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Four principles underlie the morphometrics of landmark data: (1) archiving biological form by locations of landmark points, (2) converting sets of three locations to pairs of shape coordinates, (3) processing these variables by carefully contrived multivariate statistical maneuvers, and (4) interpreting findings in the picture plane or space of the data.	
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Procrustes superposition is the best fit of one set of landmarks to a homologous set, or to an average, by a combination of translation, rotation, and rescaling. Reexpressed using the shape coordinates, the tactic may be seen to be biologically appropriate only under unusual circumstances. Application to the study of asymmetry may be valid. Example.

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