

QUANTUM FIELD THEORY AND CONDENSED MATTER

Providing a broad review of many techniques and their application to condensed matter systems, this book begins with a review of thermodynamics and statistical mechanics, before moving on to real- and imaginary-time path integrals and the link between Euclidean quantum mechanics and statistical mechanics. A detailed study of the Ising, gauge-Ising and XY models is included. The renormalization group is developed and applied to critical phenomena, Fermi liquid theory, and the renormalization of field theories. Next, the book explores bosonization and its applications to one-dimensional fermionic systems and the correlation functions of homogeneous and random-bond Ising models. It concludes with the Bohm–Pines and Chern–Simons theories applied to the quantum Hall effect. Introducing the reader to a variety of techniques, it opens up vast areas of condensed matter theory for both graduate students and researchers in theoretical, statistical, and condensed matter physics.

R. SHANKAR is the John Randolph Huffman Professor of Physics at Yale University, with a research focus on theoretical condensed matter physics. He has held positions at the Aspen Center for Physics, the American Physical Society, and the American Academy of Arts and Sciences. He has also been a visiting professor at several universities including MIT, Princeton, UC Berkeley, and IIT Madras. Recipient of both the Harwood Byrnes and Richard Sewell teaching prizes at Yale (2005) and the Julius Edgar Lilienfeld prize of the American Physical Society (2009), he has also authored several books: *Principles of Quantum Mechanics*, *Basic Training in Mathematics*, and *Fundamentals of Physics I and II*.

QUANTUM FIELD THEORY AND CONDENSED MATTER

An Introduction

R. SHANKAR

Yale University, Connecticut



CAMBRIDGE
UNIVERSITY PRESS

CAMBRIDGE
UNIVERSITY PRESS

University Printing House, Cambridge CB2 8BS, United Kingdom
One Liberty Plaza, 20th Floor, New York, NY 10006, USA
477 Williamstown Road, Port Melbourne, VIC 3207, Australia
4843/24, 2nd Floor, Ansari Road, Daryaganj, Delhi – 110002, India
79 Anson Road, #06-04/06, Singapore 079906

Cambridge University Press is part of the University of Cambridge.

It furthers the University's mission by disseminating knowledge in the pursuit of education, learning, and research at the highest international levels of excellence.

www.cambridge.org

Information on this title: www.cambridge.org/9780521592109

DOI: 10.1017/9781139044349

© R. Shankar 2017

This publication is in copyright. Subject to statutory exception and to the provisions of relevant collective licensing agreements, no reproduction of any part may take place without the written permission of Cambridge University Press.

First published 2017

Printed in the United Kingdom by Clays, St Ives plc

A catalogue record for this publication is available from the British Library.

ISBN 978-0-521-59210-9 Hardback

Cambridge University Press has no responsibility for the persistence or accuracy of URLs for external or third-party internet websites referred to in this publication and does not guarantee that any content on such websites is, or will remain, accurate or appropriate.

Cambridge University Press
978-0-521-59210-9 — Quantum Field Theory and Condensed Matter
Ramamurti Shankar
Frontmatter
[More Information](#)

Dedicated to
Michael Fisher, Leo Kadanoff, Ben Widom, and Ken Wilson
Architects of the modern RG

Contents

<i>Preface</i>	<i>page</i> xiii
1 Thermodynamics and Statistical Mechanics Review	1
1.1 Energy and Entropy in Thermodynamics	1
1.2 Equilibrium as Maximum of Entropy	3
1.3 Free Energy in Thermodynamics	4
1.4 Equilibrium as Minimum of Free Energy	5
1.5 The Microcanonical Distribution	6
1.6 Gibbs's Approach: The Canonical Distribution	10
1.7 More on the Free Energy in Statistical Mechanics	13
1.8 The Grand Canonical Distribution	18
References and Further Reading	19
2 The Ising Model in $d = 0$ and $d = 1$	20
2.1 The Ising Model in $d = 0$	20
2.2 The Ising Model in $d = 1$	23
2.3 The Monte Carlo Method	27
3 Statistical to Quantum Mechanics	29
3.1 Real-Time Quantum Mechanics	30
3.2 Imaginary-Time Quantum Mechanics	32
3.3 The Transfer Matrix	34
3.4 Classical to Quantum Mapping: The Dictionary	43
References and Further Reading	44
4 Quantum to Statistical Mechanics	45
4.1 From U to Z	45
4.2 A Detailed Example from Spin $\frac{1}{2}$	46
4.3 The τ -Continuum Limit of Fradkin and Susskind	49
4.4 Two $N \rightarrow \infty$ Limits and Two Temperatures	51
References and Further Reading	51

5	The Feynman Path Integral	52
5.1	The Feynman Path Integral in Real Time	52
5.2	The Feynman Phase Space Path Integral	58
5.3	The Feynman Path Integral for Imaginary Time	59
5.4	Classical \leftrightarrow Quantum Connection Redux	60
5.5	Tunneling by Euclidean Path Integrals	61
5.6	Spontaneous Symmetry Breaking	66
5.7	The Classical Limit of Quantum Statistical Mechanics	70
	References and Further Reading	71
6	Coherent State Path Integrals for Spins, Bosons, and Fermions	72
6.1	Spin Coherent State Path Integral	72
6.2	Real-Time Path Integral for Spin	74
6.3	Bosonic Coherent States	76
6.4	The Fermion Problem	78
6.5	Fermionic Oscillator: Spectrum and Thermodynamics	78
6.6	Coherent States for Fermions	80
6.7	Integration over Grassmann Numbers	82
6.8	Resolution of the Identity and Trace	85
6.9	Thermodynamics of a Fermi Oscillator	86
6.10	Fermionic Path Integral	86
6.11	Generating Functions $Z(J)$ and $W(J)$	94
	References and Further Reading	104
7	The Two-Dimensional Ising Model	105
7.1	Ode to the Model	105
7.2	High-Temperature Expansion	107
7.3	Low-Temperature Expansion	108
7.4	Kramer–Wannier Duality	110
7.5	Correlation Function in the tanh Expansion	112
	References and Further Reading	113
8	Exact Solution of the Two-Dimensional Ising Model	114
8.1	The Transfer Matrix in Terms of Pauli Matrices	114
8.2	The Jordan–Wigner Transformation and Majorana Fermions	115
8.3	Boundary Conditions	118
8.4	Solution by Fourier Transform	120
8.5	Qualitative Analysis in the τ -Continuum Limit	123
8.6	The Eigenvalue Problem of T in the τ -Continuum Limit	125
8.7	Free Energy in the Thermodynamic Limit	134
8.8	Lattice Gas Model	135
8.9	Critical Properties of the Ising Model	136
8.10	Duality in Operator Language	140
	References and Further Reading	142

Contents

ix

9	Majorana Fermions	143
9.1	Continuum Theory of the Majorana Fermion	143
9.2	Path Integrals for Majorana Fermions	147
9.3	Evaluation of Majorana Grassmann Integrals	149
9.4	Path Integral for the Continuum Majorana Theory	152
9.5	The Pfaffian in Superconductivity	155
	References and Further Reading	156
10	Gauge Theories	157
10.1	The XY Model	157
10.2	The Z_2 Gauge Theory in $d = 2$	163
10.3	The Z_2 Theory in $d = 3$	169
10.4	Matter Fields	173
10.5	Fradkin–Shenker Analysis	176
	References and Further Reading	182
11	The Renormalization Group	183
11.1	The Renormalization Group: First Pass	183
11.2	Renormalization Group by Decimation	186
11.3	Stable and Unstable Fixed Points	192
11.4	A Review of Wilson’s Strategy	194
	References and Further Reading	198
12	Critical Phenomena: The Puzzle and Resolution	199
12.1	Landau Theory	201
12.2	Widom Scaling	207
12.3	Kadanoff’s Block Spins	210
12.4	Wilson’s RG Program	214
12.5	The β -Function	220
	References and Further Reading	223
13	Renormalization Group for the ϕ^4 Model	224
13.1	Gaussian Fixed Point	224
13.2	Gaussian Model Exponents for $d > 4$, $\varepsilon = 4 - d = - \varepsilon $	237
13.3	Wilson–Fisher Fixed Point $d < 4$	242
13.4	Renormalization Group at $d = 4$	248
	References and Further Reading	250
14	Two Views of Renormalization	251
14.1	Review of RG in Critical Phenomena	251
14.2	The Problem of Quantum Field Theory	252
14.3	Perturbation Series in λ_0 : Mass Divergence	253
14.4	Scattering Amplitude and the Γ ’s	254
14.5	Perturbative Renormalization	259
14.6	Wavefunction Renormalization	260

x	<i>Contents</i>	
	14.7 Wilson's Approach to Renormalizing QFT	264
	14.8 Theory with Two Parameters	271
	14.9 The Callan–Symanzik Equation	273
	References and Further Reading	283
15	Renormalization Group for Non-Relativistic Fermions: I	285
	15.1 A Fermion Problem in $d = 1$	289
	15.2 Mean-Field Theory $d = 1$	291
	15.3 The RG Approach for $d = 1$ Spinless Fermions	294
	References and Further Reading	303
16	Renormalization Group for Non-Relativistic Fermions: II	305
	16.1 Fermions in $d = 2$	305
	16.2 Tree-Level Analysis of u	309
	16.3 One-Loop Analysis of u	311
	16.4 Variations and Extensions	315
	References and Further Reading	317
17	Bosonization I: The Fermion–Boson Dictionary	319
	17.1 Preamble	319
	17.2 Massless Dirac Fermion	321
	17.3 Free Massless Scalar Field	324
	17.4 Bosonization Dictionary	328
	17.5 Relativistic Bosonization for the Lagrangians	332
	References and Further Reading	333
18	Bosonization II: Selected Applications	334
	18.1 Massless Schwinger and Thirring Models	334
	18.2 Ising Correlations at Criticality	336
	18.3 Random-Bond Ising Model	340
	18.4 Non-Relativistic Lattice Fermions in $d = 1$	346
	18.5 Kosterlitz–Thouless Flow	357
	18.6 Analysis of the KT Flow Diagram	360
	18.7 The XXZ Spin Chain	363
	18.8 Hubbard Model	364
	18.9 Conclusions	367
	References and Further Reading	367
19	Duality and Triality	370
	19.1 Duality in the $d = 2$ Ising Model	370
	19.2 Thirring Model and Sine-Gordon Duality	373
	19.3 Self-Triality of the $SO(8)$ Gross–Neveu Model	376
	19.4 A Bonus Result: The $SO(4)$ Theory	382
	References and Further Reading	383

<i>Contents</i>		xi
20	Techniques for the Quantum Hall Effect	384
20.1	The Bohm–Pines Theory of Plasmons: The Goal	384
20.2	Bohm–Pines Treatment of the Coulomb Interaction	386
20.3	Fractional Quantum Hall Effect	391
20.4	FQHE: Statement of the Problem	392
20.5	What Makes FQHE So Hard?	394
20.6	Integer Filling: IQHE	398
20.7	Hall Conductivity of a Filled Landau Level: Clean Case	400
20.8	FQHE: Laughlin’s Breakthrough	408
20.9	Flux Attachment	410
20.10	Second Quantization	414
20.11	Wavefunctions for Ground States: Jain States	417
20.12	Hamiltonian Theory: A Quick Summary	420
20.13	The All- q Hamiltonian Theory	422
20.14	Putting the Hamiltonian Theory to Work	426
20.15	Chern–Simons Theory	429
20.16	The End	432
	References and Further Reading	433
	<i>Index</i>	435

Preface

Condensed matter theory is a massive field to which no book or books can do full justice. Every chapter in this book is possible material for a book or books. So it is clearly neither my intention nor within my capabilities to give an overview of the entire subject. Instead I will focus on certain techniques that have served me well over the years and whose strengths and limitations I am familiar with.

My presentation is at a level of rigor I am accustomed to and at ease with. In any topic, say the renormalization group (RG) or bosonization, there are treatments that are more rigorous. How I deal with this depends on the topic. For example, in the RG I usually stop at one loop, which suffices to make the point, with exceptions like wave function renormalization where you need a minimum of two loops. For non-relativistic fermions I am not aware of anything new one gets by going to higher loops. I do not see much point in a scheme that is exact to all orders (just like the original problem) if in practice no real gain is made after one loop. In the case of bosonization I work in infinite volume from the beginning and pay scant attention to the behavior at infinity. I show many examples where this is adequate, but point to cases where it is not and suggest references. In any event I think the student should get acquainted with these more rigorous treatments after getting the hang of it from the treatment in this book. I make one exception in the case of the two-dimensional Ising model where I pay considerable attention to boundary conditions, without which one cannot properly understand how symmetry breaking occurs only in the thermodynamic limit.

This book has been a few years in the writing and as a result some of the topics may seem old-fashioned; on the other hand, they have stood the test of time.

Ideally the chapters should be read in sequence, but if that is not possible, the reader may have to go back to earlier chapters when encountering an unfamiliar notion.

I am grateful to the Aspen Center for Physics (funded by NSF Grant 1066293) and the Indian Institute of Technology, Madras for providing the facilities to write parts of this book.

Over the years I have drawn freely on the wisdom of my collaborators and friends in acquiring the techniques described here. I am particularly grateful to my long-standing collaborator Ganpathy Murthy for countless discussions over the years.

Of all the topics covered here, my favorite is the renormalization group. I have had the privilege of interacting with its founders: Michael Fisher, Leo Kadanoff, Ben Widom, and Ken Wilson. I gratefully acknowledge the pleasure their work has given me while learning it, using it, and teaching it.

In addition, Michael Fisher has been a long-time friend and role model – from his exemplary citizenship and his quest for accuracy in thought and speech, right down to the imperative to punctuate all equations. I will always remain grateful for his role in ensuring my safe passage from particle theory to condensed-matter theory.

This is also a good occasion to acknowledge the 40+ happy years I have spent at Yale, where one can still sense the legacy of its twin giants: Josiah Willard Gibbs and Lars Onsager. Their lives and work inspire all who become aware of them.

To the team at Cambridge University Press, my hearty thanks: Editor Simon Capelin for his endless patience with me and faith in this book, Roisin Munnely and Helen Flitton for document handling, and my peerless manuscript editor Richard Hutchinson for a most careful reading of the manuscript and correction of errors in style, syntax, punctuation, referencing, and, occasionally, even equations! Three cheers for university presses, which exemplify what textbook publication is all about.

Finally I thank the three generations of my family for their love and support.