GENERAL INDEX

[The numbers refer to the pages. References to theorems contained in a few of the more important examples are given by numbers in italics]

Abel’s discovery of elliptic functions, 429, 512; inequality, 16; integral equation, 211, 229, 230; method of establishing addition theorems, 442, 496, 497, 530, 534; special form, $\phi_n(z)$, of the confluent hypergeometric function, 353; test for convergence, 17; theorem on continuity of power series, 57; theorem on multiplication of convergent series, 58, 59

Abridged notation for products of Theta-functions, 468, 469; for quotients and reciprocals of elliptic functions, 494, 498

Absolute convergence, 18, 28; Cauchy’s test for, 21; D’Alembert’s ratio test for, 22; De Morgan’s test for, 23

Absolute value, see Modulus

Absolutely convergent double series, 28; infinite products, 32; series, 18, (fundamental property of) 25, (multiple of) 29

Addition formula for Bessel functions, 357, 380; for Gegenbauer’s function, 335; for Legendre polynomials, 326, 395; for Legendre functions, 328; for the Sigma-function, 451; for Theta-functions, 467; for the Jacobian Zeta-function and for $E(u)$, 518, 534; for the third kind of elliptic integral, 533; for the Weierstrassian Zeta-function, 446

Addition formulas, distinguished from addition theorems, 519

Addition theorem for circular functions, 555; for the exponential function, 531; for Jacobian elliptic functions, 494, 497, 530; for the Weierstrassian elliptic function, 440, 457; proofs of, by Abel’s method, 442, 496, 497, 530, 534

Affix, 9

Air in a sphere, vibrations of, 399

Amplitude, 9

Analytic continuation, 95, (not always possible) 98; and Borel’s integral, 141; of the hypergeometric function, 288. See also Asymptotic expansions

Analytic functions, 92–110 (Chapter vi); defined, 83; derivatives of, 89, (inequality satisfied by) 91; distinguished from monogenic functions, 99; represented by integrals, 92; Riemann’s equations connected with, 94; values of, at points inside a contour, 88; uniformly convergent series of, 91

Angle, analytical definition of, 589; and popular conception of an angle, 589, 590

Angle, modular, 492

Area represented by an integral, 61, 589

Argand diagram, 9

Argument, 9, 588; principal value of, 9, 588; continuity of, 588

Associated function of Borel, 141; of Riemann, 183; of Legendre [$P_n^{(\alpha)}(\xi)$ and $Q_n^{(\alpha)}(\xi)$], 323–326

Asymptotic expansions, 150–159 (Chapter viii); differentiation of, 153; integration of, 153; multiplication of, 152; of Bessel functions, 368, 369, 371, 373, 374; of confluent hypergeometric functions, 342, 345; of Gamma-functions, 351, 276; of parabolic cylinder functions, 347, 348; uniqueness of, 153, 154

Asymptotic inequality for parabolic cylinder functions of large order, 354

Asymptotic solutions of Mathieu’s equation, 425

Auto-functions, 226

Automorphic functions, 455

Axioms of arithmetic and geometry, 579

Barnes’ contour integrals for the hypergeometric function, 286, 289; for the confluent hypergeometric function, 344–345

Barnes’ $G$-function, 264, 378

Barnes’ Lemma, 289

Basic numbers, 462

Bernoulli numbers, 125; polynomials, 136, 137

Bertrand’s test for convergence of infinite integrals, 71

Bessel coefficients [$J_n(\xi)$], 101, 355; addition formulae for, 357; Bessel’s integral for, 362; differential equation satisfied by, 357; expansion of, as power series, 355; expansion of
functions in series of (by Neumann), 374, 375, 384, (by Schlömilch), 377; expansion of \((t - z)^{-1}\) in series of, 374, 375, 376; expressible as a confluent form of Legendre functions, 367; expressible as confluent hypergeometric functions, 389; inequality satisfied by, 379; Neumann's function \(\Omega_n(z)\) connected with, see Neumann's function: order of, 356; recurrence formulae for, 359; special case of confluent hypergeometric functions, 358. See also Bessel functions

Bessel functions, 355–385 (Chapter xvii), \(J_n(z)\) defined, 358–360; addition formulae for, 380; asymptotic expansion of, 368, 369, 371, 373, 374; expansion of, as an ascending series, 358, 371; expansion of functions in series of, 374, 375, 377, 391; first kind of, 339; Hankel's integral for, 365; integral connecting Legendre functions with, 364, 401; integral properties of, 390, 391, 384, 385; integrals involving products of, 380, 383, 385; notations for, 356, 372, 373; order of, 356; products of, 379, 380, 383, 385, 428; recurrence formulae for, 359, 373, 374; relations between, 380, 372, 372; relation between Gegenbauer's function and, 378; Schlömilch's form of Bessel's integral for, 362, 372; second kind of, \(Y_n(z)\) (Hankel), 370; \(Y_n^{(1)}(z)\) (Neumann), 372; \(Y_n(z)\) (Weber-Schlömilch), 370; second kind of modified, \(K_n(z)\), 373; solution of Laplace's equation by, 395; solution of the wave-motion equation by, 397; tabulation of, 378; whose order is large, 366, 383; whose order is half an odd integer, 364; with imaginary argument, \(J_{\alpha}(z)\), \(K_{\alpha}(z)\), 372, 373, 385; zeros of, 361, 367, 378, 381. See also Bessel coefficients and Bessel's equation

Bessel's equation, 294, 337, 373; fundamental system of solutions of (when \(n\) is not an integer), 359, 372; second solution when \(n\) is an integer, 370, 373. See also Bessel functions

Binet's integrals for \(\log \Gamma (z)\), 248–261

Binomial theorem, 95

Bocher's theorem on linear differential equations with five singularities, 203

Bolzano's theorem on limit points, 12

Bonnet's formula of the second mean value theorem, 66

Borel's associated function, 141; integral, 140; integral and analytical continuation, 141; method of 'summing' series, 154; theorem (the modified Heine-Borel theorem), 53

Boundary, 44

Boundary conditions, 387; and Laplace's equation, 393

Bounds of continuous functions, 55

Branch of a function, 106

Branch-point, 106

Bürmann's theorem, 128; extended by Teixeira, 131

Cantor's Lemma, 183

Cauchy's condition for the existence of a limit, 13; discontinuous factor, 123; formula for the remainder in Taylor's series, 96; inequality for derivatives of an analytic function, 91; integral, 119; integral representing \(\Gamma (z)\), 243; numbers, 372; tests for convergence of series and integrals, 21, 71

Cauchy's theorem, 85; extension to curves on a cone, 87; Möbius's converse of, 87, 110

Cell, 430

Česàro's method of 'summing' series, 155; generalised, 156

Change of order of terms in a series, 35; in an infinite determinant, 37; in an infinite product, 33

Change of parameter (method of solution of Mathieu's equation), 424

Characteristic functions, 226; numbers, 219; numbers associated with symmetric nuclei are real, 226

Chartier's test for convergence of infinite integrals, 72

Circle, area of sector of, 589; limiting, 98; of convergence, 30

Circular functions, 435, 584; addition theorems for, 585; continuity of, 585; differentiation of, 585; duplication formulae, 585; periodicity of, 587; relation with Gamma-functions, 239

Circular membrane, vibrations of, 356, 396

Class, left (L), 4; right (R), 4

Closed, 44

Cluster-point, 13

Coefficients, counting, 59; in Fourier series, nature of, 167, 174; in trigonometrical series, values of, 165, 165

Coefficients of Bessel, see Bessel coefficients

Comparison theorem for convergence of integrals, 71; for convergence of series, 20

Complementary moduli, 479, 493; elliptic integrals with, 479, 501, 520

Complete elliptic integrals \([E, K, K', K']\) [first and second kinds], 498, 499, 518; Legendre's relation between, 530; properties of (qua functions of the modulus), 484, 498, 499, 501, 521;
GENERAL INDEX

series for, 299; tables of, 518; the Gaussian transformation, 533; values for small values of \(|k|\), 521; values (as Gamma-functions) for special values of \(k\), 524–527; with complementary moduli, 479, 501, 520

Complex integrals, 77; upper limit to value of, 78

Complex integration, fundamental theorem of, 78

Complex numbers, 3–10 (Chapter i), defined, 6; amplitude of, 9; argument of, 9, 598; dependence of one on another, 41; imaginary part of (I), 9; logarithm of, 589; modulus of, 8; real part of (R), 9; representative point of, 9

Complex variable, continuous function of a, 44

Computation of elliptic functions, 485; of solutions of integral equations, 211

Conditional convergence of series, 18; of infinite determinants, 415. See also Convergence and Absolute convergence

Condition of integrability (Riemann’s), 63

Conditions, Dirichlet’s, 161, 163, 164, 176

Conduction of Heat, equation of, 387

Confluence, 302, 337

Confluent form, 203, 337

Confluent hypergeometric function \([W_{k,m}(z)]\), 337–354 (Chapter xv); equation for, 337; general asymptotic expansion of, 342, 345; integral defining, 539; integrals of Barnes’ type for, 343–345; Kummer’s formulae for, 338; recurrence formulae for, 352; relations with Bessel functions, 360; the functions \(W_{k,m}(z)\) and \(W_{k,m}(e)\), 337–339; the relations between functions of these types, 346; various functions expressed in terms of \(W_{k,m}(z)\), 340, 352, 353, 360. See also Bessel functions and Parabolic cylinder functions

Confocal coordinates, 405, 547; form a triply orthogonal system, 548; in association with ellipsoidal harmonics, 552; Laplace’s equation referred to, 551; uniformising variables associated with, 549

Convergence of points in the Argand diagram, 430

Constant. Euler’s or Mascheroni’s, \(\gamma\), 235, 246, 248

Constants \(c_1, c_2, c_3\), 443; E, \(E’\), 518, 520; of Fourier, 164; \(\eta_1, \eta_2, \eta_4\), 446; \(G, \gamma, \gamma_1, \gamma_2, K, K’\), 484, 498, 499; \(K’\), 484, 501, 503

Construction of elliptic functions, 433, 478, 492; of Mathieu functions, 409, (second method) 430

Contiguous hypergeometric functions, 284

Continua, 43

Continuants, 36

Continuation, analytic, 96, (not always possible) 98; and Borel’s integral, 141; of the hypergeometric function, 288. See also Asymptotic expansions

Continuity, 41; of power series, 57; (Abel’s theorem) 57; of the argument of a complex variable, 588; of the circular functions, 585; of the exponential function, 581; of the logarithmic function, 583, 589; uniformity of, 54

Continuous functions, 41–60 (Chapter iv), defined, 41; bounds of, 55; integrability of, 63; of a complex variable, 44; of two variables, 67

Contour, 85; roots of an equation in the interior of a, 119, 123

Contour integrals, 85; evaluation of definite integrals by, 112–124; the Mellin-Barnes type of, 286, 343; see also under the special function represented by the integral

Convergence, 11–40 (Chapter ii), defined, 13, 15; circle of, 30; conditional, 18; of a double series, 27; of an infinite determinant, 36; of an infinite product, 32; of an infinite integral, 70, (tests for) 71, 72; of a series 15; (Abel’s test for) 17; (Dirichlet’s test for) 17; of Fourier series, 174–179; of the geometric series, 19; of the hypergeometric series, 24; of the series \(\sum_{n=1}^\infty 1\); of the series occurring in Mathieu functions, 422; of trigonometrical series, 161; principle of, 13; radius of, 30; theorem on (Hardy’s), 156. See also Absolute convergence, Non-uniform convergence and Uniformity of convergence

Coordinates, confocal, 405, 547; orthogonal, 401, 548

Cosine, series for, 135

Cosine, see Circular functions

Cosine-integral \([Ci(z)]\), 352; series (Fourier series), 165

Cotangents, expansion of a function in series of, 139

Cubic function, integration problem connected with, 452, 512

Cunningham’s function \([\text{se}_{m}(z)]\), 353

Curve, simple, 43; on a cone, extension of Cauchy’s theorem to, 87; on a sphere (Seifert’s spiral), 527

Cut, 291

Cylindrical functions, 355. See Bessel functions
598  GENERAL INDEX

D’Alembert’s ratio test for convergence of series, 22
Darboux’ formula, 125
Decreasing sequence, 12
Descending numbers, 4
Degree of Legendre functions, 302, 307, 324
De la Vallée Poussin’s test for uniformity of convergence of an infinite integral, 72
De Morgan’s test for convergence of series, 23
Dependence of one complex number on another, 41
Derangement of convergent series, 25; of double series, 28; of infinite determinants, 37; of
infinite products, 33, 34
Derivates of an analytic function, 89; Cauchy’s inequality for, 91; integrals for, 89
Derivates of elliptic functions, 430
Determinant, Hadamard’s, 212
Determinants, infinite, 36; convergence of, 36, (conditional) 415; discussed by Hill, 36, 415;
evaluated by Hill in a particular case, 415; rearrangement of, 37
Difference equation satisfied by the Gamma-function, 237
Differential equations satisfied by elliptic functions and quotients of Theta-functions, 436, 477,
492; (partial) satisfied by Theta-functions, 470; Weierstrass’ theorem on Gamma-functions
and 436. See also Linear differential equations and Partial differential equations
Differentiation of an asymptotic expansion, 153; of a Fourier series, 168; of an infinite
integral, 74; of an integral, 67; of a series, 79, 91; of elliptic functions, 430, 493; of the
circular functions, 585; of the exponential function, 582; of the logarithmic function, 583, 589
Dirichlet’s conditions, 161, 163, 164, 176; form of Fourier’s theorem, 161, 163, 176; formula
connecting repeated integrals, 75, 76, 77; integral, 292; integral for \( \phi(z) \), 247; integral for
Legendre functions, 314; test for convergence, 17
Discontinuities, 42; and non-uniform convergence, 47; of Fourier series, 167, 169; ordinary, 42;
regular distribution of, 212; removable, 42
Discontinuous factor, Cauchy’s, 123
Discriminant associated with Weierstrassian elliptic functions, 444, 550
Divergence of a series, 15; of infinite products, 33
Domain, 44
Double circuit integrals, 256, 293
Double integrals, 68, 254
Double series, 26; absolute convergence of, 28; convergence of (Stolz’ condition), 27; methods
of summing, 27; a particular form of, 91; rearrangement of, 28
Double periodic functions, 429–535. See also Jacobian elliptic functions, Theta-functions and
Weierstrassian elliptic functions
Duplication formula for the circular functions, 585; for the Gamma-function, 240; for the
Jacobian elliptic functions, 498; for the Sigma-function, 459, 460; for the Theta-functions,
488; for the Weierstrassian elliptic functions, 441; for the Weierstrassian Zeta-function, 459
Electromagnetic waves, equations for, 404
Elementary functions, 82
Elementary transcendental functions, 579–590 (Appendix). See also Circular functions,
Exponential function and Logarithm
Ellipsoidal harmonics, 536–578 (Chapter xxiii); associated with confocal coordinates, 552;
derived from Lamé’s equation, 538–543, 552–554; external, 576; integral equations con-
ected with, 567; linear independence of, 560; number of, when the degree is given, 546;
physical applications of, 547; species of, 537; types of, 537. See also Lamé’s equation
and Lamé functions
Elliptic cylinder functions, see Mathieu functions
Elliptic functions, 455–535 (Chapter xx–xxii); computation of, 485; construction of, 433, 478;
derivative of, 430; discovery of, by Abel, Gauss and Jacobi, 429, 512, 524; expressed by
means of Theta-functions, 473; expressed by means of Weierstrassian functions, 448–451;
general addition formula, 452; number of zeros (or poles) in a cell, 421, 438; order of,
452; periodicity of, 429, 479, 500, 509, 503; period parallelogram of, 450; relation be-
tween zeros and poles of, 453; residues of, 431, 504; transformations of, 508; with no
poles (are constant), 431; with one double pole, 432, 434; with the same periods (relations
between), 452; with two simple poles, 452, 491. See also Jacobian elliptic functions,
Theta-functions and Weierstrassian elliptic functions

© in this web service Cambridge University Press  www.cambridge.org
GENERAL INDEX

Elliptic integrals. 429, 512; first kind of, 515; function $E(u)$ and, 517; function $Z(u)$ and, 518; inversion of, 429, 452, 454, 480, 484, 512, 524; second kind of, 517; (addition formulae for) 518, 519, 534; (imaginary transformation of) 519; third kind of, 522, 524; (dynamical application of) 523; (parameter of) 522; three kinds of, 514. See also Complete elliptic integrals

Elliptic membrane, vibrations of, 404

Equating coefficients, 59, 186

Equation of degree $m$ has $m$ roots, 120

Equations, indicial, 198; number of roots inside a contour, 119, 123; of Mathematical Physics, 203, 388–403; with periodic coefficients, 412. See also Difference equation, Integral equations, Linear differential equations, and under the names of special equations

Equivalence of curvilinear integrals, 85

Error function $\int e^{-x^2}$ and $\text{erf}(x)$, 341

Essential singularity, 102; at infinity, 104

Eta-function $H(u)$, 479, 490

Eulerian integrals, first kind of $[B(m, n)]$, 253; expressed by Gamma-functions, 254; extended by Pochhammer, 256

Eulerian integrals, second kind of, 254; see Gamma-function

Euler's constant $\gamma$, 235, 246, 248; expansion (Maclaurin's), 127; method of 'summing' series, 155; product for the Gamma-function, 237; product for the Zeta-function of Riemann, 271

Evaluation of definite integrals and of infinite integrals, 111–124 (Chapter vi)

Evaluation of Hill's infinite determinant, 415

Even functions, 115, 185; of Mathieu $(ce(z, q))$, 407

Existence of derivatives of analytic function, 89; -theorems, 388

Expansions of functions, 125–149 (Chapter viii); by Barmann, 128, 131; by Darboux, 125; by Euler and Maclaurin, 127; by Fourier, see Fourier series; by Fourier (the Fourier-Bessel expansion), 149; by Lagrange, 132, 149; by Laurent, 100; by MacLaurin, 94; by Pochhammer, 149; by Plana, 145; by Taylor, 93; by Wranski, 147; in infinite products, 136; in series of Bessel coefficients or Bessel functions, 374, 375, 381, 384; in series of cotangents, 139; in series of inverse factorials, 142; in series of Legendre polynomials or Legendre functions, 310, 322, 330, 331, 333; in series of Neumann functions, 374, 375, 384; in series of parabolic cylinder functions, 351; in series of rational functions, 134. See also Asymptotic expansions, Series, and under the names of special functions

Exponential function, 581; addition theorem for, 581; continuity of, 581; differentiation of, 582; periodicity of, 585

Exponential integral $\text{Ei}(z)$, 352

Exponents at a regular point of a linear differential equation, 198

Exterior, 44

External harmonics, (ellipsoidal) 376; (spheroidal) 403

Factor, Cauchy's discontinuous, 123; -periodicity-, 463

Factorial; expansion in a series of inverse, 142

Factor-theorem of Weierstrass, 137

Fejér's theorem on the summability of Fourier series, 169, 178

Ferrers' associated Legendre functions $P_m^\alpha(z)$ and $Q_m^\alpha(z)$, 323

First kind, Bessel functions of, 359; elliptic integrals of, 515; (complete) 518; (integration of) 515; Eulerian integral of, 253; expressed by Gamma-functions 254; integral equation of, 221; Legendre functions of, 307

First mean-value theorem, 65, 96

First species of ellipsoidal harmonic, 537; (construction of) 538

Floquet's solution of differential equations with periodic coefficients, 412

Fluctuation, 56; total, 57

Foundations of arithmetic and geometry, 579

Fourier-Bessel expansion, 381; integral, 385

Fourier constants, 164

Fourier series, 160–193 (Chapter x); coefficients in, 167, 174; convergence of, 174–179; differentiation of, 165; discontinuities of, 167, 169; distinction between any trigonometrical series and, 160, 163; expansions of a function in, 163, 165, 175, 176; expansions of Jacobian elliptic functions in, 510, 511; expansion of Mathieu functions in, 409, 411, 414, 420; Fejér's theorem on, 169; Hurwitz-Liapounoff theorem on, 180; Parseval's theorem on, 182; series of sines and series of cosines, 163; summability of, 169, 178; uniformity of convergence of, 168, 179. See also Trigonometrical series

Fourier's theorem, Dirichlet's statement of, 161, 163, 176
600  GENERAL INDEX

Fourier's theorem on integrals, 188, 211
Fourth species of ellipsoidal harmonic, 537, (construction of) 542
Fredholm's integral equation, 213-217, 228
Functionality, concept of, 41
Functions, branches of, 106; identity of two, 98; limits of, 42; principal parts of, 102; without essential singularities, 105; which cannot be continued, 98. See also under the names of special functions or special types of functions, e.g. Legendre functions. Analytic functions
Fundamental formulae of Jacobi connecting Theta-functions, 467, 468
Fundamental period parallelogram, 430; polygon of (automorphic functions), 455
Fundamental system of solutions of a linear differential equation, 197, 300, 389, 559. See also under the names of special equations
Gamma-function $\Gamma(z)$, 235-264 (Chapter xiii); asymptotic expansion of, 251, 276; circular functions and, 259; complete elliptic integrals and, 524-527, 535; contour integral (Hankel's) for, 244; difference equation satisfied by, 237; differential equations and, 238; duplication formula, 240; Euler's integral of the first kind and, 254; Euler's integral of the second kind, 241, (modified by Cauchy and Staleibitz) 243, (modified by Hankel) 244; Euler's product, 237; incomplete form of, 341; integrals for, (Binet's) 248-251, (Euler's) 241; minimum value of, 253; multiplication formula, 240; series, (Kummer's) 250, (Stirling's) 251; tabulation of, 253; trigonometrical integrals and, 256; Weierstrassian product, 235, 236. See also Eulerian integrals and Logarithmic derivate of the Gamma-function
Gauss' discovery of elliptic functions, 429, 512, 524; integral for $\Gamma'(z)\Gamma'(z)$, 246; lemniscate functions; transformation of elliptic integrals, 533
Gegenbauer's function $C_n^\nu(z)$, 329; addition formula, 335; differential equation for, 329; recurrence formulae, 330; relation with Legendre functions, 329; relation involving Bessel functions and, 355; Rodrigues' formula (analogue), 329; Schlüfi's integral (analogue), 329
Genus of a plane curve, 455
Geometric series, 19
Glaisher's notation for quotients and reciprocals of elliptic functions, 494, 495
Greatest of the limits, 13
Green's functions, 395

Hadamard's lemma, 212
Half-periods of Weierstrassian elliptic functions, 444
Hankel's Bessel function of the second kind, $Y_\nu(z)$, 370; contour integral for $\Gamma(z)$, 244; integral for $J_\nu(z)$, 365
Hardy's convergence theorem, 156; test for uniform convergence, 50
Harmonics, solid and surface, 399; spheroidal, 403; tesseral, 392, 536; zonal, 302, 392, 586; Sylvester's theorem concerning integrals of, 400. See also Ellipsoidal harmonics
Heat, equation of conduction of, 387
Heine-Borel theorem (modified), 53
Heine's expansion of $(t-z)^{-1}$ in series of Legendre polynomials, 321
Hermite's equation, 204, 209, 342, 347. See also Parabolic cylinder functions
Hermite's formula for the generalised Zeta-function $\zeta(s, a)$, 269
Hermite's solution of Lamé's equation, 573-575
Henn's equation, 576, 577
Hill's equation, 406, 413-417; Hill's method of solution, 413
Hill's infinite determinant, 36, 40, 415; evaluation of, 415
Hobson's associated Legendre functions, 325
Holomorphic, 83
Homogeneity of Weierstrassian elliptic functions, 439
Homogeneous harmonics (associated with ellipsoid), 543, 576; ellipsoidal harmonics derived from (Niven's formula), 543; linear independence of, 560
Homogeneous integral equations, 217, 219
Hurwitz's definition of the generalised Zeta-function $\zeta(s, a)$, 255; formula for $\zeta(s, a)$, 268; theorem concerning Fourier constants, 180
Hypergeometric equation, see Hypergeometric functions
Hypergeometric functions, 281-301 (Chapter xvi); Barnes' integrals, 266, 289; contiguous, 294; continuation of, 299; contour integrals for, 291; differential equation for, 292, 297, 299; functions expressed in terms of, 281, 311; of two variables (Appell's), 309; relations between twenty-four expressions involving, 284, 285, 290; Riemann's $P$-equation and, 206, 283; series for (convergence of), 24, 281; squares and products of, 295; value of $F(a, b; c; 1)$, 284-285; integral representation of, 288; summation formulae for, 289; transformation of, 293; use of, 292.
GENERAL INDEX

281, 293; values of special forms of hypergeometric functions, 298, 301. See also Bessel functions, Confluent hypergeometric functions and Legendre functions

Hypergeometric series, see Hypergeometric functions

Hypothesis of Riemann on zeros of $\zeta(s)$, 272, 280

Identically vanishing power series, 58

Identity of two functions, 98

Imaginary argument, Bessel functions with $[I_n(x) \text{ and } K_n(x)]$, 372, 373, 384

Imaginary part ($i$) of a complex number, 9

Imaginary transformation (Jacobi’s) of elliptic functions, 505, 506, 535; of Theta-functions, 124, 474; of $E(u)$ and $Z(u)$, 519

Improper Integrals, 75

Incomplete Gamma functions [$\gamma(x, z)$], 341

Increasing sequence, 12

Indicial equation, 198

Inequality (Abel’s), 16; (Hadamard’s), 212; satisfied by Bessel coefficients, 379; satisfied by Legendre polynomials, 303; satisfied by parabolic cylinder functions, 354; satisfied by $\psi(x, a)$, 274, 275

Infinite determinants, see Determinants

Infinite integrals, 69; convergence of, 70, 71, 72; differentiation of, 74; evaluation of, 111–124; functions represented by, see under the names of special functions; representing analytic functions, 92; theorems concerning, 73; uniform convergence of, 70, 72, 73. See also Integrals and Integration

Infinite products, 32; absolute convergence of, 32; convergence of, 32; divergence to zero, 33; expansions of functions as, 138, 137 (see also under the names of special functions); expressed by means of Theta-functions, 472, 485; uniform convergence of, 49

Infinite series, see Series

Infinity, 11, 103; essential singularity at, 104; point at, 103; pole at, 104; zero at, 104

Integers, positive, 3; signless, 3

Integrability of continuous functions, 63; Riemann’s condition of, 63

Integral, Borel’s, 140; and analytic continuation, 141

Integral, Cauchy’s, 119

Integral, Dirichlet’s, 258

Integral equations, 211–231 (Chapter xi); Abel’s, 211, 229, 230; Fredholm’s, 213–217, 223; homogeneous, 217, 219; kernel of, 213; Liouville-Neumann method of solution of, 221; nucleus of, 215; numbers (characteristic) associated with, 219; numerical solutions of, 211; of the first and second kinds, 213, 221; satisfied by Lame functions, 564–567; satisfied by Mathieu functions, 407; satisfied by parabolic cylinder functions, 231; Schlomilch’s, 329; solutions in series, 228; Volterra’s, 221; with variable upper limit, 213, 221

Integral formulas for elliptic harmonics, 567; for the Jacobian elliptic functions, 492, 494; for the Weierstrassian elliptic function, 437

Integral equations, 108; and Lamé’s equation, 571; and Mathieu’s equation, 418

Integral properties of Bessel functions, 390, 391, 355; of Legendre functions, 225, 305, 324; of Mathieu functions, 411; of Neumann’s function, 395; of parabolic cylinder functions, 350

Integrals, 61–81 (Chapter iv); along curves (equivalence of), 87; complex, 77, 78; differentiation of, 67; double, 68, 255; double-circuit, 256, 293; evaluation of, 111–124; for derivatives of an analytic function, 69; functions represented by, see under the names of the special functions; improper, 75; lower, 61; of harmonics (Sylveste’s theorem), 460; of irrational functions, 452, 512; of periodic functions, 112; principal values of, 75, 117; regular, 201; repeated, 68, 75; representing analytic functions, 92; representing areas, 61, 589; round a contour, 85; upper, 61. See also Elliptic integrals, Infinite integrals, and Integration

Integral theorem, Fourier’s, 188, 211; of Fourier-Bessel, 385

Integration, 61; complex, 77; contour-, 77; general theorem on, 63; general theorem on complex, 78; of asymptotic expansions, 153; of integrals, 68, 74, 75; of series, 78; problem connected with cubics or quartics and elliptic functions, 452, 512. See also Infinite integrals and Integrals

Interior, 44

Internal spheroidal harmonics, 403

Invariants of Weierstrassian elliptic functions, 437

Inverse factorials, expansions in series of, 142

Inversion of elliptic integrals, 429, 452, 454, 480, 484, 512, 524

Irrational functions, integration of, 452, 512

Irrational-real numbers, 5
Irreducible set of zeros or poles, 430
Irregular points (singularities) of differential equations, 197, 202
Iterated functions, 222

Jacobian elliptic functions \([sn u, cn u, dn u]\), 482, 478, 491–535 (Chapter xxiii); addition theorems for 494, 497, 530, 535; connexion with Weierstrassian functions, 505; definitions of \(am u\), \(\Delta \phi\), \(sn u \sin am u\), \(cn u \sin am u\); \(dn u\), 478, 492, 494; differential equations satisfied by, 477, 492; differentiation of, 493; duplication formulae for, 498; Fourier series for, 510, 511, 535; geometrical illustration of, 524, 527; general description of, 504; Glaisher's notation for quotients and reciprocals of, 494; infinite products for, 508, 532; integral formulae for, 492, 494; Jacobi's imaginary transformation of, 505, 506; Lamé functions expressed in terms of, 564, 573; Landen's transformation of, 507; modular angle of, 492; modulus of, 479, 492, (complementary) 479, 493; parametric representation of points on curves by, 524, 527, 527, 523; periodicity of, 479, 500, 502, 503; poles of, 432, 503, 504; quarter periods, \(K, iK\), of, 479, 496, 499, 501; relations between, 492; residues of, 504; Seiffert's spherical spiral and, 597; triple duplication formulae, 530, 534, 535; values of, when \(u = \pm K, \pm iK\) or \(\pm (K + iK)\), 506, 507; values of, when the modulus is small, 532. See also Elliptic integrals, Lemniscate functions, Theta-functions, and Weierstrassian elliptic functions

Jacobi's discovery of elliptic functions, 429, 512; earlier notation for Theta-functions, 479; fundamental Theta-function formulae, 467, 488; imaginary transformations, 124, 474, 505, 506, 519, 535; Zeta-function, see under Zeta function of Jacobi

Jordan's lemma, 115

Kernel, 213
Klein's theorem on linear differential equations with five singularities, 203
Kummer's formulae for confluent hypergeometric functions, 338; series for \(\log \Gamma(z)\), 250

Lacunary function, 98
Lagrange's expansion, 132, 149; form for the remainder in Taylor's series, 96
Lamé functions, defined, 558; expressed as algebraic functions, 556, 577; expressed by Jacobian elliptic functions, 579–576; expressed by Weierstrassian elliptic functions, 570–572; integral equations satisfied by, 564–567; linear independence of, 559; reality and distinctness of zeros of, 557, 558, 579; second kind of, 562; values of, 558; zeros of (Stieltjes' theorem), 560. See also Lamé's equation and Ellipsoidal harmonics

Lamé's equation, 204, 536–578 (Chapter xxiii); derived from theory of ellipsoidal harmonics, 538–543, 592–554; different forms of, 554, 573; generalised, 204, 570, 573, 576, 577; series solutions of, 554, 577, 578; solutions expressed in finite form, 459, 556, 576, 577, 578; solutions of a generalised equation in finite form, 570, 573. See also Lamé functions and Ellipsoidal harmonics

Landen's transformation of Jacobian elliptic functions, 476, 507, 533
Laplace's equation, 386; its general solution, 388; normal solutions of, 553; solutions involving functions of Legendre and Bessel, 391, 395; solution with given boundary conditions, 393; symmetrical solution of, 399; transformations of, 401, 407, 561, 553
Laplace's integrals for Legendre polynomials and functions, 312, 313, 314, 319, 326, 337
Lauricella's expansion, 100
Least of limits, 15
Legendre's lemma, 172
Left (L-) class, 4
Legendre's equation, 204, 304; for associated functions, 324; second solution of, 316. See also Legendre functions and Legendre polynomials

Legendre functions, 302–336 (Chapter xv); \(P_n(z), Q_n(z), P_m^m(z), Q_m^m(z)\) defined, 306, 316, 323, 325; addition formulae for, 326, 395; Bessel functions and, 364, 387, 401; degree of, 307, 334; differential equation for, 204, 306, 324; distinguished from Legendre polynomials, 306; expansions in ascending series, 311, 326; expansions in descending series, 302, 317, 326, 334; expansion of a function as a series of, 334; expressed by Murphy as hypergeometric functions, 311, 312; expression of \(Q_n(z)\) in terms of Legendre polynomials, 312, 320, 333; Ferrers' functions associated with, 323, 324; first kind of, 307; Gegenbauer's function, \(C_n^\alpha(z)\), associated with, see Gegenbauer's function; Heine's expansion of \((1 - z)^\lambda\) as a series of, 321; Hobson's functions associated with, 325; integral connecting Bessel functions with, 364; integral properties of, 324; Laplace's integrals for, 312, 313, 319, 326, 334; Mehler-Dirichlet integral for, 314; order of, 336; recurrence formulae for, 307, 318; Schlöfli's integral for, 304, 306; second kind of, 316–320, 325, 326; summation of \(\sin^m P_n(z)\) and \(\sin^m Q_n(z)\), 302, 321; zeros of, 303, 316, 335. See also Legendre polynomials and Legendre's equation

Legendre polynomials \(P_n(z)\), 95, 302; addition formula for, 326, 387; degree of, 302; differential equation for, 204, 304; expansion in ascending series, 311; expansion in descending
GENERAL INDEX

series, 302, 334; expansion of a function as a series of, 310, 322, 330, 331, 332, 335; expressed by Murphy as a hypergeometric function, 311, 312; Heine's expansion of (t - z)^-1 as a series of, 321; integral connecting Bessel functions with, 364; integral properties of, 225, 305; Laplace's equation and, 391; Laplace's integrals for, 312, 314; Mehler-Dirichlet integral for, 314; Neumann's expansion in series of, 322; numerical inequality satisfied by, 303; recurrence formulae for, 307, 309; Rodrigues' formula for, 225, 303; Schlöfli's integral for, 303, 364; summation of 2πν Pν(2z), 302; zeros of, 303, 316. See also Legendre functions

Legendre's relation between complete elliptic integrals, 520

Lemmiscate functions [sin lemn φ and cos lemn φ], 524

Liapounoff's theorem concerning Fourier constants, 180

Limit, condition for existence of, 13

Limit of a function, 42; of a sequence, 11, 12; point (the Bolzano-Weierstrass theorem), 12

Limiting circle, 98

Limits, greatest of and least of, 13

Limit to the value of a complex integral, 78

Lindemann's theory of Mathieu's equation, 417; the similar theory of Lamé's equation, 570

Linear differential equations, 194-210 (Chapter x), 386-403 (Chapter xviii); exponents of, 198; fundamental system of solutions of, 197, 200; irregular singularities of, 197, 202; ordinary point of, 194; regular integral of, 201; regular point of, 197; singular points of, 194, 197, (confluence of) 202; solution of, 194, 197, (uniqueness of) 196; special types of equations: —Bessel's for circular cylinder functions, 204, 342, 357, 358, 373; Gauss' for hypergeometric functions, 202, 327, 283; Gegenbauer's, 329, Hermite's, 204, 209, 342, 347; Hill's, 406, 413; Jacobi's for Theta-functions, 463; Lamé's, 204, 540-543, 554-558, 570-575; Laplace's, 366, 388, 536, 551; Legendre's for zonal and surface harmonics, 204, 304, 324; Mathieu's for elliptic cylinder functions, 204, 406; Neumann's, 383; Riemann's for F-functions, 206, 258, 291, 294; Stokes', 204; Weber's for parabolic cylinder functions, 204, 209, 342, 347; Whittaker's for confluent hypergeometric functions, 387; equation for conduction of Heat, 387; equation of Telegraphy, 387; equation of wave motions, 386, 397, 402; equations with five singularities (the Klein-Böcher theorem), 203; equations with three singularities, 206; equations with two singularities, 208; equations with r singularities, 209; equation of the third order with regular integrals, 210

Lioville's method of solving integral equations, 221

Lioville's theorem, 105, 431

Logarithm, 583; continuity of, 583, 589; differentiation of, 586, 589; expansion of, 584, 589; of complex numbers, 589

Logarithmic derivative of the Gamma-function [γ(z)], 240, 241; Binet's integrals for, 248-251; circular functions and, 240; Dirichlet's integral for, 247; Gauss' integral for, 246

Logarithmic derivative of the Riemann Zeta-function, 579

Logarithmic-integral function [Li z], 341

Lower integral, 61

Lunar perigee and node, motions of, 406

Maclaurin's (and Euler's) expansion, 127; test for convergence of infinite integrals, 71; series, 94, (failure of) 104, 110

Many-valued functions, 106

Mascheroni's constant [γ], 235, 246, 248

Mathematical Physics, equations of, 203, 386-403 (Chapter xviii). See also under Linear differential equations and the names of special equations

Mathieu functions [ce(z, q), se(z, q), ce(z, q), se(z, q)], 404-428 (Chapter xxi); construction of, 409, 420; convergence of series in, 422; even and odd, 407; expansions as Fourier series, 409, 411, 420; integral equations satisfied by, 407, 409; integral formulae, 411; order of, 410; second kind of, 427

Mathieu's equation, 204, 404-428 (Chapter xxi); general form, solutions by Floquet, 412, by Lindemann and Stieljes, 417; by the method of change of parameter, 424; second solution of, 413, 420, 427; solutions in asymptotic series, 425; solutions which are periodic, see Mathieu functions; the integral function associated with, 418. See also Hill's equation

Mean-value theorems, 65, 66, 96

Mehler's integral for Legendre functions, 314

Mellin's (and Barnes') type of contour integral, 286, 343

Membranes, vibrations of, 305, 366, 396, 404, 405

Mesh, 430

Methods of 'summing' series, 154-156

Minding's formula, 119

Minimum value of λ'(z), 253
604 GENERAL INDEX

Modified Heine-Borel theorem, 53
Modular angle, 492; function, 481, (equation connected with) 489; -surface, 41
Modulus, 430; of a complex number, 8; of Jacobian elliptic functions, 479, 492, (complementary) 479, 493; periods of elliptic functions regarded as functions of the, 484, 498, 499, 501, 521
Monogenic, 83; distinguished from analytic, 99
Monotonic, 57
Morera's theorem (converse of Cauchy's theorem), 87, 110
Motions of lunar perigee and node, 406
M-test for uniformity of convergence, 49
Multiplication formula for $\Gamma(z)$, 240; for the Sigma-function, 460
Multiplication of absolutely convergent series, 29; of asymptotic expansions, 152; of convergent series (Abel's theorem), 58, 59
Multipliers of Theta-functions, 463
Murphy's formulae for Legendre functions and polynomials, 311, 312

Neumann's definition of Bessel functions of the second kind, 372; expansions in series of Legendre and Bessel functions, 322, 374; (E. E. Neumann's) integral for the Legendre function of the second kind, 320; method of solving integral equations, 221
Neumann's function $Q_{n}(z)$, 374; differential equation satisfied by, 385; expansion of, 374; expansion of functions in series of, 376, 384; integral for, 375; integral properties of, 385; recurrence formulae for, 375

Non-uniform convergence, 44; and discontinuity, 47

Normal functions, 224
Normal solutions of Laplace's equation, 558
Notations, for Bessel functions, 356, 372, 373; for Legendre functions, 325, 326; for quotients and reciprocals of elliptic functions, 494, 499; for Theta-functions, 464, 479, 497
Nucleus of an integral equation, 213; symmetric, 223, 228

Numbers. 3–10 (Chapter ii); basic, 462; Bernoulli's, 125; Cauchy's, 379; characteristico, 219; (reality of) 226; complex, 6; irrational, 6; irrational-real, 5; pairs of, 6; rational, 3, 4; rational-real, 5; real, 5

Odd functions, 115, 168; of Mathieu, $[\psi_{n}(x, q)]$, 407

Open, 44
Order (O and o), 11; of Bernoullian polynomials, 126; of Bessel functions, 356; of elliptic functions, 432; of Legendre functions, 324; of Mathieu functions, 410; of poles of a function, 102; of terms in a series, 25; of the factors of a product, 33; of zeros of a function, 94

Ordinary discontinuity, 42

Ordinary point of a linear differential equation, 194

Orthogonal coordinates, 394; functions, 224

Oscillation, 11

Parabolic cylinder functions $[D_n(z)]$, 347; contour integral for, 349; differential equation for, 204, 209, 347; expansion in a power series, 347; expansion of a function as a series of, 351; general asymptotic expansion of, 348; inequalities satisfied by, 354; integral equation satisfied by, 231; integral properties, 350; integrals involving, 353; integrals representing, 353; properties when $n$ is an integer, 350, 353, 354; recurrence formulae, 350; relations between different kinds of $[D_n(z)]$ and $D_{n+1}(\pm iz)$, 348; zeros of, 354. See also Weber's equation

Parallelogram of periods, 430

Parameter, change of (method of solving Mathieu's equation), 424; connected with Theta-functions, 463, 464; of a point on a curve, 442, 496, 497, 537, 530, 553; of members of confocal systems of quadrics, 547; of third kind of elliptic integral, 522; thermometric, 409

Parseval's theorem, 182

Partial differential equations, property of, 390, 391. See also Linear differential equations

Partition function, 462
Parts, real and imaginary, 9

Pearson's function $[\omega_{n}(x)]$, 353

P-equation, Riemann's, 206, 337; connexion with the hypergeometric equation, 208, 283; solutions of, 283, 291, (relations between) 294; transformations of, 207

Periodic coefficients, equations with (Floquet's theory of), 412

Periodic functions, integrals involving, 112, 356. See also Fourier series and Doubly periodic functions
GENERAL INDEX

Periodicity factors, 463
Periodicity of circular and exponential functions, 585–587; of elliptic functions, 429, 434, 479, 500, 502, 503; of Theta-functions, 463
Periodic solutions of Mathieu’s equation, 407
Period parallelogram, 430; fundamental, 430
Periods of elliptic functions, 429; qua functions of the modulus, 484, 498, 499, 501, 521
Phase, 9
Pincherle’s functions (modified Legendre functions), 335
Plana’s expansion, 145
Pochhammer’s extension of Eulerian integrals, 256
Point, at infinity, 108; limit-, 12; representative, 9; singular, 194, 202
Poles of a function, 102; at infinity, 104; irreducible set of, 430; number in a cell, 431; relations between zeros of elliptic functions and, 433; residues at, 492, 504; simple, 102
Polygon, (fundamental) of automorphic functions, 455
Polynomials, expressed as series of Legendre polynomials, 310; of Abel, 353; of Bernoulli, 126, 127; of Legendre, see Legendre polynomials; of Sine, 353
Popular conception of an angle, 589; of continuity, 41
Positive integers, 3
Power series, 29; circle of convergence of, 30; continuity of, 57, (Abel’s theorem) 57; expansions of functions in, see under the names of special functions; identically vanishing, 93; Maclaurin’s expansion in, 94; radius of convergence of, 30, 32; series derived from, 31; Taylor’s expansion in, 93; uniformity of convergence of, 57
Principal part of a function, 102; solution of a certain equation, 482; value of an integral, 75, 117; value of the argument of a complex number, 9, 588
Principle of convergence, 13
Pringsheim’s theorem on summation of double series, 28
Products of Bessel functions, 379, 390, 393, 385, 483; of hypergeometric functions, 298. See also infinite products
Quarter periods $K$, $iK′$, 479, 498, 499, 501. See also Elliptic Integrals
Quartic, canonical form of, 513; integration problem connected with, 492, 512
Quasi-periodicity, 445, 447, 463
Quotients of elliptic functions (Glaisher’s notation), 494, 511; of Theta-functions, 477
Radius of convergence of power series, 30, 32
Rational functions, 105; expansions in series of, 134
Rational numbers, 3, 4; real numbers, 5
Real functions of real variables, 56
Reality of characteristic numbers, 226
Real numbers, rational and irrational, 5
Real part ($R$) of a complex number, 9
Rearrangement of convergent series, 25; of double series, 28; of infinite determinants, 37; of infinite products, 33
Reciprocal functions, Volterra’s, 218
Reciprocals of elliptic functions (Glaisher’s notation), 494, 511
Recurrence formulae, for Bessel functions, 359, 373, 374; for confluent hypergeometric functions, 229; for Gegenbauer’s function, 330; for Legendre functions, 307, 309, 318; for Neumann’s function, 375; for parabolic cylinder functions, 350. See also Contiguous hypergeometric functions
Region, 44
Regular, 88; distribution of discontinuities, 212; integrals of linear differential equations, 201; (of the third order) 210; points (singularities) of linear differential equations, 197
Relations between Bessel functions, 360, 371; between confluent hypergeometric functions $W_{\pm k, m}(\pm z)$ and $M_{\pm k, m}(\pm z)$, 346; between contiguous hypergeometric functions, 294; between elliptic functions, 452; between parabolic cylinder functions $D_{\nu}(\pm z)$ and $D_{\nu-1}(\pm iz)$, 348; between poles and zeros of elliptic functions, 433; between Riemann Zeta-functions $\zeta(s)$ and $\zeta(1-s)$, 289. See also Recurrence formulae
Remainder after $n$ terms of a series, 15; in Taylor’s series, 95
Removable discontinuity, 42
Repeated integrals, 68, 75
Representative point, 9
Residues, 111–124 (Chapter vi), defined, 111; of elliptic functions, 425, 497
## General Index

<table>
<thead>
<tr>
<th>Term</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Riemann’s associated function, 183, 184, 185; condition of integrability, 63; equations satisfied by analytic functions, 84; hypothesis concerning $\zeta(s)$, 272, 280; lemmas, 172, 184, 185; $F$-equation, 206, 209, 291, 294, (transformation of) 307, (and the hypergeometric equation) 208, see also Hypergeometric functions; theory of trigonometrical series, 182–186, Zeta-function, see Zeta-function (of Riemann)</td>
<td>562</td>
</tr>
<tr>
<td>Riesz’ method of ‘summing’ series, 156</td>
<td>156</td>
</tr>
<tr>
<td>Right ($R$-) class, 4</td>
<td>4</td>
</tr>
<tr>
<td>Rodrigues’ formula for Legendre polynomials, 303; modified, for Gegenbauer’s function, 329</td>
<td>329</td>
</tr>
<tr>
<td>Roots of an equation, number of, 120, (inside a contour) 119, 123; of Weierstrassian elliptic functions ($\epsilon_1$, $\epsilon_2$, $\epsilon_3$), 443</td>
<td>443</td>
</tr>
<tr>
<td>Saalschütz’s integral for the Gamma-function, 243</td>
<td>243</td>
</tr>
<tr>
<td>Schlöfli’s Bessel function of the second kind, $Y_\nu (z)$, 370</td>
<td>370</td>
</tr>
<tr>
<td>Schlöfli’s integral for Bessel functions, 362, 372; for Legendre polynomials and functions, 303, 304, 306; modified, for Gegenbauer’s function, 329</td>
<td>329</td>
</tr>
<tr>
<td>Schläfli’s expansion in series of Bessel coefficients, 377; function, 352; integral equation, 229</td>
<td>229</td>
</tr>
<tr>
<td>Schmidt’s theorem, 223</td>
<td>223</td>
</tr>
<tr>
<td>Schwars’ lemma, 186</td>
<td>186</td>
</tr>
<tr>
<td>Second kind, Bessel function of, (Hankel’s) 370, (Neumann’s) 372, (Weber-Schlöfli), 370, (modified) 373; elliptic integral of $E(u)$, $Z(u)$, 517, (complete) 518; Eulerian integral of, 241, (extended) 244; integral equation of, 218, 221; Lamé functions of, 562; Legendre functions of, 316–320, 325, 326</td>
<td>325</td>
</tr>
<tr>
<td>Second mean-value theorem, 66</td>
<td>66</td>
</tr>
<tr>
<td>Second solution of Bessel’s equation, 370, 372, (modified) 373; of Legendre’s equation, 316; of Mathieu’s equation, 413, 427; of the hypergeometric equation, 286, (confluent form) 343; of Weber’s equation, 347</td>
<td>347</td>
</tr>
<tr>
<td>Second species of ellipsoidal harmonics, 537, (construction of) 540</td>
<td>540</td>
</tr>
<tr>
<td>Section, 4</td>
<td>4</td>
</tr>
<tr>
<td>Salfert’s spherical spiral, 527</td>
<td>527</td>
</tr>
<tr>
<td>Sequences, 11; decreasing, 13; increasing, 12</td>
<td>11</td>
</tr>
<tr>
<td>Series (infinite series), 15; absolutely convergent, 18; change of order of terms in, 25; conditionally convergent, 18; convergence of, 15; differentiation of, 31, 79, 92; divergence of, 15; geometric, 19; integration of, 32, 78; methods of summing, 154–156; multiplication of, 29, 58, 59; of analytic functions, 91; of cosines, 165; of cotangents, 139; of inverse factorials, 142; of powers, see Power series; of rational functions, 134; of sines, 166; of variable terms, 44 (see also Uniformity of convergence); order of terms in, 25; remainder of, 15; representing particular functions, see under the name of the function; solutions of differential and integral equations in, 194–202, 229; Taylor’s, 93; See also Asymptotic expansions, Convergence, Expansions, Fourier series, Trigonometrical series and Uniformity of convergence</td>
<td>194</td>
</tr>
<tr>
<td>Set, irreducible (of zeros or poles), 490</td>
<td>490</td>
</tr>
<tr>
<td>Sigma-functions of Weierstrass [$\wp (z)$, $\sigma_1 (z)$, $\sigma_2 (z)$, $\sigma_3 (z)$], 447, 448; addition formula for, 451, 455, 460; analogy with circular functions, 447; duplication formulae, 459, 460; four types of, 448; expression of elliptic functions by, 450; quasi-periodic properties, 447; singly infinite product for, 448; three-term equation involving, 451, 461; Theta-functions connected with, 448, 473, 487; tritipation formula, 459</td>
<td>459</td>
</tr>
<tr>
<td>Signless integers, 3</td>
<td>3</td>
</tr>
<tr>
<td>Simple curve, 43; pole, 102; zero, 94</td>
<td>43</td>
</tr>
<tr>
<td>Simply-connected region, 455</td>
<td>455</td>
</tr>
<tr>
<td>Sine, product for, 137; See also Circular functions</td>
<td>137</td>
</tr>
<tr>
<td>Sine-integral [$\text{Si} (z)$], 332; -series (Fourier series), 166</td>
<td>332</td>
</tr>
<tr>
<td>Singly-periodic functions, 429; See also Circular functions</td>
<td>429</td>
</tr>
<tr>
<td>Singularities, 53, 84, 102, 194, 197, 202; at infinity, 104; convenience of, 208, 337; equations with five, 203; equations with three, 206, $\pm 10$; equations with two, 208; equations with $r$, 209; essential, 102, 104, irregular, 197, 202; regular, 197</td>
<td>209</td>
</tr>
<tr>
<td>Singular points (singularities) of linear differential equations, 194, 202</td>
<td>194</td>
</tr>
<tr>
<td>Solid harmonics, 392</td>
<td>392</td>
</tr>
<tr>
<td>Solution of Riemann’s P-equation by hypergeometric functions, 283, 288</td>
<td>283</td>
</tr>
<tr>
<td>Solutions of differential equations, see Chapters x, xviii, xxiii, and under the names of special equations</td>
<td>288</td>
</tr>
<tr>
<td>Solutions of integral equations, see Chapter xi</td>
<td>288</td>
</tr>
<tr>
<td>Sine’s polynomial [$\text{Si}^m (z)$], 332</td>
<td>332</td>
</tr>
<tr>
<td>Species (various) of ellipsoidal harmonics, 537</td>
<td>537</td>
</tr>
</tbody>
</table>
### GENERAL INDEX

<table>
<thead>
<tr>
<th>Topic</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spherical harmonics, see Harmonics</td>
<td>527</td>
</tr>
<tr>
<td>Spherical spiral, Seifert’s, 527</td>
<td></td>
</tr>
<tr>
<td>Spheroidal harmonics, 408</td>
<td></td>
</tr>
<tr>
<td>Squares of Bessel functions, 379, 380; of hypergeometric functions, 298; of Jacobian elliptic functions (relations between), 492; of Theta-functions (relations between), 486</td>
<td></td>
</tr>
<tr>
<td>Statement of Fourier’s theorem, Dirichlet’s, 161, 163, 164, 176</td>
<td></td>
</tr>
<tr>
<td>Steadily tending to zero, 17</td>
<td>17</td>
</tr>
<tr>
<td>Stirling’s theorem on zeros of Lamé functions, 560, (generalised) 582; theory of Mathieu’s equation, 417</td>
<td></td>
</tr>
<tr>
<td>Stirling’s series for the Gamma-function, 251</td>
<td>251</td>
</tr>
<tr>
<td>Stokes’ equation, 204</td>
<td>204</td>
</tr>
<tr>
<td>Stolz’s condition for convergence of double series, 27</td>
<td>27</td>
</tr>
<tr>
<td>Strings, vibrations of, 160</td>
<td></td>
</tr>
<tr>
<td>Successive substitutions, method of, 221</td>
<td>221</td>
</tr>
<tr>
<td>Sum-formula and Macclaurin, 127</td>
<td>127</td>
</tr>
<tr>
<td>Summability, methods of, 154-156; of Fourier series, 169; uniform, 156</td>
<td></td>
</tr>
<tr>
<td>Surface harmonic, 392</td>
<td>392</td>
</tr>
<tr>
<td>Surface, modular, 41</td>
<td>41</td>
</tr>
<tr>
<td>Surfaces, nearly spherical, 332</td>
<td>332</td>
</tr>
<tr>
<td>Sylvester’s theorem concerning integrals of harmonics, 400</td>
<td>400</td>
</tr>
<tr>
<td>Symmetric nucleus, 228, 228</td>
<td></td>
</tr>
<tr>
<td>Tabulation of Bessel functions, 375; of complete elliptic integrals, 518; of Gamma-functions, 253</td>
<td></td>
</tr>
<tr>
<td>Taylor’s series, 93; remainder in, 95; failure of, 100, 104, 110</td>
<td></td>
</tr>
<tr>
<td>Teixeira’s extension of Bürgmann’s theorem, 131</td>
<td>131</td>
</tr>
<tr>
<td>Telegraphy, equation of, 387</td>
<td>387</td>
</tr>
<tr>
<td>Tesseral harmonics, 392; factorisation of, 538</td>
<td></td>
</tr>
<tr>
<td>Tests for convergence, see Infinite integrals, Infinite products and Series</td>
<td></td>
</tr>
<tr>
<td>Thermometric parameter, 405</td>
<td>405</td>
</tr>
<tr>
<td>Theta-functions $\phi_1(z)$, $\phi_2(z)$, $\phi_3(z)$ or $\phi(z)$, $\Theta(u)$, 469-490 (Chapter xxii); abridged notation for products, 468, 469; addition formulae, 467; connexion with Sigma-functions, 448, 473, 487; duplication formulae, 488; expression of elliptic functions by, 473; four types of, 463; fundamental formulae (Jacobi’s), 467, 468; infinite products for, 468, 473, 498; Jacob’s first notation, $\Theta(u)$ and $\Pi(u)$, 479; multipliers, 463; notations, 464, 479, 487; parameters $q$, $r$, 463; partial differential equation satisfied by, 470; periodicity factors, 463; periods, 463; quotients of, 477; quotients yielding Jacobian elliptic functions, 478; relation $\Theta(z) = \phi_2 \phi_3 \phi_4$, 470; squares of (relations between), 466; transformation of, (Jacobi’s imaginary) 224, 474, (Landen’s) 476; triplication formulae for, 490; with zero argument $(\phi_2, \phi_3, \phi_4, \phi_3)$, 464; zeros of, 465</td>
<td></td>
</tr>
<tr>
<td>Third kind of elliptic integral, II $(u, a)$, 522; a dynamical application of, 523</td>
<td></td>
</tr>
<tr>
<td>Third order, linear differential equations of, 210, 298, 418, 428</td>
<td></td>
</tr>
<tr>
<td>Third species of spheroidal harmonics, 537, (construction of) 541</td>
<td></td>
</tr>
<tr>
<td>Three kinds of elliptic integrals, 514</td>
<td></td>
</tr>
<tr>
<td>Three-term equation involving Sigma-functions, 451, 461</td>
<td></td>
</tr>
<tr>
<td>Total fluctuation, 57</td>
<td>57</td>
</tr>
<tr>
<td>Transcendental functions, see under the names of special functions</td>
<td></td>
</tr>
<tr>
<td>Transformations of elliptic functions and Theta-functions, 508; Jacobi’s imaginary, 474, 505, 506, 519; Landen’s, 476, 507; of Riemann’s $\Pi$-equation, 207</td>
<td></td>
</tr>
<tr>
<td>Trigonometrical equations, 587, 588</td>
<td></td>
</tr>
<tr>
<td>Trigonometrical Integrals, 112, 263; and Gamma-functions, 256</td>
<td></td>
</tr>
<tr>
<td>Trigonometrical series, 160-189 (Chapter xx); convergence of, 161; values of coefficients in, 163; Riemann’s theory of, 182-188; which are not Fourier series, 160, 163. See also Fourier series</td>
<td></td>
</tr>
<tr>
<td>Triplication formulae for Jacobian elliptic functions and $E(u)$, 530, 534; for Sigma-functions, 459; for Theta-functions, 490; for Zeta-functions, 459</td>
<td></td>
</tr>
<tr>
<td>Twenty-four solutions of the hypergeometric equation, 284; relations between, 285, 288, 290</td>
<td></td>
</tr>
<tr>
<td>Two-dimensional continuum, 43</td>
<td>43</td>
</tr>
<tr>
<td>Two variables, continuous functions of, 67; hypergeometric functions (Appell’s) of, 300</td>
<td></td>
</tr>
<tr>
<td>Types of spheroidal harmonics, 537</td>
<td>537</td>
</tr>
<tr>
<td>Unicursal, 455</td>
<td></td>
</tr>
<tr>
<td>Uniformisation, 454</td>
<td>454</td>
</tr>
</tbody>
</table>