

Contents

Preface	ix
I Markovian Dynamical Systems	1
1 General Dynamical Systems	3
1.1 Basic concepts	3
1.2 Ergodic Systems and the Koopman–von Neumann Theorem	5
2 Canonical Markovian Systems	11
2.1 Markovian semigroups	11
2.2 Canonical systems and their continuity	14
3 Ergodic and mixing measures	20
3.1 The Krylov–Bogoliubov existence theorem	20
3.2 Characterizations of ergodic measures	22
3.3 The strong law of large numbers	30
3.4 Mixing and recurrence	33
3.5 Limit behaviour of P_t , $t \geq 0$	39
4 Regular Markovian systems	41
4.1 Regular, strong Feller and irreducible semigroups	41
4.2 Doob’s theorem	43
II Invariant measures for stochastic evolution equations	49

5	Stochastic Differential Equations	51
5.1	Introduction	51
5.2	Wiener and Ornstein–Uhlenbeck processes	52
5.2.1	Stochastic integrals and convolutions	56
5.3	Stochastic evolution equations	65
5.4	Regular dependence on initial conditions and Kolmogorov equations	69
5.4.1	Differentiable dependence on initial datum	69
5.4.2	Kolmogorov equation	70
5.5	Dissipative stochastic systems	71
5.5.1	Generalities about dissipative mappings	72
5.5.2	Existence of solutions for deterministic equations	75
5.5.3	Existence of solutions for stochastic equations in Hilbert spaces	80
5.5.4	Existence of solutions for stochastic equations in Banach spaces	87
6	Existence of invariant measures	89
6.1	Existence from boundedness	89
6.2	Linear systems	96
6.2.1	A description of invariant measures	97
6.2.2	Invariant measures and recurrence	102
6.3	Dissipative systems	104
6.3.1	General noise	105
6.3.2	Additive noise	108
6.4	Genuinely dissipative systems	114
6.5	Dissipative systems in Banach spaces	117
7	Uniqueness of invariant measures	121
7.1	Strong Feller property for non-degenerate diffusions	121
7.2	Strong Feller property for degenerate diffusion	129
7.3	Irreducibility for non-degenerate diffusions	137
7.4	Irreducibility for equations with additive noise	140
8	Densities of invariant measures	147
8.1	Introduction	147
8.2	Sobolev spaces	149
8.3	Properties of the semigroup R_t , $t > 0$, on $L^2(H, \mu)$	153

Contents

vii

8.4	Existence and absolute continuity of the invariant measure of P_t , $t > 0$, with respect to μ	156
8.5	Locally Lipschitz nonlinearities	159
8.6	Gradient systems	160
8.7	Regularity of the density when \mathcal{L} is variational	165
8.8	Further regularity results in the diagonal case	168
III Invariant measures for specific models		175
9	Ornstein–Uhlenbeck processes	177
9.1	Introduction	177
9.2	Ornstein–Uhlenbeck processes of wave type	178
9.2.1	General properties	178
9.2.2	Second order dissipative systems	181
9.2.3	Comments on nonlinear equations	183
9.3	Ornstein–Uhlenbeck processes in finance	184
9.4	Ornstein–Uhlenbeck processes in chaotic environment	186
9.4.1	Cylindrical noise	187
9.4.2	Chaotic noise	194
10	Stochastic delay systems	199
10.1	Introduction	199
10.2	Linear case	200
10.3	Nonlinear equations	203
11	Reaction–Diffusion equations	211
11.1	Introduction	211
11.2	Finite interval. Lipschitz coefficients	213
11.2.1	Existence and uniqueness of solutions	214
11.2.2	Existence and uniqueness of invariant measures	215
11.3	Equations with non–Lipschitz coefficients	217
11.4	Reaction–diffusion equations on d dimensional spaces .	219
12	Spin systems	225
12.1	Introduction	225
12.2	Classical spin systems	227
12.3	Quantum lattice systems	235

13 Systems perturbed through the boundary	241
13.1 Introduction	241
13.2 Equations with non-homogeneous boundary conditions	243
13.3 Equations with Neumann boundary conditions	248
13.4 Ergodic solutions	254
14 Burgers equation	257
14.1 Introduction	257
14.2 Existence of solutions	260
14.3 Strong Feller property	265
14.4 Invariant measure	268
14.4.1 Existence	268
14.4.2 Uniqueness	276
15 Navier–Stokes equations	281
15.1 Preliminaries	281
15.2 Local existence and uniqueness results	284
15.3 A priori estimates and global existence	291
15.4 Existence of an invariant measure	295
IV Appendices	305
A Smoothing properties of convolutions	307
A.1	307
B An estimate on modulus of continuity	311
B.1	311
C A result on implicit functions	317
C.1	317
Bibliography	321