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Part one

Introduction to structure formation

1 Dark matter and structure formation

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Abstract

This chapter aims to present an introduction to current research on the nature of the cosmological dark matter and the origin of galaxies and large-scale structure within the standard theoretical framework: gravitational collapse of fluctuations as the origin of structure in the expanding universe. General relativistic cosmology is summarized, and the data on the basic cosmological parameters (t_0 and $H_0 \equiv 100h \text{ km s}^{-1} \text{ Mpc}^{-1}$, Ω_0 , Ω_Λ and Ω_b) are reviewed. The standard astrophysical classification of varieties of dark matter is used: *hot* and *cold*. Various particle physics candidates for hot, warm, and cold dark matter are briefly reviewed, together with current constraints and experiments that could detect or eliminate them. Also included is a very brief summary of the theory of cosmic defects, and a somewhat more extended exposition of the idea of cosmological inflation with a summary of some current models of inflation. The remainder is a discussion of observational constraints on cosmological model building, emphasizing models in which most of the dark matter is cold and the primordial fluctuations are the sort predicted by inflation. It is argued that the simplest models that have a hope of working are Cold Dark Matter with a cosmological constant (Λ CDM) if the Hubble parameter is high ($h \gtrsim 0.6$), and Cold + Hot Dark Matter (CHDM) if the Hubble parameter and age permit an $\Omega = 1$ cosmology; the most attractive variants of these models and the critical tests for each are discussed.

1.1 Introduction

The standard theory of cosmology is the Hot Big Bang, according to which the early universe was hot, dense, very nearly homogeneous, and expanding adiabatically according to the laws of general relativity (GR). This theory nicely accounts for the cosmic background radiation, and is at least roughly consistent with the abundances of the lightest nuclides. It is probably even true, as far as it goes; at least, I will assume

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so here. But as a fundamental theory of cosmology, the standard theory is seriously incomplete. One way of putting this is to say that it describes the middle of the story, but leaves us guessing about both the beginning and the end.

Galaxies and clusters of galaxies are the largest bound systems, and the filamentary or wall-like superclusters and the voids between them are the largest structures visible in the universe, but their origins are not yet entirely understood. Moreover, within the framework of the standard theory of gravity, there is compelling observational evidence that most of the mass detected gravitationally in galaxies and clusters, and especially on larger scales, is “dark” – that is, visible neither in absorption nor emission of any frequency of electromagnetic radiation. But we still do not know what this dark matter is.

Explaining the rich variety and correlations of galaxy and cluster morphology will require filling in much more of the history of the universe:

- *Beginnings*, in order to understand the origin of the fluctuations that eventually collapse gravitationally to form galaxies and large-scale structure. This is a mystery in the standard Hot Big Bang universe, because the matter that comprises a typical galaxy, for example, first came into causal contact about a year after the Big Bang. It is hard to see how galaxy-size fluctuations could have formed after that, but even harder to see how they could have formed earlier. The best solution to this problem yet discovered, and the one emphasized here, is cosmic inflation. The main alternative, discussed in less detail here, is cosmic topological defects.
- *Denouement*, since even given appropriate initial fluctuations, we are far from understanding the evolution of galaxies, clusters, and large-scale structure – or even the origins of stars and the stellar initial mass function.
- *Dark matter* is probably the key to unraveling the plot since it appears to be gravitationally dominant on all scales larger than the cores of galaxies. The dark matter is therefore crucial for understanding the evolution and present structure of galaxies, clusters, superclusters and voids.

The present chapter (updating Primack 1987, 1988, 1993–7) concentrates on the period *after* the first three minutes, during which the universe expands by a factor of $\sim 10^8$ to its present size, and all the observed structures form. This is now an area undergoing intense development in astrophysics, both observationally and theoretically. It is likely that the present decade will see the construction at last of a fundamental theory of cosmology, with perhaps profound implications for particle physics – and possibly even for broader areas of modern culture.

The current controversy over the amount of matter in the universe will be emphasized, discussing especially the two leading alternatives: a critical-density universe, i.e., with $\Omega_0 \equiv \bar{\rho}_0/\rho_c = 1$ (see Table 1.1), vs. a low-density universe having $\Omega_0 \approx 0.3$ with a positive cosmological constant $\Lambda > 0$ such that $\Omega_\Lambda \equiv \Lambda/(3H_0^2) = 1 - \Omega_0$ supplying

the additional density required for the flatness predicted by the simplest inflationary models. (The significance of the cosmological parameters Ω_0 , H_0 , t_0 , and Λ is discussed in §1.2. Note that the present-epoch matter density parameter Ω_0 is sometimes denoted Ω_m or simply Ω .) $\Omega = 1$ requires that the expansion rate of the universe, the Hubble parameter $H_0 \equiv 100h \text{ km s}^{-1} \text{ Mpc}^{-1} \equiv 50h_{50} \text{ km s}^{-1} \text{ Mpc}^{-1}$, be relatively low, $h \lesssim 0.5$, in order that the age of the universe t_0 be as large as the minimum estimate of the age of the stars in the oldest globular clusters. If the expansion rate turns out to be larger than this, we will see that GR then requires that $\Omega_0 < 1$, with a positive cosmological constant giving a larger age for any value of Ω_0 .

Although this chapter will concentrate on the implications of Cold Dark Matter (CDM) and alternative theories of dark matter for the development of galaxies and large-scale structure in the relatively “recent” universe, one can hardly avoid recalling some of the earlier parts of the story. Inflation or cosmic defects will be important in this context for the nearly constant curvature (near-“Zel’dovich”) spectrum of primordial fluctuations and as plausible solutions to the problem of generating these large-scale fluctuations without violating causality; and primordial nucleosynthesis will be important as a source of information on the amount of ordinary (“baryonic”) matter in the universe. The fact that the observational lower bound on Ω_0 —namely $0.3 \lesssim \Omega_0$ —exceeds the most conservative upper limit on baryonic mass $\Omega_b \lesssim 0.03h^{-2}$ from Big Bang Nucleosynthesis (Copi, Schramm & Turner 1995; cf. Hata *et al.* 1995) is the main evidence that there must be such nonbaryonic dark-matter particles.

Of special concern will be evidence and arguments bearing on the astrophysical properties of the dark matter, which can also help to constrain possible particle physics candidates. The most popular of these are few-eV neutrinos (the “hot” dark matter candidate), heavy stable particles such as $\sim 100 \text{ GeV}$ photinos (or whatever neutralino is the lightest supersymmetric partner particle) or $10^{-6} - 10^{-3} \text{ eV}$ invisible axions (these remain the favorite “cold” dark matter candidates), and various more exotic ideas such as keV gravitinos (“warm” dark matter) or primordial black holes (BH).

The usual astrophysical classification of the dark matter candidates is into *hot*, *warm*, or *cold*, depending on their thermal velocity in the early universe. Hot dark matter, such as few-eV neutrinos, is still relativistic when galaxy-size masses ($\sim 10^{12} M_\odot$) are first encompassed within the horizon. Warm dark matter is just becoming nonrelativistic then. Cold dark matter, such as axions or massive photinos, is nonrelativistic when even globular cluster masses ($\sim 10^6 M_\odot$) come within the horizon. As a consequence, fluctuations on galaxy scales are wiped out by the “free streaming” of the hot dark-matter particles which are moving at nearly the speed of light. But galaxy-size fluctuations are preserved with warm dark matter, and all cosmologically relevant fluctuations survive in a universe dominated by the sluggishly moving cold dark matter.

The first possibility for nonbaryonic dark matter that was examined in detail was massive neutrinos, assumed to have mass $\sim 25 \text{ eV}$ —both because that mass corresponds to closure density for $h \approx 0.5$, and because in the late 1970s the Moscow tritium β -decay experiment provided evidence (subsequently contradicted by other experiments) that the electron neutrino has that mass. Although this picture leads to superclusters

Table 1.1. *Physical constants for cosmology*

parsec	$\text{pc} = 3.09 \times 10^{18} \text{ cm} = 3.26 \text{ light years (lyr)}$
Newton's const.	$G = 6.67 \times 10^{-8} \text{ dyne cm}^2 \text{ g}^{-2}$
Hubble parameter	$H_0 = 100 h \text{ km s}^{-1} \text{ Mpc}^{-1}$, $1/2 \lesssim h \lesssim 1$
Hubble time	$H_0^{-1} = h^{-1} 9.78 \text{ Gyr}$
Hubble radius	$R_H = cH^{-1} = 3.00 h^{-1} \text{ Gpc}$
critical density	$\rho_c = 3H^2/8\pi G = 1.88 \times 10^{-29} h^2 \text{ g cm}^{-3}$ $= 10.5 h^2 \text{ keV cm}^{-3} = 2.78 \times 10^{11} h^2 M_\odot \text{ Mpc}^{-3}$
speed of light	$c = 3.00 \times 10^{10} \text{ cm s}^{-1} = 306 \text{ Mpc Gyr}^{-1}$
solar mass	$M_\odot = 1.99 \times 10^{33} \text{ g}$
solar luminosity	$L_\odot = 3.85 \times 10^{33} \text{ erg s}^{-1}$
Planck's const.	$\hbar = 1.05 \times 10^{-27} \text{ erg s} = 6.58 \times 10^{-16} \text{ eV s}$
Planck mass	$m_{\text{Pl}} = (\hbar c/G)^{1/2} = 2.18 \times 10^{-5} \text{ g} = 1.22 \times 10^{19} \text{ GeV}$

and voids of roughly the size seen, superclusters are the first structures to collapse in this theory since smaller size fluctuations do not survive. The theory foundered on this point, however, since galaxies are almost certainly older than superclusters. The standard (adiabatic) form of this theory has recently been ruled out by the COsmic Background Explorer (COBE) data: if the amplitude of the fluctuation spectrum is small enough for consistency with the COBE fluctuations, superclusters would just be beginning to form at the present epoch, and hardly any smaller-scale structures, including galaxies, could have formed by the present epoch.

A currently popular possibility is that the dark matter is cold. After Peebles (1982), my co-workers and I were among those who first proposed and worked out the consequences of the CDM model (Primack & Blumenthal 1983, 1984; Blumenthal *et al.* 1984). Its virtues include an account of galaxy and cluster formation that at first sight appeared to be very attractive. Its defects took longer to uncover, partly because uncertainty about how to normalize the CDM fluctuation amplitude allowed for a certain amount of fudging, at least until COBE measured the fluctuation amplitude. The most serious problem with CDM is probably the mismatch between supercluster-scale and galaxy-scale structures and velocities, which suggests that the CDM fluctuation spectrum is not quite the right shape—which can perhaps be remedied if the dark matter content is a mixture of hot and cold, or if there is less than a critical density of cold dark matter.

The basic theoretical framework for cosmology is reviewed first (see a summary in §11.1.2), followed by a discussion of the current knowledge about the fundamental cosmological parameters.

Table 1.1 lists the values of the most important physical constants used in this chapter (cf. Barnett *et al.* 1996). The distance to distant galaxies is deduced from their redshifts; consequently, the parameter h appears in many formulas where the distance matters.

1.2 Cosmology basics

It is assumed here that Einstein's theory of general relativity accurately describes gravity. Although it is important to appreciate that there is no observational confirmation of this on scales larger than about 1 Mpc, the tests of GR on smaller scales are becoming increasingly precise, especially with pulsars in binary star systems (Will 1981, 1986, 1990; Taylor 1994). On galaxy and cluster scales, the general agreement between the mass estimated by velocity measurements and by gravitational lensing provides evidence supporting standard gravity. There are two other reasons most cosmologists believe in GR: it is conceptually so beautifully simple that it is hard to believe it could be wrong, and anyway it has no serious theoretical competition. Nevertheless, since a straightforward interpretation of the available data in the context of the standard theory of gravity leads to the disquieting conclusion that most of the matter in the universe is dark, there have been suggestions that perhaps our theory of gravity is inadequate on large scales. The suggested alternatives are mentioned briefly in §1.2.2.

The "Copernican" or "cosmological" principle is logically independent of our theory of gravity, so it is appropriate to state it before discussing GR further. First, some definitions are necessary.

- A *comoving observer* is at rest and unaccelerated with respect to nearby material (in practice, with respect to the center of mass of galaxies within, say, $100 h^{-1}$ Mpc).
- The universe is *homogeneous* if all comoving observers see identical properties.
- The universe is *isotropic* if all comoving observers see no preferred direction.

The *cosmological principle* asserts that the universe is homogeneous and isotropic on large scales. (Isotropy about at least three points actually implies homogeneity, but the counter-example of a cylinder shows that the reverse is not true.) In reality, the matter distribution in the universe is exceedingly inhomogeneous on small scales, and increasingly homogeneous on scales approaching the entire horizon. The cosmological principle is in practice the assumption that for cosmological purposes we can neglect this inhomogeneity, or treat it perturbatively. This has now been put on an improved basis, based on the observed isotropy of the cosmic background radiation and the (partially testable) Copernican assumption that other observers also see a nearly homogeneous CMB. The "COBE–Copernicus" Theorem (Stoeger, Maartens & Ellis 1995; Maartens Ellis & Stoeger 1995; reviewed by Ellis 1996) asserts that if all comoving observers measure the cosmic microwave background radiation to be almost isotropic in a region of the expanding universe, then the universe is locally almost spatially homogeneous and isotropic in that region.

The great advantage of assuming homogeneity is that our own cosmic neighborhood becomes representative of the whole universe, and the range of cosmological models to be considered is also enormously reduced. The cosmological principle also implies the existence of a universal cosmic time, since all observers see the same sequence of

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cosmic events with which to synchronize their clocks. (This assumption is sometimes explicitly included in the statement of the cosmological principle; e.g., Rindler (1977), p. 203.) In particular, they can all start their clocks with the Big Bang.

Astronomers observe that the redshift $z \equiv (\lambda - \lambda_0)/\lambda_0$ (where λ denotes wavelength) of distant galaxies is proportional to their distance. We assume, for lack of any viable alternative explanation, that this redshift is due to the expansion of the universe. Recent evidence for this includes higher CMB temperature at higher redshift (Songaila *et al.* 1994b) and time dilation of high-redshift Type Ia supernovae (Goldhaber *et al.* 1996). The cosmological principle then implies (see, for example, Rowan-Robinson 1981, §4.3) that the expansion is homogeneous: $r = a(t)r_0$, which immediately implies Hubble's law: $v = \dot{r} = \dot{a}a^{-1}r = H_0r$. Here r_0 is the present distance of some distant galaxy (the subscript "0" in cosmology denotes the present era), r is its distance as a function of time, v is its velocity, and $a(t)$ is the scale factor of the expansion (scaled to be unity at the present: $a(t_0) = 1$). The scale factor is related to the redshift by $a = (1 + z)^{-1}$. Hubble's "constant" $H(t)$ (constant in space, but a function of time except in an empty universe) is $H(t) = \dot{a}a^{-1}$.

Finally, it can be shown (see, e.g., Weinberg 1972, Rindler 1977) that the most general metric satisfying the cosmological principle is the Robertson–Walker metric

$$ds^2 = c^2 dt^2 - a(t)^2 \left[\frac{dr^2}{1 - kr^2} + r^2(\sin^2\theta d\phi^2 + d\theta^2) \right], \quad (1.1)$$

where the curvature constant k , by a suitable choice of units for r , has the value 1, 0, or -1 , depending on whether the universe is closed, flat, or open, respectively. For $k = 1$ the spatial universe can be regarded as the surface of a sphere of radius $a(t)$ in four-dimensional Euclidean space; and although for $k = 0$ or -1 no such simple geometric interpretation is possible, $a(t)$ still sets the scale of the geometry of space.

Formally, GR consists of the assumption of the Equivalence Principle (or the Principle of General Covariance) together with Einstein's field equations, labeled (E) in Table 1.2, where the key equations have been collected together. The Equivalence Principle implies that space-time is locally Minkowskian and globally (pseudo-)Riemannian, and the field equations specify precisely how space-time responds to its contents. The essential physical idea underlying GR is that space-time is not just an arena, but rather an active participant in the dynamics, as summarized by John Wheeler: "Matter tells space how to curve, curved space tells matter how to move."

Comoving coordinates are coordinates with respect to which comoving observers are at rest. A comoving coordinate system expands with the Hubble expansion. It is convenient to specify linear dimensions in comoving coordinates scaled to the present; for example, if we say that two objects were 1 Mpc apart in comoving coordinates at a redshift of $z = 9$, their actual distance then was 0.1 Mpc. In a non-empty universe with vanishing cosmological constant, the case first studied in detail by the Russian cosmologist Alexander Friedmann in 1922–4, gravitational attraction ensures that the expansion rate is always decreasing. As a result, the Hubble radius $R_H(t) \equiv cH(t)^{-1}$ is increasing. The Hubble radius of a non-empty Friedmann universe expands even in

Table 1.2. *Theoretical framework: GR cosmology*

GR:	Matter tells space how to curve,	Curved space tells matter how to move.
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$$(E) \quad R^{\mu\nu} - \frac{1}{2}Rg^{\mu\nu} = -8\pi G T^{\mu\nu} - \Lambda g^{\mu\nu}$$

COBE–Copernicus Theorem: If all observers measure nearly isotropic CMB, then the universe is locally nearly homogeneous and isotropic – i.e., nearly FRW.

$$\text{FRW } E(00) \quad \frac{\dot{a}^2}{a^2} = \frac{8\pi}{3}G\rho - \frac{k}{a^2} + \frac{\Lambda}{3}$$

$$\text{FRW } E(ii) \quad \frac{2\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} = -8\pi Gp - \frac{k}{a^2} + \Lambda$$

$$H_0 \equiv 100 h \text{ km s}^{-1} \text{ Mpc}^{-1}$$

$$\equiv 50 h_{50} \text{ km s}^{-1} \text{ Mpc}^{-1}$$

$$\frac{E(00)}{H_0^2} \Rightarrow 1 = \Omega_0 - \frac{k}{H_0^2} + \Omega_\Lambda \text{ with } H_0 \equiv \frac{\dot{a}_0}{a_0}, a_0 \equiv 1, \Omega_0 \equiv \frac{\rho_0}{\rho_c}, \Omega_\Lambda \equiv \frac{\Lambda}{3H_0^2},$$

$$\rho_c \equiv \frac{3H_0^2}{8\pi G} = 0.70 \times 10^{11} h_{50}^2 M_\odot \text{ Mpc}^{-3}$$

$$E(ii) - E(00) \Rightarrow \frac{2\ddot{a}}{a} = -\frac{8\pi}{3}G\rho - 8\pi Gp + \frac{2}{3}\Lambda$$

$$\text{Divide by } 2E(00) \Rightarrow q_0 \equiv -\left(\frac{\ddot{a}}{a} \frac{a^2}{\dot{a}^2}\right)_0 = \frac{\Omega_0}{2} - \Omega_\Lambda$$

$$E(00) \Rightarrow t_0 = \int_0^1 \frac{da}{a} \left[\frac{8\pi}{3}G\rho - \frac{k}{a^2} + \frac{\Lambda}{3} \right]^{-\frac{1}{2}} = H_0^{-1} \int_0^1 \frac{da}{a} \left[\frac{\Omega_0}{a^3} - \frac{k}{H_0^2 a^2} + \Omega_\Lambda \right]^{-\frac{1}{2}}$$

$$t_0 = H_0^{-1} f(\Omega_0, \Omega_\Lambda) \quad H_0^{-1} = 9.78 h^{-1} \text{ Gyr} \quad \begin{aligned} f(1, 0) &= \frac{2}{3} \\ f(0, 0) &= 1 \\ f(0, 1) &= \infty \end{aligned}$$

$$\frac{d}{dt}[E(00)a^3] \text{ vs. } E(ii) \Rightarrow \frac{d}{da}(\rho a^3) = -3p a^2 \text{ ("continuity")}$$

Given eq. of state $p = p(\rho)$, integrate to determine $\rho(a)$,
 integrate $E(00)$ to determine $a(t)$

Examples: $p = 0 \Rightarrow \rho = \rho_0 a^{-3}$ (assumed above in q_0, t_0 eqs.)

$$p = \frac{\rho}{3}, k = 0 \Rightarrow \rho \propto a^{-4}$$

comoving coordinates. Our backward lightcone encompasses more of the universe as time goes on.

1.2.1 Friedmann–Robertson–Walker universes

For a homogeneous and isotropic fluid of density ρ and pressure p in a homogeneous universe with curvature k and cosmological constant Λ , Einstein’s system of partial

differential equations reduces to the two ordinary differential equations labeled in Table 1.2 as FRW E(00) and E(ii), for the diagonal time and spatial components (see, e.g., Rindler 1977, §9.9). Dividing E(00) by H_0^2 , and subtracting E(00) from E(ii) puts these equations into more familiar forms. Dividing the latter by $2E(00)$ and evaluating all expressions at the present epoch then gives the familiar expression for the deceleration parameter q_0 in terms of Ω_0 and Ω_Λ .

Multiplying E(00) by a^3 , differentiating with respect to a , and comparing with E(ii) gives the equation of continuity. Given an equation of state $p = p(\rho)$, this equation can be integrated to determine $\rho(a)$; then E(00) can be integrated to determine $a(t)$.

Consider, for example, the case of vanishing pressure $p = 0$, which is presumably an excellent approximation for the present universe since the contribution of radiation and massless neutrinos (both having $p = \rho c^2/3$) to the mass-energy density is at the present epoch much less than that of nonrelativistic matter (for which p is negligible). The continuity equation reduces to $(4\pi/3)\rho a^3 = M = \text{constant}$, and E(00) yields *Friedmann's equation*

$$\dot{a}^2 = \frac{2GM}{a} - kc^2 + \frac{\Lambda c^2 a^2}{3}. \quad (1.2)$$

This gives an expression for the age of the universe t_0 which can be integrated in general in terms of elliptic functions, and for $\Lambda = 0$ or $k = 0$ in terms of elementary functions (cf. standard textbooks, e.g., Peebles 1993, §13, and Felton & Isaacman 1986).

Figure 1.1 (a) plots the evolution of the scale factor a for three interesting examples: $(\Omega_0, \Omega_\Lambda) = (1, 0)$, $(0.3, 0)$, and $(0.3, 0.7)$. Figure 1.1 (b) shows how t/t_H depends on Ω_0 both for $\Lambda = 0$ (dashed) and $\Omega_\Lambda = 1 - \Omega_0$ (solid). Notice that for $\Lambda = 0$, t_0/t_H is somewhat greater for $\Omega_0 = 0.3$ ($t_0/t_H = 0.81$) than for $\Omega = 1$ ($t_0/t_H = 2/3$), while for $\Omega_0 = 1 - \Omega_\Lambda = 0.3$ it is substantially greater ($t_0/t_H = 0.96$). In the last case, the competition between the attraction of the matter and the repulsion of space by space represented by the cosmological constant results in a slowing of the expansion at $a \sim 0.5$; the cosmological constant subsequently dominates, resulting in an accelerated expansion (negative deceleration $q_0 = -0.55$ at the present epoch), corresponding to an inflationary universe. In addition to increasing t_0 , this behavior has observational implications that we will explore in §1.3.3.

1.2.2 *Is the gravitational force $\propto r^{-1}$ at large r ?*

Back to the question whether our conventional theory of gravity is trustworthy on large scales. The reason for raising this question is that interpreting modern observations within the context of the standard theory leads to the conclusion that at least 90% of the matter in the universe is dark. Moreover, there is no observational confirmation that the gravitational force falls as r^{-2} on large scales.

Tohline (1983) pointed out that a modified gravitational force law, with the gravitational acceleration given by $a' = (GM_{\text{lum}}/r^2)(1 + r/d)$, could be an alternative to dark matter galactic halos as an explanation of the constant-velocity rotation curves of

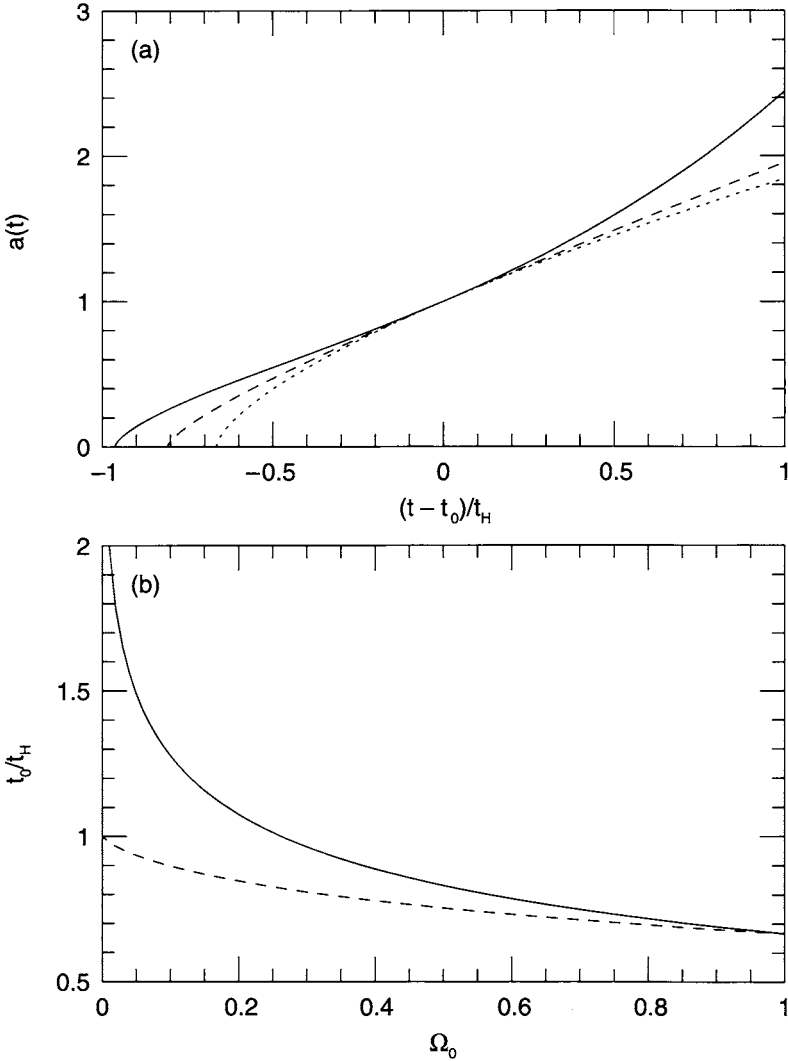


Figure 1.1. (a) Evolution of the scale factor $a(t)$ plotted vs. the time after the present $(t - t_0)$ in units of Hubble time $t_H \equiv H_0^{-1} = 9.78h^{-1}$ Gyr for three different cosmologies: Einstein-de Sitter ($\Omega_0 = 1, \Omega_\Lambda = 0$ dotted curve), negative curvature ($\Omega_0 = 0.3, \Omega_\Lambda = 0$: dashed curve), and low- Ω_0 flat ($\Omega_0 = 0.3, \Omega_\Lambda = 0.7$: solid curve). (b) Age of the universe today t_0 in units of Hubble time t_H as a function of Ω_0 for $\Lambda = 0$ (dashed curve) and flat $\Omega_0 + \Omega_\Lambda = 1$ (solid curve) cosmologies.

spiral galaxies. (The mass is written as M_{lum} , where “lum” is shorthand for ‘luminous’, to emphasize that there is not supposed to be any dark matter.) Indeed, this equation implies $v^2 = GM_{\text{lum}}/d = \text{constant}$ for $r \gg d$. However, with the distance scale d where the force shifts from r^{-2} to r^{-1} taken to be a physical constant, the same for all