

Contents

<i>Foreword</i>	<i>page</i> xiii
<i>Preface</i>	xv
Part One Equilibrium and Arbitrage	1
1 Equilibrium in Security Markets	3
1.1 Introduction	3
1.2 Security Markets	4
1.3 Agents	6
1.4 Consumption and Portfolio Choice	6
1.5 First-Order Conditions	7
1.6 Left and Right Inverses of a Matrix	8
1.7 General Equilibrium	9
1.8 Existence and Uniqueness of Equilibrium	10
1.9 Representative Agent Models	11
1.10 Notes	11
2 Linear Pricing	15
2.1 Introduction	15
2.2 The Law of One Price	15
2.3 The Payoff Pricing Functional	15
2.4 Linear Equilibrium Pricing	17
2.5 State Prices in Complete Markets	18
2.6 Recasting the Optimization Problem	19
2.7 Notes	21
3 Arbitrage and Positive Pricing	22
3.1 Introduction	22
3.2 Arbitrage and Strong Arbitrage	22
3.3 Diagrammatic Representation	23
3.4 Positivity of the Payoff Pricing Functional	26
3.5 Positive State Prices	27

vi	<i>Contents</i>	
	3.6 Arbitrage and Optimal Portfolios	27
	3.7 Positive Equilibrium Pricing	30
	3.8 Notes	30
4	Portfolio Restrictions	33
	4.1 Introduction	33
	4.2 Short Sales Restrictions	33
	4.3 Portfolio Choice under Short Sales Restrictions	35
	4.4 The Law of One Price	36
	4.5 Limited and Unlimited Arbitrage	37
	4.6 Diagrammatic Representation	38
	4.7 Bid-Ask Spreads	39
	4.8 Bid-Ask Spreads in Equilibrium	40
	4.9 Notes	42
	Part Two Valuation	45
5	Valuation	47
	5.1 Introduction	47
	5.2 The Fundamental Theorem of Finance	48
	5.3 Bounds on the Values of Contingent Claims	49
	5.4 The Extension	52
	5.5 Uniqueness of the Valuation Functional	54
	5.6 Notes	55
6	State Prices and Risk-Neutral Probabilities	56
	6.1 Introduction	56
	6.2 State Prices	56
	6.3 Farkas–Stiemke Lemma	59
	6.4 Diagrammatic Representation	60
	6.5 State Prices and Value Bounds	60
	6.6 Risk-Free Payoffs	61
	6.7 Risk-Neutral Probabilities	61
	6.8 Notes	63
7	Valuation under Portfolio Restrictions	65
	7.1 Introduction	65
	7.2 Payoff Pricing under Short Sales Restrictions	65
	7.3 State Prices under Short Sales Restrictions	67
	7.4 Diagrammatic Representation	70
	7.5 Bid-Ask Spreads	70
	7.6 Notes	73
	Part Three Risk	75
8	Expected Utility	77
	8.1 Introduction	77
	8.2 Expected Utility	77

	<i>Contents</i>	vii
8.3	Von Neumann–Morgenstern	78
8.4	Savage	79
8.5	Axiomatization of State-Dependent Expected Utility	79
8.6	Axiomatization of Expected Utility	80
8.7	Nonexpected Utility	82
8.8	Expected Utility with Two-Date Consumption	83
8.9	Notes	84
9	Risk Aversion	87
9.1	Introduction	87
9.2	Risk Aversion and Risk Neutrality	87
9.3	Risk Aversion and Concavity	88
9.4	Arrow–Pratt Measures of Absolute Risk Aversion	90
9.5	Risk Compensation	90
9.6	The Pratt Theorem	92
9.7	Decreasing, Constant, and Increasing Risk Aversion	94
9.8	Relative Risk Aversion	94
9.9	Utility Functions with Linear Risk Tolerance	95
9.10	Risk Aversion with Two-Date Consumption	96
9.11	Notes	97
10	Risk	99
10.1	Introduction	99
10.2	Greater Risk	99
10.3	Uncorrelatedness, Mean Independence, and Independence	100
10.4	A Property of Mean Independence	101
10.5	Risk and Risk Aversion	101
10.6	Greater Risk and Variance	104
10.7	A Characterization of Greater Risk	106
10.8	Notes	108
	Part Four Optimal Portfolios	109
11	Optimal Portfolios with One Risky Security	111
11.1	Introduction	111
11.2	Portfolio Choice and Wealth	111
11.3	Optimal Portfolios with One Risky Security	112
11.4	Risk Premium and Optimal Portfolios	114
11.5	Optimal Portfolios When the Risk Premium Is Small	116
11.6	Notes	117
12	Comparative Statics of Optimal Portfolios	118
12.1	Introduction	118
12.2	Wealth	118
12.3	Expected Return	120
12.4	Risk	122

viii	<i>Contents</i>	
	12.5 Optimal Portfolios with Two-Date Consumption	123
	12.6 Notes	126
13	Optimal Portfolios with Several Risky Securities	128
	13.1 Introduction	128
	13.2 Optimal Portfolios	128
	13.3 Risk–Return Trade-Off	129
	13.4 Optimal Portfolios under Fair Pricing	130
	13.5 Risk Premia and Optimal Portfolios	131
	13.6 Optimal Portfolios under Linear Risk Tolerance	133
	13.7 Optimal Portfolios with Two-Date Consumption	136
	13.8 Notes	136
	Part Five Equilibrium Prices and Allocations	139
14	Consumption-Based Security Pricing	141
	14.1 Introduction	141
	14.2 Risk-Free Return in Equilibrium	141
	14.3 Expected Returns in Equilibrium	142
	14.4 Volatility of Marginal Rates of Substitution	144
	14.5 A First Pass at the CAPM	145
	14.6 Notes	146
15	Complete Markets and Pareto-Optimal Allocations of Risk	147
	15.1 Introduction	147
	15.2 Pareto-Optimal Allocations	147
	15.3 Pareto-Optimal Equilibria in Complete Markets	149
	15.4 Complete Markets and Options	150
	15.5 Pareto-Optimal Allocations under Expected Utility	151
	15.6 Pareto-Optimal Allocations under Linear Risk Tolerance	153
	15.7 Notes	155
16	Optimality in Incomplete Security Markets	157
	16.1 Introduction	157
	16.2 Constrained Optimality	157
	16.3 Effectively Complete Markets	158
	16.4 Equilibria in Effectively Complete Markets	159
	16.5 Effectively Complete Markets with No Aggregate Risk	162
	16.6 Effectively Complete Markets with Options	163
	16.7 Effectively Complete Markets with Linear Risk Tolerance	163
	16.8 Multifund Spanning	165
	16.9 A Second Pass at the CAPM	166
	16.10 Notes	166
	Part Six Mean-Variance Analysis	169
17	The Expectations and Pricing Kernels	171
	17.1 Introduction	171

	<i>Contents</i>	<i>ix</i>
17.2	Hilbert Spaces and Inner Products	171
17.3	The Expectations Inner Product	172
17.4	Orthogonal Vectors	172
17.5	Orthogonal Projections	173
17.6	Diagrammatic Methods in Hilbert Spaces	175
17.7	Riesz Representation Theorem	176
17.8	Construction of the Riesz Kernel	177
17.9	The Expectations Kernel	178
17.10	The Pricing Kernel	179
17.11	Notes	181
18	The Mean-Variance Frontier Payoffs	183
18.1	Introduction	183
18.2	Mean-Variance Frontier Payoffs	183
18.3	Frontier Returns	184
18.4	Zero-Covariance Frontier Returns	189
18.5	Beta Pricing	190
18.6	Mean-Variance Efficient Returns	191
18.7	Volatility of Marginal Rates of Substitution	191
18.8	Notes	193
19	Capital Asset Pricing Model	194
19.1	Introduction	194
19.2	Security Market Line	194
19.3	Mean-Variance Preferences	197
19.4	Equilibrium Portfolios under Mean-Variance Preferences	199
19.5	Quadratic Utilities	200
19.6	Normally Distributed Payoffs	201
19.7	Notes	201
20	Factor Pricing	204
20.1	Introduction	204
20.2	Exact Factor Pricing	204
20.3	Exact Factor Pricing, Beta Pricing, and the CAPM	206
20.4	Factor Pricing Errors	207
20.5	Factor Structure	208
20.6	Mean-Independent Factor Structure	210
20.7	Options as Factors	212
20.8	Notes	214
	Part Seven Multidate Security Markets	217
21	Equilibrium in Multidate Security Markets	219
21.1	Introduction	219
21.2	Uncertainty and Information	219
21.3	Multidate Security Markets	222

x	<i>Contents</i>	
	21.4 The Asset Span	223
	21.5 Agents	224
	21.6 Portfolio Choice and the First-Order Conditions	224
	21.7 General Equilibrium	225
	21.8 Notes	226
22	Multidate Arbitrage and Positivity	228
	22.1 Introduction	228
	22.2 Law of One Price and Linearity	228
	22.3 Arbitrage and Positive Pricing	229
	22.4 One-Period Arbitrage	230
	22.5 Positive Equilibrium Pricing	230
	22.6 Notes	232
23	Dynamically Complete Markets	233
	23.1 Introduction	233
	23.2 Dynamically Complete Markets	233
	23.3 Binomial Security Markets	234
	23.4 Event Prices in Dynamically Complete Markets	235
	23.5 Event Prices in Binomial Security Markets	236
	23.6 Equilibrium in Dynamically Complete Markets	237
	23.7 Pareto-Optimal Equilibria	238
	23.8 Notes	239
24	Valuation	241
	24.1 Introduction	241
	24.2 The Fundamental Theorem of Finance	241
	24.3 Uniqueness of the Valuation Functional	244
	24.4 Notes	244
	Part Eight Martingale Property of Security Prices	245
25	Event Prices, Risk-Neutral Probabilities, and the Pricing Kernel	247
	25.1 Introduction	247
	25.2 Event Prices	247
	25.3 Risk-Free Return and Discount Factors	250
	25.4 Risk-Neutral Probabilities	250
	25.5 Expected Returns under Risk-Neutral Probabilities	252
	25.6 Risk-Neutral Valuation	253
	25.7 Value Bounds	254
	25.8 The Pricing Kernel	255
	25.9 Notes	256
26	Security Gains as Martingales	258
	26.1 Introduction	258
	26.2 Gain and Discounted Gain	258

	<i>Contents</i>	xi
26.3	Discounted Gains as Martingales	260
26.4	Gains as Martingales	261
26.5	Notes	262
27	Conditional Consumption-Based Security Pricing	264
27.1	Introduction	264
27.2	Expected Utility	264
27.3	Risk Aversion	265
27.4	Conditional Covariance and Variance	266
27.5	Consumption-Based Security Pricing	266
27.6	Security Pricing under Time Separability	268
27.7	Volatility of Intertemporal Marginal Rates of Substitution	269
27.8	Notes	269
28	Conditional Beta Pricing and the CAPM	271
28.1	Introduction	271
28.2	Two-Date Security Markets at a Date- t Event	271
28.3	Conditional Beta Pricing	272
28.4	Conditional CAPM with Quadratic Utilities	274
28.5	Multidate Market Return	275
28.6	Conditional CAPM with Incomplete Markets	276
28.7	Notes	276
	<i>Index</i>	277