QUANTUM FIELD THEORY AND CONDENSED MATTER

Providing a broad review of many techniques and their application to condensed matter systems, this book begins with a review of thermodynamics and statistical mechanics, before moving on to real- and imaginary-time path integrals and the link between Euclidean quantum mechanics and statistical mechanics. A detailed study of the Ising, gauge-Ising and *XY* models is included. The renormalization group is developed and applied to critical phenomena, Fermi liquid theory, and the renormalization of field theories. Next, the book explores bosonization and its applications to one-dimensional fermionic systems and the correlation functions of homogeneous and random-bond Ising models. It concludes with the Bohm–Pines and Chern–Simons theories applied to the quantum Hall effect. Introducing the reader to a variety of techniques, it opens up vast areas of condensed matter theory for both graduate students and researchers in theoretical, statistical, and condensed matter physics.

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An Introduction

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Dedicated to
Michael Fisher, Leo Kadanoff, Ben Widom, and Ken Wilson
Architects of the modern RG

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Preface

Condensed matter theory is a massive field to which no book or books can do full justice. Every chapter in this book is possible material for a book or books. So it is clearly neither my intention nor within my capabilities to give an overview of the entire subject. Instead I will focus on certain techniques that have served me well over the years and whose strengths and limitations I am familiar with.

My presentation is at a level of rigor I am accustomed to and at ease with. In any topic, say the renormalization group (RG) or bosonization, there are treatments that are more rigorous. How I deal with this depends on the topic. For example, in the RG I usually stop at one loop, which suffices to make the point, with exceptions like wave function renormalization where you need a minimum of two loops. For non-relativistic fermions I am not aware of anything new one gets by going to higher loops. I do not see much point in a scheme that is exact to all orders (just like the original problem) if in practice no real gain is made after one loop. In the case of bosonization I work in infinite volume from the beginning and pay scant attention to the behavior at infinity. I show many examples where this is adequate, but point to cases where it is not and suggest references. In any event I think the student should get acquainted with these more rigorous treatments after getting the hang of it from the treatment in this book. I make one exception in the case of the two-dimensional Ising model where I pay considerable attention to boundary conditions, without which one cannot properly understand how symmetry breaking occurs only in the thermodynamic limit.

This book has been a few years in the writing and as a result some of the topics may seem old-fashioned; on the other hand, they have stood the test of time.

Ideally the chapters should be read in sequence, but if that is not possible, the reader may have to go back to earlier chapters when encountering an unfamiliar notion.

I am grateful to the Aspen Center for Physics (funded by NSF Grant 1066293) and the Indian Institute of Technology, Madras for providing the facilities to write parts of this book.

Over the years I have drawn freely on the wisdom of my collaborators and friends in acquiring the techniques described here. I am particularly grateful to my long-standing collaborator Ganpathy Murthy for countless discussions over the years.

Of all the topics covered here, my favorite is the renormalization group. I have had the privilege of interacting with its founders: Michael Fisher, Leo Kadanoff, Ben Widom, and Ken Wilson. I gratefully acknowledge the pleasure their work has given me while learning it, using it, and teaching it.

In addition, Michael Fisher has been a long-time friend and role model – from his exemplary citizenship and his quest for accuracy in thought and speech, right down to the imperative to punctuate all equations. I will always remain grateful for his role in ensuring my safe passage from particle theory to condensed-matter theory.

This is also a good occasion to acknowledge the 40+ happy years I have spent at Yale, where one can still sense the legacy of its twin giants: Josiah Willard Gibbs and Lars Onsager. Their lives and work inspire all who become aware of them.

To the team at Cambridge University Press, my hearty thanks: Editor Simon Capelin for his endless patience with me and faith in this book, Roisin Munnelly and Helen Flitton for document handling, and my peerless manuscript editor Richard Hutchinson for a most careful reading of the manuscript and correction of errors in style, syntax, punctuation, referencing, and, occasionally, even equations! Three cheers for university presses, which exemplify what textbook publication is all about.

Finally I thank the three generations of my family for their love and support.