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0521565839 - Relativistic Quantum Mechanics: With Applications in Condensed Matter and Atomic Physics
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This graduate text introduces relativistic quantum theory, emphasizing its important applications in condensed matter physics.

Basic theory, including special relativity, angular momentum and particles of spin zero are first reprised. The text then goes on to discuss the Dirac equation, symmetries and operators, and free particles. Physical consequences of solutions including hole theory and Klein's paradox are considered. Several model problems are solved. Important applications of quantum theory to condensed matter physics then follow. Relevant theory for the one-electron atom is explored. The theory is then developed to describe the quantum mechanics of many electron systems, including Hartree–Fock and density functional methods. Scattering theory, band structures, magneto-optical effects and superconductivity are amongst other significant topics discussed. Many exercises and an extensive reference list are included.

This clear account of relativistic quantum theory will be valuable to graduate students and researchers working in condensed matter physics and quantum physics.

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Relativistic Quantum Mechanics

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RELATIVISTIC QUANTUM MECHANICS

WITH APPLICATIONS IN CONDENSED MATTER AND ATOMIC PHYSICS

Paul Strange

Keele University

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Dedication

Unlike so many other supportive families, mine did not proof-read, or type this manuscript, or anything else. In fact they played absolutely no part in the preparation of this book, and distracted me from it at every opportunity. They are not the slightest bit interested in physics and know nothing of relativity and quantum theory. Their lack of knowledge in these areas does not worry them at all. Furthermore they undermine one of the central tenets of the theory of relativity, by providing me with a unique frame of reference. Nonetheless, I would like to dedicate this book to them, Jo, Jessica, Susanna and Elizabeth.

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Preface

I always thought I would write a book and this is it. In the end, though, I hardly wrote it at all, it evolved from my research notes, from essays I wrote for postgraduates starting work with me, and from lecture handouts I distribute to students taking the relativistic quantum mechanics option in the Physics department at Keele University. Therefore the early chapters of this book discuss pure relativistic quantum mechanics and the later chapters discuss applications of relevance in condensed matter physics. This book, then, is written with an audience ranging from advanced students to professional researchers in mind. I wrote it because anyone aiming to do research in relativistic quantum theory applied to condensed matter has to pull together information from a wide range of sources using different conventions, notation and units, which can lead to a lot of confusion (I speak from experience). Most relativistic quantum mechanics books, it seems to me, are directed towards quantum field theory and particle physics, not condensed matter physics, and many start off at too advanced a level for present day physics graduates from a British university. Therefore, I have tried to start at a sufficiently elementary level, and have used the SI system of units throughout.

When I started preparing this book I thought I might be able to write everything I knew in around fifty pages. It soon became apparent that that was not the case. Indeed it now appears to me that the principal decisions to be taken in writing a book are about what to omit. I have written this much quantum mechanics and not used the word Lagrangian. This saddens me, but surely must make me unique in the history of relativistic quantum theory. I have not discussed the very interesting quantum mechanics describing the neutrino and its helicity, another topic that invariably appears in other relativistic quantum mechanics texts. However, as we are leaning towards condensed matter physics in this book, there are sections on topics such as magneto-optical effects and

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magnetic anisotropy which don't appear in other books despite being intrinsically relativistic and quantum mechanical in nature. In the end, what is included and what is omitted is just a question of taste, and it is up to the reader to decide whether such decisions were good or bad.

This book is very mathematical, containing something like two thousand equations. I make no apology for that. I think the way the mathematics works is the great beauty of the subject. Throughout the book I try to make the mathematics clear, but I do not try to avoid it. Paraphrasing Niels Bohr I believe that "If you can't do the maths, you don't understand it." If you don't like maths, you are reading the wrong book.

There are a lot of people I would like to thank for their help with, and influence on, my understanding of quantum mechanics, particularly the relativistic version of the theory. They are Dr E. Arola, Dr P.J. Durham, Professor H. Ebert, Professor W.M. Fairbairn, Professor J.M.F. Gunn, Prof B.L. Györffy, Dr R.B. Jones, Dr P.M. Lee, Dr J.B. Staunton, Professor J.G. Valatin, and Dr W. Yeung.

Several of the examples and problems in this book stem from projects done by undergraduate students during their time at Keele, and from the work of my Ph.D students. Thanks are also due to them, C. Blewitt, H.J. Gotsis, O. Gratton, A.C. Jenkins, P.M. Mobit, and E. Pugh, and to the funding agencies who supported them (Keele University physics department, the EPSRC, and the Nuffield foundation).

There are several other people I would like to thank for their general influence, encouragement and friendship. They are Dr T. Ellis, Professor M.J. Gillan, Dr M.E. Hagen, Dr P.W. Haycock, Mr J. Hodgeson and Mr B.G. Locke-Scobie. I would also like to thank R. Neal and L. Nightingale of Cambridge University Press for their encouragement of, and patience with, me. Finally, my parents do not have a scientific background, nonetheless they have always supported me in my education and have taken a keen interest in the writing of this book. Thanks are also due to them, R.J. and V.A. Strange.

I hope you enjoy this book, although I am not sure 'enjoy' is the right word to describe the feeling one has when reading a quantum mechanics textbook. Perhaps it would be better to say that I hope you find this book informative and instructive. What I would really like would be for you to be inspired to look deeper into the subject, as I was by my undergraduate lectures many years ago. Many people think quantum mechanics is not relevant to everyday life, but it has certainly influenced my life for the better! I hope it will do the same for you.

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