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Leszek Plaskota
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NOISY INFORMATION AND COMPUTATIONAL COMPLEXITY

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NOISY INFORMATION AND COMPUTATIONAL COMPLEXITY

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To
my wife
Aleksandra,
and our daughters
Kinga,
Klaudia,
and Ola.

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Preface

In the modern world, the importance of information can hardly be overestimated. Information also plays a prominent role in scientific computations. A branch of computational complexity which deals with problems for which information is partial, noisy and priced is called *information-based complexity*.

In a number of information-based complexity books, the emphasis was on partial and exact information. In the present book, the emphasis is on *noisy* information. We consider deterministic and random noise. The analysis of noisy information leads to a variety of interesting new algorithms and complexity results.

The book presents a theory of computational complexity of continuous problems with noisy information. A number of applications is also given. It is based on results of many researchers in this area (including the results of the author) as well as new results not published elsewhere.

This work would not have been completed if I had not received support from many people. My special thanks go to H. Woźniakowski who encouraged me to write such a book and was always ready to offer his help. I appreciate the considerable help of J.F. Traub. I would also like to thank M. Kon, A. Werschulz, E. Novak, K. Ritter and other colleagues for their valuable comments on various portions of the manuscript.

I wish to express my thanks to the Institute of Applied Mathematics and Mechanics at the University of Warsaw, where the book was almost entirely written. Some parts of the book were prepared in the Mathematical Institute of the Erlangen-Nuremberg University, and in the Columbia University Computer Science Department.

And finally, I am pleased to acknowledge the substantial support of my wife Aleksandra, my daughters Kinga, Klaudia and Ola, as well as my whole family, who patiently waited for completion of this work.

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List of symbols

$S : F \to G$	solution operator
$g = S(f)$	exact solution
$E \subset F$	set of problem elements
μ	a priori distribution (measure) on F
$N : F \to Y$	exact information operator
$N(f)$	exact information about f
Δ	precision vector
Σ	correlation matrix
$\mathbb{N} : F \to 2^Y, \mathbb{N} = \{N, \Delta\}$	(noisy) information operator
$\mathbb{III} = \{\pi_f\}_f, \mathbb{III} = \{N, \Sigma\}$	information distribution
$y \in \mathbb{N}(f)$	information about f (deterministic)
$y \sim \pi_f$	information about f (random)
$x = y - N(f)$	information noise
δ, σ^2	noise level (bound and variance)
$\varphi : Y \to G$	algorithm
$\varphi(y)$	approximate solution (approximation)
$e^{wor}(\mathbb{N}, \varphi), e^{wor}(\mathbb{N}, \varphi; E)$	worst case error of the algorithm φ
$e^{ave}(\mathbb{III}, \varphi), e^{ave}(\mathbb{III}, \varphi; \mu)$	average error of φ
$e^{w-a}(\mathbb{III}, \varphi), e^{w-a}(\mathbb{III}, \varphi; E)$	worst-average error of φ
$e^{a-w}(\mathbb{N}, \varphi), e^{a-w}(\mathbb{N}, \varphi; \mu)$	average-worst error of φ
$rad^{wor}(\mathbb{N}), rad^{wor}(\mathbb{N}; E)$	worst case radius of information $\mathbb{N}$
$rad^{ave}(\mathbb{III}), rad^{ave}(\mathbb{III}; \mu)$	average radius of information $\mathbb{III}$

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$\operatorname{diam}(\mathbb{N})$	diameter of information
Λ	class of permissible functionals
$\mathcal{N}_n$	a class of exact information
$r_n^{\mathrm{wor}}(\delta), r_n^{\mathrm{wor}}(\Delta)$	minimal radius (worst case)
$r_n^{\mathrm{ave}}(\sigma^2), r_n^{\mathrm{ave}}(\Sigma)$	minimal radius (average case)
$\mathbf{c}(\delta), \mathbf{c}(\sigma^2)$	cost function
$\mathbf{g}$	cost of operations in G
$\mathrm{R}(T)$	Tth minimal radius
$\mathcal{P}$	program
$\mathrm{cost}^{\mathrm{wor}}(\mathcal{P}), \mathrm{cost}^{\mathrm{ave}}(\mathcal{P})$	cost of executing $\mathcal{P}$
$\mathrm{comp}^{\mathrm{wor}}(\mathbb{N}, \varphi)$	worst case complexity of φ using $\mathbb{N}$
$\mathrm{comp}^{\mathrm{ave}}(\mathbb{\Pi}, \varphi)$	average complexity of φ using $\mathbb{\Pi}$
$\mathrm{IC}^{\mathrm{non}}(\varepsilon)$	(nonadaptive) information ε-complexity
$\mathrm{Comp}^{\mathrm{wor}}(\varepsilon), \mathrm{Comp}^{\mathrm{ave}}(\varepsilon)$	ε-complexity (of a problem)
$\mathbb{R}$	reals
A^*	adjoint operator to A
App	approximation problem
Int	integration problem
$R(\cdot,\cdot)$	reproducing kernel (covariance kernel)
G_N	Gram matrix for exact information N
$\mathrm{tr}(\cdot)$	trace of an operator
$\{H, F\}$	abstract Wiener space
w, w_r	classical and r-fold Wiener measure
$C_\mu : F^* \to F$	correlation operator of the measure μ
$\mu_2(\cdot\|y)$	conditional distribution on$F$
W_r^0	r-fold square integrable functions
C_r^0	r-fold continuous functions
$\mathcal{N}(\cdot,\cdot)$	Gaussian (normal) distribution on $\mathbb{R}^n$
$a_n \asymp b_n$	weak equivalence of sequences
$a(\varepsilon) \asymp b(\varepsilon)$	weak equivalence functions
$a_n \approx b_n$	strong equivalence of sequences
$a(\varepsilon) \approx b(\varepsilon)$	strong equivalence of functions

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