier transform developed by Gabor 50 years ago. The continuous wavelet transform is used mostly for analysis, but the discrete wavelet transform allows very fast compression and transmission of data and speeds up numerical calculation, and is applied, for example, in the solution of partial differential equations in physics. This book will be of interest to graduate students and researchers in many fields of physics, and to applied mathematicians and engineers interested in physical application.

J. C. van den Berg studied physics and mathematics at the University of Amsterdam. He graduated in high energy physics, doing some work on the automatization of the analysis of bubble chamber films exhibiting the paths of elementary particles in collision experiments. He later took a degree in philosophy of science and logic at the same university, doing his masters thesis on quantum logic. He became a mathematics instructor at Wageningen University in 1973 and is now an Assistant Professor of Applied Mathematics at the Biometris group of Wageningen University and Research Center.

After being interested in the foundations of quantum mechanics for many years, he moved on to non-linear dynamics, especially the concept of multifractals and the difficulties of analysing them. In the writings of Alain Arnéodo on multifractals, he came across the wavelet transform for the first time, taking his first technical course on the subject in 1991 at the CWI in Amsterdam. Soon after, discovering the pioneering works of Marie Farge in turbulence and Gerald Kaiser in electromagnetism, he became convinced that wavelets were important for physics at large. Gradually wavelets overshadowed all his other interests and have remained a main focus ever since. This book is a result of that continuing interest and he hopes it may stimulate others to explore the possibilities of the new tools wavelet analysis continues to deliver.

Wavelets in Physics

Edited by
J.C. VAN DEN BERG
Wageningen University and Research Center,
Wageningen, The Netherlands

CAMBRIDGE
UNIVERSITY PRESS

PUBLISHED BY THE PRESS SYNDICATE OF THE UNIVERSITY OF CAMBRIDGE
The Pitt Building, Trumpington Street, Cambridge, United Kingdom

CAMBRIDGE UNIVERSITY PRESS
The Edinburgh Building, Cambridge CB2 2RU, UK
40 West 20th Street, New York, NY 10011-4211, USA
477 Williamstown Road, Port Melbourne, VIC 3207, Australia
Ruiz de Alarcón 13, 28014 Madrid, Spain
Dock House, The Waterfront, Cape Town 8001, South Africa
http://www.cambridge.org

First published 1999
First paperback edition 2004

Printed in the United Kingdom at the University Press, Cambridge

Typset in Times 11/14 [KW]

A catalogue record for this book is available from the British Library

ISBN 0 521 59311 5 hardback
ISBN 0521 53353 8 paperback

Contents

Contributors

J.-P. Antoine
Institut de Physique Théorique
Université Catholique de Louvain
Louvain-la-Neuve, Belgium
antoine@fyma.ucl.ac.be

Ph. Antoine
Alcatel
Antwerp, Belgium
philippe.antoine@alcatel.be

A. Arnéodo
Laboratoire de Physique
Ecole Normale Supérieure de Lyon
Lyon, France
alain.arneodo@ens-lyon.fr

E. Bacry
Centre de Mathématiques Appliquées
Ecole Polytechnique
Palaiseau Cedex, France
emmanuel.bacry@polytechnique.fr

J.C. van den Berg (Editor)
Biometris
Wageningen University and Research Center
Wageningen, The Netherlands
hansc.vandenberg@wur.nl

A. Bijaoui
Observatoire de la Côte d'Azur
Dpt. CERGA – UMR CNRS 6527
Nice, France
albert.bijaoui@obs-nice.fr

M. Farge
Laboratoire de Météorologie Dynamique du CNRS
Ecole Normale Supérieure
Paris, France
farge@lmd.ens.fr

A. Fournier
National Center for Atmospheric Research
Climate and Global Dynamics Division
Climate Dynamics and Predictability
Boulder, Colorado, USA
fournier@ucar.edu

A.L. Goldberger
Beth Israel Deaconess Medical Center
Harvard Medical School
Boston, Massachusetts, USA
agoldber@bidmc.harvard.edu

Ch.-A. Guérin
Institut Fresnel
Université Aix-Marseille III
Marseille, France
charles-antoine.guerin@fresnel.fr

S. Havlin
Department of Physics
Bar-Ilan University
Ramat Gan, Israel
havlin@ophir.ph.biu.ac.il

M. Holschneider
Universität Potsdam
Applied and Industrial Mathematics
Potsdam, Germany
hols@rz.uni-potsdam.de

L. Hudgins
Northrop Grumman Electronic Systems
Space Systems Division
Azusa, California, USA
lonnie.hudgins@northropgrumman.com

P.Ch. Ivanov
Center for Polymer Studies, Boston University and
Beth Israel Deaconess Medical Center, Harvard Medical School
Boston, Massachusetts, USA
plamen@argento.bu.edu

J.H. Kaspersen
SINTEF Unimed Ultrasound
7465 Trondheim, Norway
Jon.H.Kaspersen@sintef.no

N.K.-R. Kevlahan
Department of Mathematics and Statistics
McMaster University
Hamilton, Canada
kevlahan@mcmaster.ca

B.Ph. van Milligen
Laboratorio Nacional de Fusión
Asociación EURATOM-CIEMAT
Madrid, Spain
boudewijn, vanmilligen@ciemat.es

J.F. Muzy
Laboratoire SPE, CNRS UMR 6134
Université de Corse
Corte, France
muzy@univ-corse.fr

C.-K. Peng
ReyLab and Harvard Medical School
Boston University
Boston, Massachusetts, USA
peng@chaos.bidmc.harvard.edu

V. Perrier
Laboratoire de Modélisation et Calcul
IMAG
Grenoble, France
Valerie.Perrier@imag.fr

B. Piraux
Laboratoire de Physique Atomique et Moléculaire
Université Catholique de Louvain
Louvain-la-Neuve, Belgium
piraux@fyam.ucl.ac.be

M.G. Rosenblum
Nonlinear Dynamics Group at the Department of Physics
Potsdam University
Potsdam, Germany
MRos@agnld.uni-potsdam.de

K. Schneider
Centre de Mathématiques et d'Informatiques
Université de Provence (Aix-Marseille I)
Marseille, France
kschneid@cmi.univ.mrs.fr

H.E. Stanley
Center for Polymer Studies and Department of Physics
Boston University
Boston, Massachusetts, USA
hes@bu.edu

Preface to the paperback edition

Since the hardback edition of this book was put together wavelets have continued to flourish both in mathematics and in applications in ever more diverse branches of science and engineering. A standard library electronic alert system now easily produces more than fourteen hundred references to papers per year, developing or using wavelet techniques. These are published in a very broad array of journals. Here we can point to only a few of the recently developed methods, in particular as they have been used in physics.

In recent years many variations on the wavelet theme have appeared. One tries to go 'beyond wavelets'. In this context there is a whole family of new animals in the wavelet zoo. Its members carry names like *bandelets*, *beamlets*, *chirplets*, *contourlets*, *curvelets*, *fresnelets*, *ridgelets* . . . These are new bases or frames of functions, customized to handle 2D or 3D data processing better. In [23] for example, it is explained how ridgelets and curvelets can be used in astrophysics. It turns out that noise filtering, contrast enhancement and morphological component analysis of galaxy images are performed much better by a skilful combination of the new transforms than by mere wavelet transforms. More examples can be found on the 'curvelet homepage' [24], maintained by J.L. Starck.

It seems that the applications of the *discrete wavelet transform* (DWT) far outnumber those of the *continuous wavelet transform* (CWT), although the latter started the modern development of wavelet theory in the early eighties. Of the more than two hundred books on wavelet theory that have been published since the early nineties, most are focussed on the DWT and sometimes omit to mention the CWT altogether. This, I think, is unfortunate because both transforms have a lot to offer. A drawback of the CWT is that its computation is much more time consuming than that of the DWT. However, progress has been made in this area too. For example, in [20] a fast

algorithm is described for the computation of the CWT at any real scale a and integer time localization b.

The 2D CWT described in detail in Ch. 2 has been further developed by J.-P. Antoine *et al.* and now also covers the case of wavelets living on a sphere instead of on a flat plane [1], [2]. These *spherical wavelets* have been used for instance in astrophysics [4], and also in the recently emerged field of cosmic topology [21], the study of the global shape of the universe. How much richer the world of the 2D transform has become since Ch. 2 was written the reader may see in great detail in the volume especially devoted to this topic [3].

In turbulence studies M. Farge, the earliest promotor of wavelet methods in that field, together with K. Schneider and N. Kevlahan proposed the method of *Coherent Vortex Simulation* (CVS), initially applied to 2D flows, which is already briefly mentioned here in Ch. 4, p. 189. This method was much further developed in the following years, and was recently applied also to 3D flows [7]. More results of Farge and her increasingly productive team, which she set up together with K. Schneider, can be found at [8].

The *Wavelet Transform Modulus Maximim* (WTMM) method and its use for the computation of singularity spectra of multifractals, pioneered by A. Arnéodo's group and described here in Ch. 9, has recently been extended to image analysis [5] and to 3D fields [19]. Another application continuing to produce interesting results is the wavelet-based study of correlations in DNA [6].

The authors of Ch. 10, using wavelet techniques for the study of cardiac dynamics, more recently also adopted the WTMM method [15], [12] to expose the multifractal character of cardiovascular and several other human physiological signals.

It is of interest to note here that M. Haase and B. Lehle [13], using wavelets that are derivatives of the Gaussian function, have been able to derive differential equations for the maxima lines used in the WTMM method. Thus they produce an algorithm for the singularity spectra that is more accurate. More applications can be found at [14].

A. Fournier has advanced the research described in Ch. 7 in at least three ways: by establishing the wavelet-energetics interpretation for idealized fluid models [9], by enlarging the observational dataset to obtain statistically significant results [10] and by inventing customized representations of blocking using 'best shift' wavelets [11].

Let me finish by mentioning some interesting recent examples not directly related to the material of this volume. An application to chaos control was published by G. W. Wei *et al.* [25]. They study a set of chaotic Lorenz

oscillators, synchronized by nearest neighbour couplings. Using wavelets to decompose the coupling matrix, they show they can vastly reduce the minimally necessary coupling strength for synchronization to occur.

A. Romeo *et al.* [22] published an appealing N-body simulation of disc galaxies, where N ranges between 10^5 and 9×10^6, in which the initial symmetry is broken after initial fluctuations have been amplified sufficiently by gravitational instability. They show that their use of wavelets to denoise the calculation at each timestep makes their simulations become equivalent to simulations with two orders of magnitude more bodies. Their wavelet method is expected to produce a comparable improvement in performance for cosmological and plasma simulations.

G. Kaiser, well known for his book on wavelets [16], has extended his very interesting programme of finding 'physical wavelets', i.e. wavelets that are also solutions of physical equations such as the Maxwell equations or the wave equation. Initially these were solutions of source-free equations, but now sources have been included in the treatment as well [17].

There are many more interesting recent examples, but reasons of space unfortunately force me to stop here. I hope I have made it clear that wavelets are continuing to inspire physicists in many disciplines to improve existing methods and to explore new territory as well.

At the beginning of this year the wavelet community witnessed the relaunch, after one year of silence, of its popular electronic news bulletin the Wavelet Digest [18], started by Wim Sweldens in 1992, in a modernized format, with an enlarged readership of about 20,000 people, a sure sign, I think, of the vigour of the wavelet enterprise.

HANS VAN DEN BERG

Wageningen University and Research Centre
April 2003

References

[1] J.-P. Antoine and P. Vandergheynst. Wavelets on the 2-sphere: a group-theoretical approach, *Appl. Comput. Harmon. Anal.*, **7:**262–291, 1999.
[2] J.-P. Antoine, L. Demanet, L. Jacques and P. Vandergheynst. Wavelets on the sphere: implementation and approximations, *Applied Comput. Harmon. Anal.*, **13:**177–200, 2002.
[3] J.-P. Antoine, R. Murenzi, P. Vandergheynst and S.T. Ali. *Two-dimensional wavelets and their relatives*. (Cambridge University Press, forthcoming).
[4] J.-P. Antoine, L. Demanet, J.-F. Hochedez, L. Jacques, R. Terrier and E. Verwichte. Application of the 2-D wavelet transform to astrophysical images, *Physicalia Magazine*, **24:**93–116, 2002.

[5] A. Arnéodo, N. Decoster and S.G. Roux. A wavelet-based method for multifractal image analysis. I. Methodology and test applications on isotropic and anisotropic random rough surfaces. *Eur. Phys. J. B*, **15:**567–600, 2000.
[6] B. Audit, C. Thermes, C. Vaillant, Y. d'Aubenton-Carafa, J.F. Muzy and A. Arnéodo. Long-range correlations in genomic DNA: a Signature of the Nucleosomal Structure. *Phys. Rev. Lett.*, **86**(11):2471–2472, 2001.
[7] M. Farge, G. Pellegrino and K. Scheider. Coherent vortex extraction in 3D turbulent flows using orthogonal wavelets. *Phys. Rev. Lett.*, **87**(5):054501, 2001.
[8] wavelets.ens.fr.
[9] A. Fournier. Atmospheric energetics in the wavelet domain. Part I: Governing equations and interpretation for idealized flows. *J. Atmos. Sci.*, **59:**1182–1197, 2002.
[10] A. Fournier. Atmospheric energetics in the wavelet domain. Part II: Time-averaged observed atmospheric blocking. *J. Atmos. Sci.* **60:**319–338, 2003.
[11] A. Fournier. Instantaneous wavelet energetic transfers between atmopsheric blocking and local eddies. *Journal of Climate*, in press 2003.
[12] A.L. Goldberger, L.A. Amaral, J.M. Hausdorff, P.Ch. Ivanov, C.-K. Peng and H.E. Stanley. Fractal dynamics in physiology: alterations with disease and aging. *Proc. Nat. Ac. Sc.* **99** suppl.1:2466–2472, 2002.
[13] M. Haase and B. Lehle. Tracing the Skeleton of Wavelet Maxima lines for the characterization of fractal distributions. In: *Fractals and beyond* (M.M. Novak, Ed.) (World Scientific, Singapore, 1998), pp. 241–250.
[14] www.csv.ica.uni-stuttgart.de/homes/mh/home.html.
[15] P.Ch. Ivanov, L.A. Nunes Amaral, A.L. Goldberger, S. Havlin, M.G. Rosenblum, H.E. Stanley and Z.R. Struzik. From 1/f noise to Multifractal Cascades in Heartbeat Dynamics. *Chaos* **11:**641–652, 2001.
[16] G. Kaiser. *A Friendly Guide to Wavelets*. (Birkhäuser, Boston, 1994).
[17] G. Kaiser. Physical wavelets and their sources: real physics in complex spacetime. arxiv.org/abs/math-ph/0303027, invited 'Topical review' for *Journal of Physics A: Mathematical and General*, www.iop.org/journals/jphysa.
[18] www.wavelet.org.
[19] P. Kestener and A. Arnéodo. A three-dimensional wavelet based multifractal method: about the need of revisiting the multifractal description of turbulence dissipation data. arxiv.org/abs/cond-mat/0302602.
[20] A. Muñoz, R. Ertlé and M. Unser. Continuous wavelet transform with arbitrary scales and O(N) complexity. *Signal Processing*, **82:**749–757, 2002.
[21] G. Rocha, L. Cayón, R. Bowen, A. Canavezes, J. Silk, A.J. Banday and K.M. Górski. Topology of the universe from *COBE*-DMR; a wavelet approach. arxiv.org/abs/astro-ph/0205155.
[22] A.B. Romeo, C. Horellou and J. Bergh. N-body simulations with two-orders-of-magnitude higher performance using wavelets. arxiv.org/abs/astro-ph/0302343, in press with *Mon. Not. R. Astron. Soc.*
[23] J.L. Starck, D.L. Donoho and E.J. Candès. Astronomical Image Representation by the Curvelet Transform. *Astr. Astroph.*, **398:**785–800, 2003.
[24] www-stat.stanford.eu/~jstarck.
[25] G.W. Wei, M. Zhan and C.-H. Lai. Tailoring wavelets for chaos control. *Phys. Rev. Lett.*, **89**(28):284103, 2002.

Preface to the first edition

Why should physicists bother about wavelets? Why not leave them to the mathematicians and engineers?

Physicists are sometimes reluctant to learn about wavelets because they cannot be interpreted in physical terms as easily as sines and cosines and their frequencies. This is understandable enough: the 'harmonic oscillator' has been with us for more than three centuries, and continues to play its important role. But as we hope to show in the chapters that follow, wavelets can also be of great help in uncovering the presence or absence of certain frequencies in a physical phenomenon. Wavelet analysis is not replacing frequency analysis, but is rather an important refinement and expansion of it: Fourier analysis analyses a signal *globally*, whereas wavelet analysis looks into the signal *locally*.

Let us illustrate this is in musical terms. If you listen to a classical symphony you hear several parts, usually three to four. Each of them has its own main key: e.g. C minor, E♭ major, etc. The Fourier power spectrum of the symphony will of course reveal the dominating keys: groundtones, and their harmonics. Frequencies of other chords which occur more fleetingly during modulations and variations in the piece of music, will also show up. If you would play the parts in a different order, the power spectrum would not change at all, but to the listener it becomes a very different piece, and more so if you interchange parts within the parts, at an ever finer scale: you have changed the musical score drastically. A musical score is a still coarse (the ear catches much more information than the composer writes down in the score) but time-localized frequency analysis of the symphony. This is what a wavelet analysis also supplies you with: it not only gives the main frequencies used, but also, in contrast to the Fourier Transform, indicates *when* they occur, and what

their duration is. In the words of Lau and Went (Ch. 1, ref. 18) wavelets 'make a time series sing'.

To be fair, this was already tried with some success in Fourier analysis also: as explained by Antoine in Chapter 1, in 1946 Gabor introduced the Windowed Fourier Transform, by placing a Gaussian time window with constant width over the signal to be analysed, and shifting the window through the signal. Wavelets, springing up in the early 1980s, generalize this in two respects: there is a large and ever growing family of different wavelet functions, and their time resolution is not fixed, but is variable with the frequency, so that high frequencies have a better time resolution. Moreover, one has been able to construct orthonormal bases for many different types of wavelets. Instead of considering signals $f(t)$ to be composed of everlasting oscillations (Fourier Transform) or oscillations within a fixed time window (Windowed Fourier Transform) one considers the signal as being composed of oscillations which arise and die out in time, more rapidly the higher their frequencies. The Wavelet Transform uses a time window which may be shortened or stretched adaptively, thus giving much more flexibility in representing non-stationary signals. This is why the Wavelet Transform is sometimes called a *mathematical microscope*: it allows you to 'zoom' in and out at any desired magnification (inversely proportional to the scale), at any point of time in the signal. It is precisely this kind of flexibility that makes the Wavelet Transform such a useful and efficient analysis tool. Of course the transform can also be performed in two (image analysis) and more dimensions, and even in space-time.

A further reason to learn about wavelets is that wavelets are fast. How fast? For a one-dimensional signal with n data points the Fourier Transform requires $\sim n^2$ operations. This was reduced by the Fast Fourier Transform to $\sim n \log n$, which after its implementation in software packages, made the application of Fourier analysis an industry in many fields of science and technology. Orthornormal wavelets reduce this even further: here one needs only $\sim cn$ computations where the constant c depends only on the type of wavelet used. As already mentioned, wavelets exist in a variety of shapes and one can pick any particular one to work with according to one's need. This is in marked contrast to Fourier analysis, where everything is always analysed in terms of sines and cosines. The computational efficiency is fine for data compression and transmission, and for numerical calculations, and turns out to produce more accurate and/or faster solutions for partial differential equations occurring in physics, as the reader will see for instance in Chapter 4 and Chapter 8.

Every student learning about the Fourier Transform and the power spectrum should now at least be made aware of some of the possibilities wavelets have to offer. From scientists of various disciplines one still sometimes hears the complaint that the mathematics of wavelets is so much more complicated than Fourier analysis that they don't really want to try. This feeling is caused partly because the first generation of good books about the subject is thoroughly mathematical. But the time has arrived that undergraduate books are appearing to serve those people who have only basic mathematical training. To mention only one here: R. Todd Ogden's little book (see Ch. 1), ref. 26). Moreover journals in many fields have published tutorials that deal with the mathematical basics only. Also there are now quite a number of software toolboxes available which can give the beginner a hands-on feeling for the subject without a deep mathematical understanding. The reader will find more on this material in the last paragraphs of Chapter 1 and the references therein.

The first time I myself met wavelets was in 1991, when I read work by Arneodo, Holschneider and others, about the analysis of (multi)fractal measures arising in certain non-linear dynamical systems. My understanding of it was much stimulated by a wavelet course given at the Amsterdam Center of Mathematics and Computer Science at the end of 1991. The closing lecture was given by Michiel Hazewinkel: 'Wavelets Understand Fractals'. He reported on the work by those scientists, and since that lecture I was hooked onto wavelets. Arneodo and Holschneider both contribute to this volume (Ch. 9 and Ch. 11). One of the other speakers in the course was Tom Koornwinder, who later introduced me further into the theory of wavelets. During that period he came up with the suggestion that I produce a book like the present one, for which I am still grateful to him. I soon became aware of the use of wavelets in other areas of physics, in particular by Farge, in turbulence research, and by Kaiser in electromagnetism (applications in radar) and acoustics. Farge and some of her colleagues contribute Chapter 4 of this work, whereas Kaiser's investigations are published in the second part of his fine textbook on wavelets (Ch. 1, ref. 16).

The material you find in this book does not by any means exhaust the applications of wavelets in physics, but I do hope that the reader finds representative examples of good work in this area, and that it stimulates further exploration and application in the fields covered, and elsewhere. Before the book starts, Chapter 0 gives you a brief 'guided tour' through the chapters.

Acknowledgements

First of all I want to thank all the authors for the generous way in which they dealt with my comments and criticisms, and for the patience they sometimes had to have with my lack of understanding of their work. I have learned enormously from going through this process with them. Apart from many discussions with the authors, I have also had the benefit of feedback from other people who read some of the chapters. In particular I want to thank Rob Zuidwijk and Maria Haase for their remarks and suggestions for improvements. Aimé Fournier not only contributed his chapter but offered a number of very welcome suggestions along the way, and some were also supplied by my dear and always critical friend Taco Visser. Many thanks also to Albert Otten at our department, for indefatigably helping me out with my file handling and other computer problems. Without his help I could not have done this job. During the final stages of preparation I was offered the hospitality of the Max-Planck-Institute for the Physics of Complex Systems in Dresden, for which I am very grateful. Continuous encouragement and support I received all the time from Simon Capelin, Publishing Director for the Physical Sciences at Cambridge University Press. Simon, it was a pleasure to work with you and your staff.

August 1998 HANS VAN DEN BERG
Wageningen Agricultural University