

Physics of Crystal Growth

This text introduces the physical principles of how and why crystals grow.

The first three chapters recall the fundamental properties of crystal surfaces at equilibrium. The next six chapters describe simple models and basic concepts of crystal growth including diffusion, thermal smoothing of a surface, and applications to semiconductors. Following chapters examine more complex topics such as kinetic roughness, growth instabilities, and elastic effects. A brief closing chapter looks back at the crucial contributions of crystal growth in electronics during this century. The book focuses on growth using molecular beam epitaxy. Throughout, the emphasis is on the role played by modern statistical physics. Informative appendices, interesting exercises and an extensive bibliography reinforce the text.

This book will be of interest to graduate students and researchers in statistical physics, materials science, surface physics and solid state physics. It will also be suitable for use as a coursebook at beginning graduate level.

Cambridge University Press
978-0-521-55198-4 - Physics of Crystal Growth
Alberto Pimpinelli and Jacques Villain
Frontmatter
[More information](#)

Cambridge University Press
978-0-521-55198-4 - Physics of Crystal Growth
Alberto Pimpinelli and Jacques Villain
Frontmatter
[More information](#)

**Collection Aléa-Saclay:
Monographs and Texts in Statistical Physics**

General editor: Claude Godrèche

- C. Godrèche (ed.): *Solids far from equilibrium*
P. Peretto: *An introduction to the modeling of neural networks*
C. Godrèche and P. Manneville (eds.): *Hydrodynamics and nonlinear instabilities*
A. Pimpinelli and J. Villain: *Physics of crystal growth*
D. H. Rothman and S. Zaleski: *Lattice-gas cellular automata*
B. Chopard and M. Droz: *Cellular automata modeling of physical systems*

Cambridge University Press
978-0-521-55198-4 - Physics of Crystal Growth
Alberto Pimpinelli and Jacques Villain
Frontmatter
[More information](#)

Cambridge University Press
978-0-521-55198-4 - Physics of Crystal Growth
Alberto Pimpinelli and Jacques Villain
Frontmatter
[More information](#)

Physics of Crystal Growth

Alberto Pimpinelli
Université Blaise Pascal – Clermont-Ferrand II

Jacques Villain
Centre d'Études Nucléaires de Grenoble



Cambridge University Press
 978-0-521-55198-4 - Physics of Crystal Growth
 Alberto Pimpinelli and Jacques Villain
 Frontmatter
[More information](#)

CAMBRIDGE UNIVERSITY PRESS
 Cambridge, New York, Melbourne, Madrid, Cape Town,
 Singapore, São Paulo, Delhi, Tokyo, Mexico City

Cambridge University Press
 The Edinburgh Building, Cambridge CB2 8RU, UK

Published in the United States of America by
 Cambridge University Press, New York

www.cambridge.org
 Information on this title: www.cambridge.org/9780521440677

© Cambridge University Press 1998

This publication is in copyright. Subject to statutory exception
 and to the provisions of relevant collective licensing agreements,
 no reproduction of any part may take place without the written
 permission of Cambridge University Press.

First published 1998

A catalogue record for this publication is available from the British Library

Library of Congress cataloguing in publication data

Pimpinelli, Alberto.

Physics of crystal growth / Alberto Pimpinelli, Jacques Villain.

p. cm. – (Collection Alea – Saclay: 4)

Includes bibliographical references and index.

ISBN 0 521 55198 6. – ISBN 0 521 55855 7 (pbk.)

1. Solids – Surfaces. 2. Crystal growth. 3. Statistical physics.

I. Villain, Jacques. II. Title. III. Series.

QC176.8.S8P56 1997

548'.5 – dc21 96-51772 CIP

ISBN 978-0-521-55198-4 Hardback

ISBN 978-0-521-55855-6 Paperback

Cambridge University Press has no responsibility for the persistence or
 accuracy of URLs for external or third-party internet websites referred to in
 this publication, and does not guarantee that any content on such websites is,
 or will remain, accurate or appropriate. Information regarding prices, travel
 timetables, and other factual information given in this work is correct at
 the time of first printing but Cambridge University Press does not guarantee
 the accuracy of such information thereafter.

Contents

| | |
|--|-----------|
| <i>Preface</i> | xiii |
| <i>List of symbols</i> | xv |
| 1 Morphology of a crystal surface | 1 |
| 1.1 A high-symmetry surface observed with a microscope | 2 |
| 1.2 <i>In situ</i> microscopy and diffraction | 7 |
| 1.3 Step free energy and thermal roughness of a surface | 7 |
| 1.4 The roughening transition | 10 |
| 1.5 Smooth and rough surfaces | 12 |
| 1.6 The SOS model and other models | 15 |
| 1.7 Roughening transition of a vicinal crystal surface | 16 |
| 1.8 The roughening transition: a very weak transition | 17 |
| 1.9 Conclusion | 21 |
| 2 Surface free energy, step free energy, and chemical potential | 23 |
| 2.1 Surface tension | 23 |
| 2.2 Small surface fluctuations and surface rigidity | 26 |
| 2.3 Singularities of the surface tension | 27 |
| 2.4 Chemical potential | 28 |
| 2.5 Step line tension and step chemical potential | 33 |
| 2.6 Step line tension close to a high-symmetry direction at low T | 35 |
| 2.7 Thermal fluctuations at the midpoint of a line of tension γ | 37 |
| 2.8 The case of a compressible solid body | 38 |
| 2.9 Step-step interactions | 39 |
| 3 The equilibrium crystal shape | 43 |
| 3.1 Equilibrium shape: general principles | 43 |
| 3.2 The Wulff construction | 46 |
| 3.3 The Legendre transformation | 47 |

| | | |
|----------|---|------------|
| viii | <i>Contents</i> | |
| 3.4 | Legendre transform and crystal shape | 48 |
| 3.5 | Surface shape near a facet | 50 |
| 3.6 | The double tangent construction | 51 |
| 3.7 | The shape of a two-dimensional crystal | 53 |
| 3.8 | Adsorption of impurities and the equilibrium shape | 55 |
| 3.9 | Pedal, <i>taupins</i> and Legendre transform | 56 |
| 3.10 | Conclusion | 57 |
| 4 | Growth and dissolution crystal shapes: Frank's model | 60 |
| 4.1 | Frank's model | 61 |
| 4.2 | Kinematic Wulff's construction | 62 |
| 4.3 | Stability of self-similar shapes | 64 |
| 4.4 | Frank's theorem | 66 |
| 4.5 | Examples | 66 |
| 4.6 | Experiments | 69 |
| 4.7 | Roughening and faceting | 69 |
| 4.8 | Conclusion | 69 |
| 5 | Crystal growth: the abc | 70 |
| 5.1 | The different types of heterogrowth | 70 |
| 5.2 | Wetting | 73 |
| 5.3 | Commensurate and incommensurate growth | 73 |
| 5.4 | Effects of the elasticity of the solid | 75 |
| 5.5 | Nucleation, steps, dislocations, Frank-Read sources | 75 |
| 5.6 | Supersaturation | 78 |
| 5.7 | Kinetic roughening during growth at small supersaturation | 79 |
| 5.8 | Evaporation rate, saturating vapour pressure, cohesive energy, and sticking coefficient | 81 |
| 5.9 | Growth from the vapour | 83 |
| 5.10 | Segregation, interdiffusion, buffer layers | 84 |
| 6 | Growth and evaporation of a stepped surface | 88 |
| 6.1 | General equations | 89 |
| 6.2 | The quasi-static approximation | 90 |
| 6.3 | The case without evaporation: validity of the BCF model | 92 |
| 6.4 | The Schwoebel effect | 94 |
| 6.5 | Advacancies and evaporation | 97 |
| 6.6 | Validity of the BCF model in the case of evaporation | 98 |
| 6.7 | Step bunching and macrosteps | 100 |
| 7 | Diffusion | 111 |
| 7.1 | Mass diffusion and tracer diffusion | 112 |
| 7.2 | Conservation law and current density | 114 |

| <i>Contents</i> | | ix |
|-----------------|--|-----|
| 7.3 | Vacancies and interstitial defects in a bulk solid | 114 |
| 7.4 | Surface diffusion | 116 |
| 7.5 | Diffusion under the effect of a chemical potential gradient | 117 |
| 7.6 | Diffusion of radioactive tracers | 119 |
| 7.7 | Activation energy | 119 |
| 7.8 | Surface melting | 123 |
| 7.9 | Calculation of the diffusion constant | 124 |
| 7.10 | Diffusion of big adsorbed clusters | 124 |
| 8 | Thermal smoothing of a surface | 130 |
| 8.1 | General features | 131 |
| 8.2 | The three ways to transport matter | 132 |
| 8.3 | Smoothing of a surface above its roughening transition | 133 |
| 8.4 | Thermal smoothing due to diffusion in the bulk solid | 136 |
| 8.5 | Smoothing below the roughening transition | 137 |
| 8.6 | Smoothing of a macroscopic profile below T_R | 140 |
| 8.7 | Grooves parallel to a high-symmetry orientation | 142 |
| 9 | Silicon and other semiconducting materials | 144 |
| 9.1 | The crystal structure | 144 |
| 9.2 | The (001) face of semiconductors | 146 |
| 9.3 | Surface reconstruction | 147 |
| 9.4 | Anisotropy of surface diffusion and sticking at steps | 148 |
| 9.5 | Crystalline growth vs. amorphous growth | 148 |
| 9.6 | The binary compounds AB | 149 |
| 9.7 | Step and kink structure | 150 |
| 9.8 | The (111) face of semiconductors | 150 |
| 9.9 | Orders of magnitude | 151 |
| 10 | Growth instabilities of a planar front | 156 |
| 10.1 | Diffusion limited aggregation: shape instabilities | 156 |
| 10.2 | Linear stability analysis: the Bales-Zangwill instability | 158 |
| 10.3 | Stabilizing effects: line or surface tension | 162 |
| 10.4 | Stability of a regular array of straight steps: the general case | 164 |
| 10.5 | The case of MBE growth without evaporation | 167 |
| 10.6 | Beyond the linear stability analysis: cellular instabilities | 168 |
| 10.7 | The Mullins-Sekerka instability | 170 |
| 10.8 | Growth instabilities in metallurgy | 174 |
| 10.9 | Dendrites | 176 |
| 10.10 | Conclusion | 177 |
| 11 | Nucleation and the adatom diffusion length | 181 |
| 11.1 | The definition of the diffusion length | 182 |

| | | |
|-------|--|-----|
| x | <i>Contents</i> | |
| 11.2 | The nucleation process | 183 |
| 11.3 | Adatom lifetime and adatom density | 185 |
| 11.4 | Adatom-adatom and adatom-island collisions | 185 |
| 11.5 | The case $i^* = 1$ | 186 |
| 11.6 | Numerical simulations and controversies | 187 |
| 11.7 | Experiments | 188 |
| 11.8 | The case $i^* = 2$ | 190 |
| 11.9 | Generalization | 194 |
| 11.10 | Diffraction oscillations in MBE | 195 |
| 11.11 | Critical nucleus size and numerical simulations | 196 |
| 11.12 | Surfactants | 197 |
| 12 | Growth roughness at long lengthscales in the linear approximation | 201 |
| 12.1 | What is a rough surface? | 201 |
| 12.2 | Random fluctuations and healing mechanisms | 202 |
| 12.3 | A subject of fundamental, rather than technological, interest | 204 |
| 12.4 | The linear approximation (Edwards & Wilkinson 1982) | 205 |
| 12.5 | Lower and upper critical dimensions | 206 |
| 12.6 | Correlation length | 207 |
| 12.7 | Scaling behaviour and exponents | 208 |
| 13 | The Kardar-Parisi-Zhang equation | 211 |
| 13.1 | The most general growth equation | 211 |
| 13.2 | Relevant and irrelevant terms in (13.1): the KPZ equation | 213 |
| 13.3 | Upper critical dimension and exponents | 215 |
| 13.4 | Behaviour of λ near solid-fluid equilibrium | 217 |
| 13.5 | A relation between the exponents of the KPZ model | 218 |
| 13.6 | Numerical values of the coefficients λ and ν | 219 |
| 13.7 | The KPZ model without fluctuations ($\delta f = 0$) | 219 |
| 14 | Growth without evaporation | 221 |
| 14.1 | Where λ is shown to vanish in the KPZ equation | 221 |
| 14.2 | Diffusion bias and the Eaglesham-Gilmer instability | 222 |
| 14.3 | A theorist's problem: the case $\lambda = \nu = 0$ | 224 |
| 14.4 | Calculation of the roughness exponents | 225 |
| 14.5 | Numerical simulations | 227 |
| 14.6 | The Montréal model | 227 |
| 14.7 | Conclusion | 228 |
| 15 | Elastic interactions between defects on a crystal surface | 230 |
| 15.1 | Introduction | 231 |
| 15.2 | Elastic interaction between two adatoms at a distance r | 232 |

| <i>Contents</i> | | xi |
|--|--|------------|
| 15.3 | Interaction between two parallel rows of adatoms | 234 |
| 15.4 | Interaction between two semi-infinite adsorbed layers | 236 |
| 15.5 | Steps on a clean surface | 238 |
| 15.6 | More general formulae for elastic interactions | 241 |
| 15.7 | Instability of a constrained adsorbate | 243 |
| 16 | General equations of an elastic solid | 249 |
| 16.1 | Memento of elasticity in a bulk solid | 249 |
| 16.2 | Elasticity with an interface | 252 |
| 16.3 | The isotropic solid | 255 |
| 16.4 | Homogeneous solid under uniform hydrostatic pressure | 256 |
| 16.5 | Free energy | 257 |
| 16.6 | The equilibrium free energy as a surface integral | 259 |
| 16.7 | Solid adsorbate in epitaxy with a semi-infinite crystal | 261 |
| 16.8 | The Grinfeld instability | 264 |
| 16.9 | Dynamics of the Grinfeld instability | 268 |
| 16.10 | Surface stress and surface tension: Shuttleworth relation | 268 |
| 16.11 | Force dipoles, adatoms and steps | 270 |
| 17 | Technology, crystal growth and surface science | 277 |
| 17.1 | Introduction | 277 |
| 17.2 | The first half of the twentieth century: the age of the radio | 278 |
| 17.3 | The third quarter of the twentieth century: the age of transistors | 278 |
| 17.4 | The last quarter of the twentieth century: the age of chips | 281 |
| 17.5 | MOSFETS and memories | 282 |
| 17.6 | From electronics to optics | 284 |
| 17.7 | Semiconductor lasers | 285 |
| 17.8 | Quantum wells | 287 |
| Appendix A – From the discrete Gaussian model to the two-dimensional Coulomb gas | | 289 |
| Appendix B – The renormalization group applied to the two-dimensional Coulomb gas | | 293 |
| Appendix C – Entropic interaction between steps or other linear defects | | 296 |
| Appendix D – Wulff’s theorem finally proved | | 300 |
| Appendix E – Proof of Frank’s theorem | | 304 |
| Appendix F – Step flow with a Schwoebel effect | | 309 |
| Appendix G – Dispersion relations for the fluctuations of a train of steps | | 312 |

| | | |
|-----|--|-----|
| xii | <i>Contents</i> | |
| | Appendix H – Adatom diffusion length ℓ_s and nucleation | 316 |
| | Appendix I – The Edwards-Wilkinson model | 319 |
| | Appendix J – Calculation of the coefficients of (13.1) for a stepped surface | 322 |
| | Appendix K – Molecular beam epitaxy, the KPZ model, the Edwards-Wilkinson model, and similar models | 324 |
| | Appendix L – Renormalization of the KPZ model | 326 |
| | Appendix M – Elasticity in a discrete lattice | 332 |
| | Appendix N – Linear response of a semi-infinite elastic, homogeneous medium | 335 |
| | Appendix O – Elastic dipoles in the z direction | 342 |
| | Appendix P – Elastic constants of a cubic crystal | 345 |
| | <i>References</i> | 347 |
| | <i>Index</i> | 374 |

Preface

In writing a preface, an author is faced with the question: what is this book of mine? Of course, in the end only the reader will decide what it really is. The scope of this preface, as of all prefaces, is to say what it was intended to be.

This book tries to offer a reasonably complete description of the physical phenomena which make solid materials grow in a certain way, homogeneous or not, rough or smooth. These phenomena belong to chemistry, quantum physics, mechanics, statistical mechanics. However, chemistry, mechanics and quantum physics are essentially the same during growth as they are at equilibrium. The statistical aspects are quite different. For this reason, the authors have insisted on statistical mechanics.

Another reason to emphasize the statistical mechanical concepts is that they will probably survive. The concepts developed many years ago by Frank, or more recently by Kardar, Parisi and Zhang are still valid while, for instance, quantum mechanical calculations of the relevant energy parameters will certainly evolve a lot in the next few years. We have not considered it useful to devote too many pages to them, but we have tried to present the frame in which the data can be inserted, as soon as they are known.

However, although emphasis is on statistical mechanics, other aspects are not ignored, even though they may have been treated somewhat superficially. The reader will find more detailed information in an extensive bibliography, where all titles are given in extenso, thus making its use much easier.

This book is mainly devoted to growth, and therefore to non-equilibrium processes. Nevertheless, we have tried to make it self-contained and to incorporate some elements of equilibrium surface physics, for instance the roughening transition and the equilibrium shape. The reader eager to know more will again find the necessary references in the bibliography.

The authors are theorists and their book is mainly devoted to theory. Few details are given on experimental methods, but many experimental pictures (mostly from scanning tunneling microscopy) show how real materials do behave. In this domain, too, an abundant bibliography is available.

Although the responsibility for all which is written here—good or bad—is completely ours, we owe a lot to all those who contributed to our understanding of the subject. We wish to thank J.M. Bermond, H. Bonzel, J.-P. Bucher, J. Chevrier, G. Comsa, J. Ernst, J. Frenken, M. Hanbücken, J.C. Heyraud, K. Kern, R. Kern, M. Lagally, J. Lapujoulade, J.J. Métois, B. Mutafschiev and E. Williams, whose experimental works revealed to us all the beauty of Surface Physics and Crystal Growth—and often contributed to the iconographic asset of the book. We are also very grateful to D. Wolf, Ph. Nozières, R. Kern again, J. Krug, P. Jensen, J. Langer, C. Misbah, L. Sander, D. Vvedensky and A. Zangwill, who shared with us some of their secrets. Special thanks are due to P. Politi and M. Schroeder, whose untiring reading of preliminary versions has been a source of most valuable suggestions and improvements. A good share of the chapters on elasticity has much profited from the competence of C. Duport, who corrected all our formulae, and even explained some of them to us!

We thank and beg pardon to all who are omitted here either for space or memory limitations. A final thank is due to the people in Cambridge University Press, and most of all to R. Neal, for waiting patiently for the completion of this work.

List of symbols

- DLA: *Diffusion limited aggregation* (section 11.2)
 MBE: *Molecular beam epitaxy*
 ML: *Monolayer*
 RHEED: *Reflection high-energy electron diffraction*
- a : *lattice constant or atomic distance (usually taken equal to 1 in this book).*
 A_x, A_{xy}, A : *kinetic coefficients in sections 13.1 and 13.2*
 $A(t)$: *amplitude appearing in section 13.7*
 B : *a thermodynamic coefficient in (2.38)*
 B_{xy} : *kinetic coefficient in chapter 13*
 C : *a constant (section 15.7)*
 d : *dimension of the space (usually 3)*
 d' : *surface dimension $d - 1$*
 $d' = D/D', d'' = D/D''$: *see (6.20)*
 d_c^u : *upper critical dimension (section 12.4)*
 d_c^l : *lower critical dimension (section 12.4)*
 d_f : *fractal dimension of a fractal terrace (section 11.2)*
 \tilde{D} : *Fick diffusion constant (sections 7.1, 8.4)*
 \tilde{D}_s, \tilde{D} : *surface Fick diffusion constant*
 D^* : *tracer diffusion constant (section 7.1)*
 D_{int} : *diffusion constant of interstitials*
 D_{vac} : *diffusion constant of vacancies in section 7.3.*
 D_s^0 : *see eq. (7.13)*
 D_0 : *see eq. (11.28)*
 D_2 : *diffusion constant of dimers at a surface (sections 7.10 and 11.8)*
 D', D'' : *step kinetic coefficients (section 6.4)*
 D_s (D when no ambiguity is possible): *surface diffusion constant of adatoms*
 e : *basis of Napierian logarithms*
 E : *Young modulus*
 E_2 : *binding energy of a dimer (section 11.8)*

- $\delta \mathbf{f}^{\text{ext}}, \delta \mathbf{F}^{\text{ext}}$: elementary external forces (chapter 16)
 F_{α}^{ext} : External force
 F : beam intensity in MBE
 $\mathbf{F}_{\mathbf{R}}$: force acting at point \mathbf{R}
 g : gravity (in chapter 1)
 $G(\mathbf{R})$: height-height correlation function at equilibrium, eq. (1.4)
 $G(\mathbf{r}, t)$: height-height correlation function during growth, eq. (12.1)
 h : amplitude of the modulation of a surface, height of a defect
 i^* : critical size of a cluster of adatoms (section 11.2)
 \mathbf{j}_s : surface current density of adatoms
 $\mathbf{j}_s^{\text{adat}}$: the current density of adatoms (section 7.5)
 $\mathbf{j}_s^{\text{advac}}$: current density of advacancies (section 7.5)
 \mathbf{j}_k : two-dimensional Fourier transform of the surface adatom current density
 $\mathbf{k} = (k_x, k_y)$: a vector of the two-dimensional reciprocal space
 k_B : the Boltzmann constant
 K : bulk modulus (section 16.4)
 K : kinetic coefficient defined by (12.6)
 L_x, L_y : sample sizes in the x and y directions
 L : linear size of a system
 L : wavelength of a modulation (chapter 8)
 $m_{\alpha\gamma}, m$: force dipole moment, eq. (15.2)
 M : a point at the surface of a solid
 \mathbf{n} : unit vector normal to a surface at a point, directed outward
 N : number of particles in a physical system
 N : quantity related to the size of the surface in section 2.6.
 $N(t)$: cluster density (section 11.6)
 $p_{\alpha\gamma}, p_0$: stress
 $p_j(\mathbf{r}, t)$: probability that the j -th impurity is at \mathbf{r} at time t (section 7.1)
 $\mathbf{q} = (q_x, q_y)$: a vector of the two-dimensional reciprocal space
 r_1 : characteristic length (chapter 8)
 R : radius of a terrace (section 8.5)
 R : radius of curvature of a line or of a surface (chapter 10)
 \mathbf{R}, \mathbf{r} : a point of the two-dimensional (chapter 1) or three-dimensional space
 dS : surface element (chapter 2)
 S : surface of a crystal (chapters 3 and 4)
 t : time
 T : temperature
 T_M : melting temperature.
 T_R : roughening transition temperature
 $\mathbf{u}(\mathbf{r})$: atomic displacement at point \mathbf{r}
 $\mathbf{v}, \mathbf{v}(\mathbf{n})$: velocity of the surface of a crystal (chapter 5)
 v : velocity of a step
 $v = V/N$: volume per atom (chapter 2)

List of symbols

xvii

- v_g : volume per atom in the vapour (chapter 2)
 v_l : volume per atom in the liquid (chapter 2).
 V : volume
 δV : volume element (section 16.1)
 W : energy or free energy barrier
 W_0 : energy of a kink on a step (section 1.8)
 W_0 : energy barrier (section 15.7)
 w_0 : energy of a chemical bond (section 15.5)
 W_a : adatom creation energy or free energy, see eq. (7.14a)
 W_{adv} : see eq. (7.14b)
 W_1 : energy of a step per bond
 W_{sd} : activation energy for surface diffusion (eq. (7.13) and section 11.8)
 $W_{int}(r)$: interaction energy between defects at distance r (chapter 15)
 W_{coh} : cohesive energy
 x, y, z : coordinates of a point in the three-dimensional space.
 $x_s = \sqrt{D_s \tau_v} = 1/\kappa$: average adatom diffusion length before desorption
 y' : derivative dy/dx
 \tilde{z} : dynamical “critical exponent” for the correlation length (eq. 12.14)
 $z_q(t)$: Fourier transform of $z(\mathbf{r}, t)$
 $j_s, D_s, \Lambda_s, \tilde{D}_s$: section 8.4
 α, γ : coordinates x, y or z
 α : kinetic coefficient (eq. 7.8)
 α : critical exponent for the spatial decay of the correlation function (eq. 12.18)
 β : critical exponent for the temporal decay of the correlation function (eq. 12.18)
 $\beta = 1/(k_B T)$
 γ : free energy of a step per unit length or per atom (line tension)
 $\tilde{\gamma} \equiv \gamma(\theta) + d^2\gamma/d\theta^2$: step stiffness (section 2.4)
 γ : exponent defined in section 11.9
 $\Gamma(\mathbf{r} - \mathbf{r}')$: elastic Green function (eq. 15.10)
 Γ : a step, in chapter 10
 δa : misfit (section 15.4)
 $\delta \mathcal{F}, \delta \mu$, etc.: increment of \mathcal{F}, μ , etc. from some reference value
 $\delta \Sigma$: surface element (section 16.1)
 ΔF : “supersaturation” $F - \rho_0/\tau_v$
 Δ' and Δ'' : kinetic coefficients defined in section 6.4
 ϵ : Interaction energy between steps (section 1.7)
 $\epsilon_{\alpha\gamma}(\mathbf{r})$: strain
 $\epsilon(t)$: amplitude of a modulation in eq. (10.14) (called η in eq. (10.7))
 ζ : Poisson ratio (eq. 6.12)
 ζ : in section 10.6, a control parameter
 η : amplitude of a modulation in eq. (10.7) (called ϵ in eq. (10.14))

xviii *List of symbols*

- θ : value of an angle
- $\kappa = 1/x_s$: inverse evaporation length of an adatom (eq. 6.6)
- λ : capillary length (chapter 1)
- λ : Lamé coefficient (chapters 15 and 6).
- λ : kinetic coefficient in the KPZ equation (13.4)
- λ : a constant (e.g. in section 4.2)
- Λ_s, Λ : surface diffusion constant of advacancies (section 6.5)
- μ : Lamé coefficient (chapters 15 and 16)
- μ : chemical potential (elsewhere)
- $v(t)$: number of sites visited by a diffusing adatom in time t (sections 6.3, 11.4)
- v : kinetic coefficient defined by (12.6)
- $\xi(t)$: correlation length (section 12.6)
- (Π) : tangent plane to the crystal surface (chapter 3)
- $\rho(\mathbf{r}, t), \rho_1, \rho_s$: adatom density (occupied sites/surface sites)
- ρ_n : surface density of clusters of n atoms (section 11.4).
- ρ_0 : equilibrium density of adatoms. (section 6.5)
- $\bar{\rho}$: see section 6.3
- $\rho_{\text{int}}(\mathbf{r}, t)$: concentration of interstitials
- σ_0 : equilibrium density of advavancies (section 6.5)
- $\sigma, \sigma(\mathbf{n})$: free energy of a surface per unit length or per atom (surface tension)
- $\sigma(\mathbf{r})$: density of advacancies
- Σ : Wulff's plot (section 3.2)
- $\tilde{\sigma}$: surface rigidity, formula (2.5)
- $1/\tau_v$: evaporation probability of an adatom per unit time (section 6.1)
- $1/\tau_{\text{nuc}}$: nucleation rate of new terraces (section 11.2)
- $1/\tau_k$: the rate of emission of "gradatoms" (problem 10.6)
- $\phi(z_x, z_y)$: the projected free energy density per unit area (eq. 2.2)
- $\varphi(\{\epsilon(\mathbf{r})\})$: elastic free energy density (section 16.5)
- φ_p : components of an eigenvector of the transfer matrix in Appendix C
- Φ : Gibbs free energy or free enthalpy $\mathcal{F} + PV$ (chapters 2 and 16)
- ω : decay rate of a modulation, see eq. (10.15)
- Ω : atomic area, generally set equal to 1 in this book
- Ω : Grand potential $\mathcal{F} + PV - \mu N$
- $\Omega_{xy}^{\xi\xi}$: elastic constants
- $\tilde{\Omega}_{RR'}^{\alpha\gamma}$: "discretized elastic constants", eq. (16.25)
- ℓ : distance between steps
- ℓ_s : adatom diffusion length before nucleation of a terrace (section 11.1)
- ℓ_s : length on which diffusion is able to heal the surface (chapter 12)
- $d\mathcal{A}$: elementary projected area $dx dy$ (section 16.8)
- \mathcal{A} : total projected area $L_x L_y$

List of symbols

xix

- \mathcal{F} : free energy
- \mathcal{G} : Gibbs free energy $\mathcal{G} = \mathcal{F} + PV$.
- \mathcal{N} : number of lattice sites on a surface in section 14.6
- \simeq : nearly equal to (e.g. $\sin x \simeq x$ for small x)
- \sim : proportional to (e.g. $10x \sim x$)
- \approx : of the order of magnitude of (e.g. $10 \approx 1$)
- cot: cotangent
- z_x, z_α : partial derivative $\partial z / \partial x, \partial z / \partial x_\alpha$
- $v_\alpha, j_\alpha, R_\alpha$: the components of the vectors $\mathbf{v}, \mathbf{j}, \mathbf{R}$
- $\langle \chi \rangle$: average value of a quantity χ
- $|z|$: absolute value of z
- $\dot{z}, \dot{\rho}$, etc.: derivative of z, ρ , etc., with respect to time
- $(hk\ell)$: orientation of a crystal surface. E.g. the (001) face, the (111) orientation
- $[hk\ell]$: orientation of a crystal axis. E.g. [110] steps
- $\{hk\ell\}$: set of crystal planes which are crystallographically equivalent. E.g. (cubic crystal): the $\{001\}$ orientations are the (001), (010) and (100) planes.