PRINCIPLES OF FINANCIAL ECONOMICS

The subfield of financial economics is generally understood to be a branch of microeconomic theory and, more broadly, of general equilibrium theory. Finance methods are increasingly used to analyze problems involving time and uncertainty in such fields as monetary and environmental economics.

This book introduces graduate students in economics to the subfield. It stresses the link between financial economics and equilibrium theory, devoting less attention to purely financial topics such as valuation of derivatives. Because students often find this link hard to grasp, the treatment aims to make the connection explicit in each stage of the exposition. Emphasis is placed on detailed study of two-date models because almost all of the key ideas in financial economics can be developed in the two-date setting. The analysis aims to be comparable in rigor to the best work in microeconomics; at the same time, the authors provide enough discussion and examples to make the ideas readily understandable.

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Foreword

For quite some time there has been a need for a modern treatment of the principles of finance suitable for the beginning Ph.D. graduate student. This rigorous, thoughtfully constructed, and thoroughly excellent text more than fills the bill. The ideas are carefully developed in a discrete time setting with both rigor and intuition – no easy pedagogic task since it requires an uncommonly sensitive balance between the Scylla of sterile formalism and the Charybdis of blowsy chat. The decision to work in a strictly discrete time setting is entirely appropriate for a beginning course and is to be much applauded; the seeming ease of analysis in continuous time comes at the expense of a deeper mathematical foundation that can obscure the underlying economic principles.

The order of development is natural. The text begins with the cornerstone of modern finance, the absence of arbitrage and the implications of this absence for pricing. It moves on to a careful development of risk and utility theory and from there goes naturally into the basic portfolio problem. Spanning and completeness are given a thorough airing and the authors share my view of the importance of this topic, which is seldom treated in the fullness it deserves. When the mean variance analysis and the CAPM are introduced, starting with a very nice tutorial on the Hilbert space approach, the student is ready for it as a special case of the more general development that has preceded. The beginning student will no doubt be pleasantly surprised by both the elegance and the generality of the structure, and this is all to the good; too many students of economics who have only a passing familiarity with finance fail to appreciate the value of the mean variance analysis. Intertemporal analysis and the martingale pricing approach ends the text, leaving the student perfectly positioned for advanced pricing courses and for the empirical world of modern finance.

The student who has gone through a course based on this book has been exposed to all of the tools and ideas necessary for a solid foundation in finance. From here it is an easy step to continuous-time valuation methods and that step will now seem very natural and make the ensuing mathematical requirements seem natural as well. The student is also well prepared to take advanced courses in empirical finance and in corporate finance. This tree structure beginning with a basic theory course and branching into subsequent courses in advanced pricing theory, i.e., continuous-time finance, empirical finance, and corporate finance, is emerging as the preferred model for Ph.D. sequences in finance.

This book is perfectly positioned for the first course of such a sequence. In fact, I teach such a course at MIT, and I have been searching in vain for an appropriate textbook. I will be assigning LeRoy and Werner in my class this term with enthusiasm and with a sigh of relief.

Stephen A. Ross
May 2000

Preface

Financial economics plays a far more prominent role in the training of economists than it did even a few years ago. This change is generally attributed to the parallel transformation in financial markets that has occurred in recent years. Assets worth trillions of dollars are traded daily in markets for derivative securities, such as options and futures, that hardly existed a decade ago. However, the importance of these changes is less obvious than the changes themselves. Insofar as derivative securities can be valued by arbitrage, such securities only duplicate primary securities. For example, to the extent that the assumptions underlying the Black–Scholes model of option pricing (or any of its more recent extensions) are accurate, the entire options market is redundant because by assumption the payoff of an option can be duplicated using stocks and bonds. The same argument applies to other derivative securities markets. Thus it is arguable that the variables that matter most – consumption allocations – are not greatly affected by the change in financial markets. Along these lines one would no more infer the importance of financial markets from their volume of trade than one would make a similar argument for supermarket clerks or bank tellers based on the fact that they handle large quantities of cash.

In questioning the appropriateness of correlating the expanding role of finance theory to the explosion in derivatives trading, we are in the same position as the physicist who demurs when journalists express the opinion that Einstein's theories are important because they led to the development of television. Similarly, in his appraisal of John Nash's contributions to economic theory, Myerson [13] protested the tendency of journalists to point to the FCC bandwidth auctions as indicating the importance of Nash's work. At least to those curious about the physical and social sciences, Einstein's and Nash's work has a deeper importance than television and the FCC auctions! The same is true of finance theory: its increasing prominence has little to do with the expansion of derivatives markets, which, in any case, owes more to developments in telecommunications and computing than to finance theory.

A more plausible explanation for the expanded role of financial economics is found in the rapid development of the field itself. A generation ago, finance theory was little more than institutional description combined with practitioner-generated rules of thumb that had little analytical basis and, for that matter, little validity. Financial economists agreed that, in principle, security prices ought to be amenable to analysis using serious economic theory. In practice, however, most did not devote much effort to developing economics in this direction.

Today, in contrast, financial economics is increasingly occupying center stage in the economic analysis of problems that involve both time and uncertainty. Many of the problems formerly studied using nonfinance methods now are seen as finance topics. The term structure of interest rates is a good example: formerly this was a topic in monetary economics; now it is a topic in finance. There can be little doubt that the quality of the analysis has improved immensely as a result of this change. Increasingly finance methods are used to analyze problems beyond those involving securities prices or portfolio selection, particularly when these involve both time and uncertainty. An example is the real options literature, in which finance tools initially developed for the analysis of options are applied to areas like environmental economics. Such areas do not deal with options per se, but do involve problems to which the idea of an option is very much relevant.

Financial economics lies at the intersection of finance and economics. The two disciplines are different culturally, more so than one would expect given their substantive similarity. Partly this reflects the fact that finance departments are in business schools and are oriented toward finance practitioners, whereas economics departments typically are in liberal arts divisions of colleges and universities and usually are not oriented toward any single nonacademic community. From the perspective of economists starting out in finance, the most important difference is that finance scholars typically use continuous-time models, whereas economists use discrete-time models. Students notice that continuous-time finance is much more difficult mathematically than discrete-time finance, leading them to ask why finance scholars prefer it. The question is seldom discussed. Certainly product differentiation is part of the explanation, and the possibility that entry deterrence plays a role cannot be dismissed. However, for the most part the preference of finance scholars for continuous-time methods is because the problems most distinctively financial rather than economic – valuation of derivative securities, for example – are best handled using continuous-time methods. The technical reason relates to the effect of risk aversion on equilibrium security prices in models of financial markets. In many settings risk aversion is most conveniently handled by imposing a certain distortion on the probability measure used to value payoffs. Under very weak restrictions, in continuous time the distortion affects the drifts of the stochastic processes characterizing the evolution of security prices, but not their volatilities

(Girsanov's theorem). This is evident in the derivation of the Black-Scholes option pricing formula.

In contrast, it is easy to show using examples that in discrete-time models distorting the underlying measure affects volatilities as well as drifts. Furthermore, given that the effect disappears in continuous time, the effect in discrete time is second-order in the length of time interval. The presence of these higher-order terms often makes the discrete-time versions of valuation problems intractable. It is far easier to perform the underlying analysis in continuous time, even when one must ultimately discretize the resulting partial differential equations in order to obtain numerical solutions. For serious students of finance, the conclusion from this is that there is no escape from learning continuous-time methods, however difficult they may be.

Despite this, the appropriate place to begin is with discrete-time and discrete-state models – the maintained framework in this book – where the economic ideas can be discussed in a setting that requires mathematical methods that are standard in economic theory. For most of this book (Parts One–Six) we assume that there is one time interval (two dates) and a single consumption good. This setting is most suitable for the study of the relation between risk and return on securities and the role of securities in allocation of risk. In the remaining parts (Parts Seven–Eight), we assume that there are multiple dates (a finite number). The multidate model allows for gradual resolution of uncertainty and retrading of securities as new information becomes available.

A little more than ten years ago the beginning student in doctoral-level financial economics had no alternative but to read journal articles. There are several obvious disadvantages to such sources. The ideas are not presented systematically, so that authors typically presuppose, often unrealistically, that the reader already understands prior material. Alternatively, familiar material may be reviewed, often in painful detail. Furthermore, typically notation varies from one article to the next. The inefficiency of this process is evident.

Now the situation is the reverse: there are about a dozen excellent books that can serve as texts in introductory courses in financial economics. Books that have an orientation similar to ours include Krouse [9], Milne [12], Ingersoll [8], Huang and Litzenberger [5], Pliska [16], and Ohlson [15]. Books that are oriented more toward finance specialists, and therefore include more material on valuation by arbitrage and less material on equilibrium considerations, include Hull [7], Dothan [3], Baxter and Rennie [1], Wilmott, Howison, and DeWynne [18], Nielsen [14], and Shiryaev [17]. Of these, Hull emphasizes the practical use of continuous-finance tools rather than their mathematical justification. Wilmott, Howison, and DeWynne approach continuous-time finance via partial differential equations rather than through risk-neutral probabilities, which has some advantages and some disadvantages. Baxter and Rennie give an excellent intuitive presentation of the mathematical ideas of continuous-time finance, but do not discuss the economic ideas at length. Campbell, Lo, and MacKinlay [2] stress empirical and econometric issues. The most authoritative text is Duffie [4]. However, because Duffie presumes a very thorough mathematical preparation, that source may not be the place to begin.

Several excellent books exist on subjects closely related to financial economics such as the introductions to the economics of uncertainty by Laffont [10], and Hirshleifer and Riley [6]. Magill and Quinzii [11] is a fine exposition of the economics of incomplete markets in a more general setting than that adopted here.

Our opinion is that none of the finance books cited above adequately emphasizes the connection between financial economics and general equilibrium theory, or sets out the major ideas in the simplest and most direct way possible. We attempt to do so. However, we understand that some readers have a different orientation. For example, finance practitioners often have little interest in making the connection between security pricing and general equilibrium, and therefore want to proceed to continuous-time finance by the most direct route possible. Such readers might do better to begin with studies other than ours.

This book is based on material used in the introductory finance field sequence in the economics departments of the University of California, Santa Barbara, the University of Minnesota, and the Carlson School of Management at the University of Minnesota. The second author has also taught material from this book at Pompeu Fabra University and the University of Bonn. At the University of Minnesota the book is now the basis for a two-semester sequence, and at the University of California, Santa Barbara, it is the basis for a one-quarter course. In a one-quarter course it is unrealistic to expect that students will master the material; rather, the intention is to introduce the major ideas at an intuitive level. Students writing dissertations in finance typically sit in on the course again in years following the year they take it for credit, at which time they digest the material more thoroughly. It is not obvious which method of instruction is more efficient.

Our students have had good preparation in doctoral-level microeconomics, but have not had enough experience with economics to have developed strong intuitions about how economic models work. Typically they have had no previous exposure to finance or the economics of uncertainty. When that has been the case we have encouraged them to read undergraduate-level finance texts and the introductions to the economics of uncertainty cited above. Rather than emphasizing technique, we have tried to discuss results so as to enable students to develop intuition.

After some hesitation we decided to adopt a theorem-proof expository style. A less formal writing style might make the book more readable, but it would also make it more difficult for us to achieve the level of analytical precision that we believe is appropriate in a book such as this. We have provided examples wherever appropriate. However, readers will find that they will assimilate the material best if they make up their own examples. The simple models we consider lend themselves well to numerical solution using *Mathematica* or *Mathcad*. Although not strictly necessary, it is a good idea for readers to develop facility with methods for numerical solution of these models.

We are painfully aware that the placid financial markets modeled in these pages bear little resemblance to the turbulent markets one reads about in the *Wall Street Journal*. Furthermore, attempts to test empirically the models described in these pages have not had favorable outcomes. There is no doubt that much is missing from these models; the question is how to improve them. There is little consensus on the best method, so we restrict our attention to relatively elementary and noncontroversial material. We believe that when improved models come along, the themes discussed here – allocation and pricing of risk – will still play a central role. We hope that readers of this volume will be in a good position to develop these improved models.

We wish to acknowledge conversations about these ideas with many of our colleagues at the University of California, Santa Barbara, and the University of Minnesota. Jack Kareken read successive drafts of parts of this book and made many valuable comments. The book has benefited enormously from his attention. However, we do not entertain any illusions that he believes our writing is as clear as it could and should be. Our greatest debt is to several generations of Ph.D. students at the University of California, Santa Barbara, and the University of Minnesota. Comments from Alexandre Baptista have been particularly helpful. Students assure us that they enjoy the material and think they benefit from it. Remarkably, the assurances continue even after grades have been recorded and dissertations signed. Our students have repeatedly and with evident pleasure accepted our invitations to point out errors in the text. We are grateful for these corrections. Several ex-students, we are pleased to report, have gone on to make independent contributions to the body of material introduced here. Our hope and expectation is that this book will enable others whom we have not taught to do the same.

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