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As the first advanced textbook on stellar astrophysics for nearly three decades, this long-awaited volume provides a thorough introduction for graduate students and an up-to-date review for researchers.

Advanced Stellar Astrophysics

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William K. Rose University of Maryland College Park



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## Preface

Remarkable progress in understanding stellar phenomena has occurred in recent decades. This textbook discusses in some detail those equations and physical processes that are of greatest relevance to stellar interiors and atmospheres and closely related astrophysics. Motivation for writing this book came from my own research interests and also from teaching graduate astrophysics courses, especially a course on stellar interiors at the University of Maryland. Although the text emphasizes physical principles, astronomical results and unresolved issues are also described.

Introductory material on the history of stellar astrophysics, astronomical observations, star formation and stellar evolution are given in Chapter 1, which also contains a discussion of spectroscopic binaries. Differences between single and binary star evolution have explained a number of interesting observations that are described further in later chapters.

Stellar interiors is one of the most fundamental subjects in astrophysics. Although complicated physical processes are decisive in explaining some predictions of stellar model calculations, the basic principles of stellar interiors do not require a comprehensive knowledge of them. Chapter 2 gives an introductory discussion of the physics and equations of stellar interiors. It also includes a short description of numerical methods.

Statistical physics provides the theoretical basis for much of stellar astrophysics. In Chapter 3 those aspects of statistical physics that are of greatest relevance are developed in some detail. Stellar opacities play a vital role in interpreting observations. Absorption processes are described in Chapter 4. This latter chapter includes self-contained discussions of bound-bound absorption by hydrogenic ions, bound-free absorption and free-free absorption. The basic principles of stellar atmospheres and a discussion of stellar winds are presented in Chapter 5.

Thermonuclear reactions generate most of the energy that is radiated from the photospheres of stars. They also synthesize most elements. Much of the research effort in stellar atmospheres and interiors is motivated by determining abundances and providing theoretical explanations for observations. Chapter 6 discusses low-energy nuclear reactions, some important concepts of nuclear physics and nucleosynthesis. Weak interactions are described in Chapter 7.

Although most stars are in hydrostatic equilibrium, important classes of stars such as Cepheids, RR Lyrae variables and long-period variables are unstable to radial pulsations, whereas some other stars are unstable to nonradial oscillations. Asymptotic-giant-branch

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stars become thermally unstable. Type II supernovae are caused by dynamical core collapse. Chapter 8 discusses the theory of stellar instabilities and gives a brief description of spherical hydrodynamics.

By definition close binary systems are binary stars in which mass transfer between the two components becomes important. Chapter 9 contains discussions of binary stars and mass accretion. Stellar rotation and the related phenomenon of meridional circulation are also included. Several topics related to stellar magnetic fields are described in Chapter 10.

The end states of stellar evolution are white dwarfs, neutron stars and black holes. White dwarfs, classical novae and supernovae are discussed in Chapter 11. The occurrence of supernovae is closely connected to the Chandrasekhar limit, which is the upper limit to the mass of a white dwarf. General relativity is essential in any treatment of neutron stars and black holes. Chapter 12 contains a concise introduction to general relativity, whereas most of Chapter 13 is concerned with neutron stars and black holes. This latter chapter emphasizes observations as well as theory. X-ray astronomy has played a particularly important role in the development of neutron-star and black hole astrophysics.

The writing of this book was substantially influenced by teaching graduate-level courses and participating on various research efforts. I am very grateful to students who took my stellar astrophysics courses and research collaborators. I also wish to thank J. Hall, S. Lehr and S. Smith for performing the difficult task of typing the manuscript.