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Chapter 1

Introduction

1.1 Synchronization in historical perspective

The Dutch researcher Christiaan Huygens (Fig. 1.1), most famous for his studies in optics and the construction of telescopes and clocks, was probably the first scientist who observed and described the synchronization phenomenon as early as in the seventeenth century. He discovered that a couple of pendulum clocks hanging from a common support had synchronized, i.e., their oscillations coincided perfectly and the pendula moved always in opposite directions. This discovery was made during a sea trial of clocks intended for the determination of longitude. In fact, the invention and design of pendulum clocks was one of Huygens' most important achievements. It made a great impact on the technological and scientific developments of that time and increased the accuracy of time measurements enormously. In 1658, only two years after Huygens obtained a Dutch Patent for his invention, a clock-maker from Utrecht, Samuel Coster, built a church pendulum clock and guaranteed its weekly deviation to be less than eight minutes.

After this invention, Huygens continued his efforts to increase the precision and stability of such clocks. He paid special attention to the construction of clocks suitable for use on ships in the open sea. In his memoirs *Horologium Oscillatorium* (*The Pendulum Clock, or Geometrical Demonstrations Concerning the Motion of Pendula as Applied to Clocks*), where he summarized his theoretical and experimental achievements, Huygens [1673] gave a detailed description of such clocks.

> In these clocks the length of the pendulum was nine inches and its weight one-half pound. The wheels were rotated by the force of weights and were enclosed together with the weights in a case which was four feet long. At the

> bottom of the case was added a lead weight of over one hundred pounds so that the instrument would better maintain a perpendicular orientation when suspended in the ship.
>
> Although the motion of the clock was found to be very equal and constant in these experiments, nevertheless we made an effort to perfect it still further in another way as follows. . . . the result is still greater equality of clocks than before.

Furthermore, Huygens shortly, but extremely precisely, described his observation of synchronization as follows.

> . . . It is quite worth noting that when we suspended two clocks so constructed from two hooks imbedded in the same wooden beam, the motions of each

Figure 1.1. Christiaan Huygens (1629–1695), the famous Dutch mathematician, astronomer and physicist. Among his main achievements are the discovery of the first moon and the true shape of the rings of Saturn; the first printed work on the calculus of probabilities; the investigation of properties of curves; the formulation of a wave theory of light including what is well-known nowadays as the Huygens principle. In 1656 Christiaan Huygens patented the first pendulum clock, which greatly increased the accuracy of time measurement and helped him to tackle the longitude problem. During a sea trial, he observed synchronization of two such clocks (see also the introduction to the English translation of his book [Huygens 1673] for a historical survey). Photo credit: Rijksmuseum voor de Geschidenis der Natuuringtenschappen, courtesy American Institute of Physics Emilio Segrè Visual Archives.

> pendulum in opposite swings were so much in agreement that they never receded the least bit from each other and the sound of each was always heard simultaneously. Further, if this agreement was disturbed by some interference, it reestablished itself in a short time. For a long time I was amazed at this unexpected result, but after a careful examination finally found that the cause of this is due to the motion of the beam, even though this is hardly perceptible. The cause is that the oscillations of the pendula, in proportion to their weight, communicate some motion to the clocks. This motion, impressed onto the beam, necessarily has the effect of making the pendula come to a state of exactly contrary swings if it happened that they moved otherwise at first, and from this finally the motion of the beam completely ceases. But this cause is not sufficiently powerful unless the opposite motions of the clocks are exactly equal and uniform.

The first mention of this discovery can be found in Huygens' letter to his father of 26 February 1665, reprinted in a collection of papers [Huygens 1967a] and reproduced in Appendix A1. According to this letter, the observation of synchronization was made while Huygens was sick and stayed in bed for a couple of days watching two clocks hanging on a wall (Fig. 1.2). Interestingly, in describing the discovered phenomenon, Huygens wrote about "*sympathy of two clocks*" (*le phénoméne de la sympathie, sympathie des horloges*).

Thus, Huygens had given not only an exact description, but also a brilliant qualitative explanation of this effect of **mutual synchronization**; he correctly understood that the conformity of the rhythms of two clocks had been caused by an imperceptible motion of the beam. In modern terminology this would mean that the clocks were synchronized in anti-phase due to **coupling** through the beam.

In the middle of the nineteenth century, in his famous treatise *The Theory of Sound*, William Strutt (Fig. 1.3) [Lord Rayleigh 1945] described the interesting phenomenon of synchronization in acoustical systems as follows.

> When two organ-pipes of the same pitch stand side by side, complications ensue which not unfrequently give trouble in practice. In extreme cases the pipes may

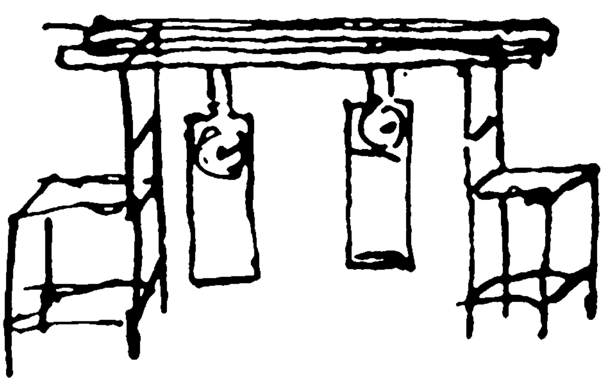

Figure 1.2. Original drawing of Christiaan Huygens illustrating his experiments with two pendulum clocks placed on a common support.

> almost reduce one another to silence. Even when the mutual influence is more moderate, it may still go so far as to cause the pipes to speak in absolute unison, in spite of inevitable small differences.

Thus, Rayleigh observed not only mutual synchronization when two distinct but similar pipes begin to sound in unison, but also the effect of **quenching** (**oscillation death**) when the coupling results in suppression of oscillations of interacting systems.

A new stage in the investigation of synchronization was related to the development of electrical and radio engineering. On 17 February 1920 W. H. Eccles and J. H. Vincent applied for a British Patent confirming their discovery of the synchronization property of a triode generator – a rather simple electrical device based on a vacuum tube that produces a periodically alternating electrical current [Eccles and Vincent 1920]. The frequency of this current oscillation is determined by the parameters of the elements of the scheme, e.g., of the capacitance. In their

Figure 1.3. Sir John William Strutt, Lord Rayleigh (1842–1919). He studied at Trinity College, Cambridge University, graduating in 1864. His first paper in 1865 was on Maxwell's electromagnetic theory. He worked on the propagation of sound and, while on an excursion to Egypt taken for health reasons, Strutt wrote *Treatise on Sound* (1870–1871). In 1879 he wrote a paper on traveling waves, this theory has now developed into the theory of solitons. His theory of scattering (1871) was the first correct explanation of the blue color of the sky. In 1873 he succeeded to the title of Baron Rayleigh. From 1879 to 1884 he was the second Cavendish professor of experimental physics at Cambridge, succeeding Maxwell. Then in 1884 he became the secretary of the Royal Society. Rayleigh discovered the inert gas argon in 1895, the work which earned him a Nobel Prize in 1904. Photo credit: Photo Gen. Stab. Lit. Anst., courtesy AIP Emilio Segrè Visual Archives.

experiments, Eccles and Vincent coupled two generators which had slightly different frequencies and demonstrated that the coupling forced the systems to vibrate with a common frequency.

A few years later Edward Appleton (Fig. 1.4) and Balthasar van der Pol (Fig. 1.5) replicated and extended the experiments of Eccles and Vincent and made the first step in the theoretical study of this effect [Appleton 1922; van der Pol 1927]. Considering the simplest case, they showed that the frequency of a generator can be entrained, or synchronized, by a weak external signal of a slightly different frequency. These studies were of great practical importance because triode generators became the basic elements of radio communication systems. The synchronization phenomenon was used to stabilize the frequency of a powerful generator with the help of one which was weak but very precise.

Synchronization in living systems has also been known for centuries. In 1729 Jean-Jacques Dortous de Mairan, the French astronomer and mathematician, who

Figure 1.4. Sir Edward Victor Appleton (1892–1965). Educated at Cambridge University, he began research at the Cavendish Laboratory with W. L. Bragg. During the First World War he developed an interest in valves and "wireless" signals, which inspired his subsequent research career. He returned to the Cavendish Laboratory in 1919, continuing to work on valves and, with B. van der Pol, on nonlinearity, and on atmospherics. In 1924, in collaboration with M. F. Barnett, he performed a crucial experiment which enabled a reflecting layer in the atmosphere to be identified and measured. In 1936 he succeeded C. T. R. Wilson in the Jacksonian Chair of Natural Philosophy at Cambridge, where he continued collaborative research on many ionospheric problems. He was awarded the Nobel Prize for Physics in 1947 for his investigations of the ionosphere. Photo credit: AIP Emilio Segrè Visual Archives, E. Scott Barr Collection.

was later the Secretary of the Académie Royale des Sciences in Paris, reported on his experiments with a haricot bean. He noticed that the leaves of this plant moved up and down in accordance with the change of day into night. Having made this observation, de Mairan put the plant in a dark room and found that the motion of the leaves continued even without variations in the illuminance of the environment. Since that time these and much more complicated experiments have been replicated in different laboratories, and now it is well-known that all biological systems, from

Figure 1.5. Balthasar van der Pol (1889–1959). He studied physics and mathematics in Utrecht and then went to England, where he spent several years working at the Cavendish Laboratory in Cambridge. There he met E. Appleton, and they started to work together in radio science. In 1919 van der Pol returned to Holland and in 1920 he was awarded a doctorate from Utrecht University. In 1922 he accepted an offer from the Philips Company and began work at Philips Research Laboratories in Eindhoven; soon he became Director of Fundamental Radio Research. Van der Pol acquired his international reputation due to his pioneering work on the propagation of radio waves and nonlinear oscillations. His studies of oscillation in a triode circuit led to the derivation of the van der Pol equation, a paradigmatic model of oscillation theory and nonlinear dynamics (see Eq. (7.2)). Together with van der Mark he pioneered the application of oscillation theory to physiological systems. Their work on modeling and hardware simulation of the human heart by three coupled relaxation oscillators [van der Pol and van der Mark 1928] remains a masterpiece of biological physics. Photo credit: Philips International B.V., Company Archives, Eindhoven, The Netherlands (see Bremmer [1960/61] for details).

rather simple to highly organized ones, have internal biological clocks that provide their "owners" with information on the change between day and night. The origin of these clocks is still a challenging problem, but it is well established that they can adjust their circadian rhythms (from *circa* = about and *dies* = day) to external signals: if the system is completely isolated from the environment and is kept under controlled constant conditions (constant illuminance, temperature, pressure, parameters of electromagnetic fields, etc.), its internal cycle can essentially differ from a 24-hour cycle. Under natural conditions, biological clocks tune their rhythms in accordance with the 24-hour period of the Earth's daily cycle.

As the last historical example, we cite another Dutchman, the physician Engelbert Kaempfer [1727][1] who, after his voyage to Siam in 1680 wrote:

> The glowworms . . . represent another shew, which settle on some Trees, like a fiery cloud, with this surprising circumstance, that a whole swarm of these insects, having taken possession of one Tree, and spread themselves over its branches, sometimes hide their Light all at once, and a moment after make it appear again with the utmost regularity and exactness . . . .

To our knowledge, this is the first reported observation of synchronization in a large population of oscillating systems.

We end our historical excursus in the 1920s. Since then many interesting synchronization phenomena have been observed and reported in the literature; some of them are described in the following chapters. More importantly, it gradually became clear that diverse effects which at first sight have nothing in common, obey some universal laws. A great deal of research carried out by mathematicians, engineers, physicists, and scientists from other fields, has led to the development of an understanding that, say, the conformity of the sounds of organ-pipes or the songs of the snowy tree cricket is not occasional, but can be described by a unified theory. In the following chapters we intend to demonstrate that these and a variety of other seemingly different effects have common characteristic features and can be understood within a unified framework.

1.2 Synchronization: just a description

We have shown with a few introductory examples (and we will illustrate it with further examples below) that synchronization is encountered in various fields of science, in engineering and in social behavior. We do not intend to give any rigorous definition of this phenomenon now. Before we discuss this notion in detail, although without mathematical methods, in Part I, and before we present the theoretical description in Parts II and III, we just give here a simple qualitative description of the effect; this section can be skipped by readers with a basic knowledge of physics

[1] Citation taken from [Buck and Buck 1968].

and nonlinear dynamics. Using several characteristic examples, we explain what synchronization is, and outline the common properties of systems that allow this effect to occur. However, the answer to the question "Why does it take place?" is left to Chapter 2.

1.2.1 What is synchronization?

We understand synchronization as an **adjustment of rhythms of oscillating objects due to their weak interaction**. Except for rare cases when it is said explicitly otherwise, this concept is used throughout the book. To explain this concept in qualitative terms we will concentrate on the following four questions.

- What is an oscillating object?
- What do we understand by the notion "rhythm"?
- What is an interaction of oscillating systems?
- What is an adjustment of rhythms?

To illustrate this general definition we take the classical example – a pendulum clock.

Self-sustained oscillator: a model of natural oscillating objects

Let us discuss how a clock works. Its mechanism transforms the potential energy of the lifted weight (or compressed spring, or electrical battery) into the oscillatory motion of the pendulum. In its turn, this oscillation is transferred into the rotation of the hands on the clock's face (Fig. 1.6a). We are not interested in the particular design of the mechanism; what is important is only that it takes energy from the source and maintains a steady oscillation of the pendulum, which continues without any change until the supply of energy expires. The next important property is that the exact form of the oscillatory motion is entirely determined by the internal parameters of the clock and does not depend on how the pendulum was put into motion. Moreover, after being slightly perturbed, following some transient process the pendulum restores its previous internal rhythm.

These features are typical not only of clocks, but also of many oscillating objects of diverse nature. The set of these features constitutes the answer to the first of the questions above. In physics such oscillatory objects are denoted **self-sustained oscillators**; below we discuss their properties in detail. Further on we often omit the word "self-sustained", but by default we describe only systems of this class. Here we briefly summarize the properties of self-sustained oscillatory systems.

- This oscillator is an **active system**. It contains an internal **source of energy** that is transformed into oscillatory movement. Being isolated, the oscillator continues to generate the same rhythm until the source of energy expires.

Mathematically, it is described by an **autonomous** (i.e., without explicit time dependence) dynamical system.

- The form of oscillation is determined by the parameters of the system and does not depend on how the system was "switched on", i.e., on the transient to steady oscillation.
- The oscillation is stable to (at least rather small) perturbations: being disturbed, the oscillation soon returns to its original shape.

Examples of self-sustained oscillatory systems are electronic circuits used for the generation of radio-frequency power, lasers, Belousov–Zhabotinsky and other oscillatory chemical reactions, pacemakers (sino-atrial nodes) of human hearts or artificial pacemakers that are used in cardiac pathologies, and many other natural and artificial systems. As we will see later, an outstanding common feature of such systems is their ability to be synchronized.

Characterization of a rhythm: period and frequency

Self-sustained oscillators can exhibit rhythms of various shapes, from simple sine-like waveforms to a sequence of short pulses. Now we quantify such rhythms using our particular example – the pendulum clock. The oscillation of the pendulum is periodic (Fig. 1.6b), and the **period** T is the main characteristic of the clock. Indeed, the mechanism that rotates the hands actually counts the number of pendulum oscillations, so that its period constitutes the base time unit.

Often it is convenient to characterize the rhythm by the number of oscillation cycles per time unit, or by the oscillation **cyclic frequency**

$$f = \frac{1}{T}.$$

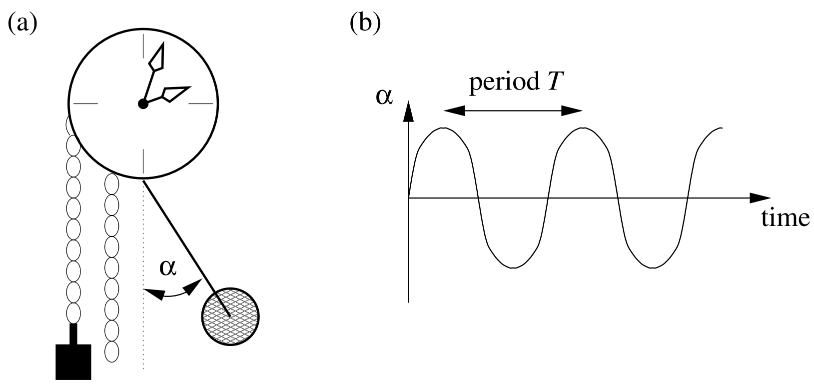

Figure 1.6. (a) An example of a self-sustained oscillator, the pendulum clock. The potential energy of the lifted weight is transformed into oscillatory motion of the pendulum and eventually in the rotation of hands. (b) The motion of the pendulum is periodic, i.e., its angle α with respect to the vertical varies in time with the period T.

In the theoretical treatment of synchronization, the **angular frequency** $\omega = 2\pi f = 2\pi/T$ is often more convenient; below we often omit the word "angular" and call it simply the frequency. Later on we will see that the frequency can be changed because of the external action on the oscillator, or due to its interaction with another system. To avoid ambiguity, we call the frequency of the autonomous (isolated) system the *natural frequency*.

Coupling of oscillating objects

Now suppose that we have not one clock, but two. Even if they are of the same type or are made by the same fabricator, the clocks seem to be identical, but they are not. Some fine mechanical parameters always differ, probably by a tenth of a per cent, but this tiny difference causes a discrepancy in the oscillatory periods. Therefore, these two clocks show a slightly different time, and if we look at them at some instant of time, then typically we find the pendula in different positions (Fig. 1.7).

Let us now assume that these two nonidentical clocks are not independent, but interact weakly. There might be different forms of interaction, or **coupling**, between these two oscillators. Suppose that the two clocks are fixed on a common support, and let this be a not absolutely rigid beam (Fig. 1.8), as it was in the original observation of Huygens. This beam can bend, or it may vibrate slightly, moving from left to right, this does not matter much. What is really important is only that the motion of each pendulum is transmitted through the supporting structure to the other pendulum and, as a result, both clocks "feel" each other: they interact through the vibration of the common support. This vibration might be practically imperceptible; in order to detect and visualize it one has to perform high-precision mechanical measurements. However, in spite of its weakness, it may alter the rhythms of both clocks!

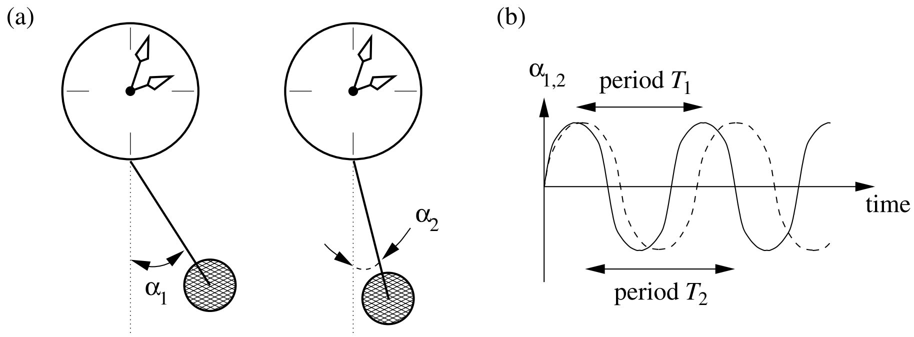

Figure 1.7. Two similar pendulum clocks (a) cannot be perfectly identical; due to a tiny parameter mismatch they have slightly different periods (here $T_2 > T_1$) (b). Therefore, if we look at them at some arbitrary moment of time, the pendula are, generally speaking, in different positions: $\alpha_1 \neq \alpha_2$.