CONTINUUM MECHANICS IN THE EARTH SCIENCES

Continuum mechanics lies at the heart of studies into many geological and geophysical phenomena, from earthquakes and faults to the fluid dynamics of the Earth. Based on the author's extensive teaching and research experience, this interdisciplinary book provides geoscientists, physicists and applied mathematicians with a class-tested, accessible overview of continuum mechanics.

Starting from thermodynamic principles and geometrical insights, the book surveys solid, fluid, and gas dynamics. In later review chapters, it explores new aspects of the field emerging from nonlinearity and dynamical complexity, and provides a brief introduction to computational modeling. Simple, yet rigorous, derivations are used to review the essential mathematics, assuming an undergraduate geophysics, physics, engineering or mathematics background. The author emphasizes the full three-dimensional geometries of real-world examples, enabling students to apply these in deconstructing solid-Earth and planet-related problems.

Problem sets and worked examples are provided, making this a practical resource for graduate students in geophysics, planetary physics and geology, as well as a beneficial tool for professional scientists seeking a better understanding of the mathematics and physics underlying problems in the Earth sciences.

William I. Newman is a Professor in the Department of Earth and Space Sciences as well as the Departments of Physics and Astronomy, and Mathematics at UCLA, where he has taught for over 30 years. A unifying feature of his research is the role of chaos and complexity in nature, in applications ranging from geophysics to astrophysics as well as mathematical, statistical, and computational modeling in condensed matter physics, climate dynamics, and theoretical biology. Professor Newman is a former member of the Institute for Advanced Study in Princeton, a John Simon Guggenheim Memorial Foundation Fellow, and has held appointments as the Stanislaw Ulam Distinguished Scholar at the Center for Nonlinear Studies, Los Alamos National Laboratory, and as the Morris Belkin Visiting Professor in Computational and Applied Mathematics at the Weizmann Institute of Science, Israel. For six months he also worked with colleagues in the Russian Academy of Sciences in Moscow on modeling earthquake events. He has served in an editorial capacity for the *Journal of Geophysical Research* and *Nonlinear Processes in Geophysics*, and has held elective office as chair of the Division of Dynamical Astronomy of the American Astronomical Society.

CONTINUUM MECHANICS IN THE EARTH SCIENCES

WILLIAM I. NEWMAN

CAMBRIDGE
UNIVERSITY PRESS

CAMBRIDGE UNIVERSITY PRESS
Cambridge, New York, Melbourne, Madrid, Cape Town,
Singapore, São Paulo, Delhi, Mexico City

Cambridge University Press
The Edinburgh Building, Cambridge CB2 8RU, UK

Published in the United States of America by Cambridge University Press, New York

www.cambridge.org
Information on this title: www.cambridge.org/9780521562898

First published 2012

Printed in the United Kingdom at the University Press, Cambridge

A catalog record for this publication is available from the British Library

Library of Congress Cataloging in Publication data

ISBN 978-0-521-56289-8 Hardback

To my children, Ted and Jenny.

Cambridge University Press
978-0-521-56289-8 - Continuum Mechanics in the Earth Sciences
William I. Newman
Frontmatter
More information

Contents

	Preface	*page* ix
	Acknowledgements	xii
1	**Some mathematical essentials**	1
	1.1 Scalars, vectors, and Cartesian tensors	1
	1.2 Matrices and determinants	7
	1.3 Transformations of Cartesian tensors	9
	1.4 Eigenvalues and eigenvectors	12
	1.5 Simplified approach to rotation	16
	1.6 Curvature, torsion, and kinematics	18
	Exercises	23
2	**Stress principles**	27
	2.1 Body and surface forces	27
	2.2 Cauchy stress principle	29
	2.3 Stress tensor	31
	2.4 Symmetry and transformation laws	33
	2.5 Principal stresses and directions	34
	2.6 Solving the cubic eigenvalue equation problem	37
	2.7 Maximum and minimum stress values	39
	2.8 Mohr's circles	42
	2.9 Plane, deviator, spherical, and octahedral stress	44
	Exercises	46
3	**Deformation and motion**	49
	3.1 Coordinates and deformation	49
	3.2 Strain tensor	53
	3.3 Linearized deformation theory	54
	3.4 Stretch ratios	58
	3.5 Velocity gradient	59
	3.6 Vorticity and material derivative	61
	Exercises	64

4 Fundamental laws and equations 67
4.1 Terminology and material derivatives 67
4.2 Conservation of mass and the continuity equation 71
4.3 Linear momentum and the equations of motion 73
4.4 Piola–Kirchhoff stress tensor 74
4.5 Angular momentum principle 75
4.6 Conservation of energy and the energy equation 76
4.7 Constitutive equations 79
4.8 Thermodynamic considerations 82
Exercises 86

5 Linear elastic solids 89
5.1 Elasticity, Hooke's law, and free energy 89
5.2 Homogeneous deformations 94
5.3 Role of temperature 97
5.4 Elastic waves for isotropic bodies 100
5.5 Helmholtz's decomposition theorem 102
5.6 Statics for isotropic bodies 104
5.7 Microscopic structure and dislocations 106
Exercises 109

6 Classical fluids 112
6.1 Stokesian and Newtonian fluids: Navier–Stokes equations 113
6.2 Some special fluids and flows 117
Exercises 129

7 Geophysical fluid dynamics 134
7.1 Dimensional analysis and dimensionless form 134
7.2 Dimensionless numbers 138
Exercises 143

8 Computation in continuum mechanics 147
8.1 Review of partial differential equations 148
8.2 Survey of numerical methods 153

9 Nonlinearity in the Earth 159
9.1 Friction 163
9.2 Fracture 165
9.3 Percolation and self-organized criticality 168
9.4 Fractals 171

References 175
Index 180

Preface

This book is the outcome of an introductory graduate-level course that I have given at the University of California, Los Angeles for a number of years as part of our program in Geophysics and Space Physics. Our program is physics-oriented and draws many of its students from the ranks of undergraduate physics and, sometimes, mathematics majors, in addition to geophysics and occasionally geology majors. Accordingly, this text approaches the subject by promoting a physics-based understanding of the basic principles with a relatively rigorous mathematical approach. Since the needs of this course were rather unique, blending concepts in physics and mathematics with the Earth sciences, I approached teaching the subject by drawing on many sources in developing the necessary material. (Throughout this volume, I refer to materials that provide more complete treatments of the topics which we only have time to overview.) In contrast to other sources, I wanted this course to treat not only classical methods but survey some of the ideas emerging in the geosciences that were drawn directly from current ideas in physics, especially nonlinear dynamics. Over time, the material developed more coherence and my lecture notes for this academic quarter-long course evolved into this text.

The subject of continuum mechanics is predicated on the notion that many natural phenomena have a fundamentally smooth, continuous nature. This constitutes the basis for solid and fluid mechanics, major components of this course. However, there are also some important departures from the continuum assumption which must be reviewed and this is done in the later chapters. Thus, this text surveys a mixture of traditional as well as contemporary topics.

I begin by introducing the mathematical notation, stress principles, and kinematic description that is common to almost all books on this subject. However, I emphasize the three-dimensionality that prevails in many geoscience problems and the techniques needed to accommodate that geometric complexity. The fundamental laws and equations are introduced employing a thermodynamic basis for their derivation, rather than simply assuming an underlying linearity, as is done in most engineering texts. Mathematical derivations are provided where they have proven to be helpful during lecture presentations. Linear elasticity is then introduced, covering many issues encountered in statics and dynamics, including the equations for seismic wave propagation. Exercises are provided at the end of each of these chapters.

Since this graduate course was designed to be given during an academic quarter, rather than a semester, I chose to develop four additional chapters to survey a number of themes that emerge from continuum mechanics that could be introduced if sufficient time were available. (During a semester-long offering, the four added survey chapters can be augmented by readings from the various books and other materials cited.) Because each of these chapters present brief surveys, I preface each of them by citing representative books in the literature that can provide much greater detail. Classical fluid flow is introduced, including some elements of perturbation theory as well as the application of eddy viscosities and the scaling laws often encountered in turbulent flows. Additional attention is placed on geophysical fluid flows and their associated power laws, and we consider how complications in the underlying physics can sometimes break these simple scalings. Since the complexity of these real-world problems often compels us to use the computer to simulate these continua, the penultimate chapter provides a brief survey of some of the current computational methodologies available, so that students pursuing the current literature will have a basic understanding of how numerical results are obtained. Finally, I go on to survey a number of themes germane to nonlinearity in the Earth, beginning with some of the phenomenology of earthquakes, including self-similar scaling and fractals, and some current thinking on models including percolation and self-organized criticality. While not designed to be an in-depth introduction, these additional chapters have been constructed to provide graduate students relatively early in their studies a taste for some of the topics that have grown to be important. At the same time, this material is designed to help them learn enough of the prevailing ideas to be able to benefit from seminars and other venues where these topics are discussed. While the final two chapters do not have exercises provided, they have proven useful in relating some of the more conventional topics associated with continuum mechanics to real geoscience problems and, therefore, help provide an entrée for graduate students into these arenas.

The Earth and space environment provide both an important testing ground for many ideas in continuum mechanics, as well as a cogent need for the development of this subject. Indeed, it is the art of bridging the physics and mathematics of continuum mechanics with their application to the geosciences that poses the greatest challenge and provides the greatest benefits.

While completing this volume, my editors at Cambridge University Press, Susan Francis and Laura Clark, asked me to provide a cover image for this book and recommended that a photograph be adopted, instead of a geometrical design or blank cover as is so often employed in technical books. For this purpose, I selected a photograph that my daughter, Jenny, and I had taken several years ago of Devils Tower in northeastern Wyoming. This picture provides a beautiful illustration of the competition of two very different processes encountered in continuum mechanics in the formation of this remarkable geologic feature. Devils Tower is an igneous intrusion that rises 386 m above the surrounding sedimentary terrain and the Belle Fourche River. As the basalt cooled, the outer surface cooled at a more rapid pace than the flow material inside, and thermal conduction from the interior could not keep up with the exterior cooling and circumferential contraction. Columnar fractures and joints emerged during the cooling process and yielded a set of parallel prismatic columns presenting a hexagonal pattern. A number of aspects relating to the formation of Devils Tower remain controversial. Nevertheless, this American national monument serves as a striking example of the interplay between the different kinds of geology, physics, mathematics, and geometry that define continuum mechanics.

Acknowledgements

I gratefully acknowledge the advice and comments provided by the many students to whom I have had the privilege of teaching this material. Their comments and advice were invaluable. I have benefited as well from the advice of many of my colleagues, notably Donald Turcotte, Andrei Gabrielov, Jonathan Aurnou, Gerald Schubert, Vladimir Keilis-Borok, Gerhard Oertel, Paul Davis, and Peter Bird, among others. I also wish to thank William Kaula and Leon Knopoff, who had a profound influence upon my appreciation for this subject. I want to thank several of our graduate students who took time out to help proofread the manuscript, especially Nathaniel Hamlin, Jon Harrington, Shantanu Naidu, Sebastiano Padovan, and Igor Stubailo. Finally, I wish to thank my editors at Cambridge University Press, Susan Francis and Laura Clark, for their steadfast support.