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978-0-521-56511-0 - Topics in Advanced Econometrics: Estimation, Testing, and Specification of Cross-Section and Time Series Models

Herman J. Bierens

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In this book Herman Bierens provides a mathematically rigorous treatment of a number of timely topics in advanced econometrics. His subjects include nonlinear estimation, maximum likelihood theory, ARMA and ARMAX models, unit roots and cointegration, and nonparametric regression, together with an extensive and thorough treatment of the necessary probability theory. The book is uniquely self-contained, providing the reader with a selection of the latest developments in econometric theory, plus the required introductory material on each topic. It will be used by graduate students of econometrics and statistics, and is particularly suitable for self-tuition.

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Topics in advanced econometrics

**Estimation, testing, and specification of
cross-section and time series models**

HERMAN J. BIERENS

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CAMBRIDGE UNIVERSITY PRESS

Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore, São Paulo, Delhi

Cambridge University Press

The Edinburgh Building, Cambridge CB2 8RU, UK

Published in the United States of America by Cambridge University Press, New York

www.cambridge.org

Information on this title: www.cambridge.org/9780521419000

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First published 1994

First paperback edition published 1996

A catalogue record for this publication is available from the British Library

Library of Congress Cataloguing in Publication data

Bierens, Herman J., 1943–

Topics in advanced econometrics / Herman J. Bierens.

p. cm.

Includes bibliographical references.

ISBN 0 521 41900 X (hard)

I. Econometrics. I. Title.

HB139.B533 1993

330'.01'5195–dc20 92-47068 CIP

ISBN 978-0-521-41900-0 hardback

ISBN 978-0-521-56511-0 paperback

Transferred to digital printing 2008

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Preface

This book covers topics in advanced econometrics that I have taught in graduate econometrics programs of the University of California at San Diego, Southern Methodist University, Dallas, the Netherlands Network of Quantitative Economics, Tinbergen Institute, and the Free University, Amsterdam. The selection of the topics is based on my personal interest in the subjects, as well as lack of availability of suitable textbooks in these areas.

Rather than providing an encyclopedic survey of the literature, I have chosen a presentation which fills the gap between intermediate statistics and econometrics (including linear time series analysis) and the level necessary to gain access to the recent econometric literature; in particular, the literature on nonlinear and nonparametric regression, and advanced time series analysis. The ultimate goal is to provide the student with tools for independent research in these areas. This book is particularly suitable for self-tuition, and may prove useful in a graduate course in mathematical statistics and advanced econometrics.

The first four chapters contain enough material to fill a half-semester graduate course in asymptotic theory and nonlinear inference if one skips some of the material involved, and a full semester course if not. In teaching such a half-semester course I usually skip the details of the proofs in chapter 2, and focus on the relations between the various modes of convergence only. Also, I usually skip the sections of chapter 2 and chapter 4 dealing with non-identically distributed samples, and only sketch the proof of the uniform law of large numbers for the i.i.d. case (theorem 2.5.7). Section 3.3 is related to chapter 5, so if you skip chapter 5 then you may skip section 3.3 as well.

Chapter 5 requires knowledge of the material in section 3.3 and chapter 4. I have used this chapter in a short (12 hours) graduate course on model misspecification testing.

Chapters 6, 7 and 8 form a unity, and I have used them in a half-semester course on (non)linear time series modeling, without the sections on heterogeneous time series. Chapters 1–3 are prerequisite for this course.

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I have used the parts of chapter 9 on unit roots for a short (12 hours) topics course. Chapter 1 is prerequisite for this course, together with an understanding of the α -mixing concept explained in chapter 6.

Finally, chapter 10 stands on its own, except section 10.7 which requires knowledge of the α -mixing and ν -stability concepts and related convergence results in chapter 6. I have used this chapter, without section 10.7, in a short (12 hours) graduate course on nonparametric estimation.

The first five chapters of this book have been disseminated in draft form as working papers. I am grateful to Anil Bera, Alexander Georgiev, and Jan Magnus for suggesting additional references, and to Lourens Broersma, Shengyi Guo, Helmut Lütkepohl, Johan Smits, Ton Steerneman and Baoyuan Wang, for suggesting various improvements.

Most of the work on this book was done while affiliated with the Free University, Amsterdam, and the sections on maximum likelihood and cointegration were written while enjoying the hospitality of the Center for Economic Research, Tilburg University.