Nonlinear Dynamics: A Primer

This book provides a systematic and comprehensive introduction to the study of nonlinear dynamical systems, in both discrete and continuous time, for nonmathematical students and researchers working in applied fields including economics, physics, engineering, biology, statistics and linguistics. It includes a review of linear systems and an introduction to the classical theory of stability, as well as chapters on the stability of invariant sets, bifurcation theory, chaotic dynamics and the transition to chaos. In the final chapters the authors approach the study of dynamical systems from a measure-theoretical point of view, comparing the main notions and results to their counterparts in the geometrical or topological approach. Finally, they discuss the relations between deterministic systems and stochastic processes.

The book is part of a complete teaching unit. It includes a large number of pencil and paper exercises, and an associated website offers, free of charge, a Windows-compatible software program, a workbook of computer exercises coordinated with chapters and exercises in the book, answers to selected book exercises, and further teaching material.

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To the memory of my father
who taught me to love books

To my mother and father

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Preface

Over the years we have had the rare opportunity to teach small classes of intelligent and strongly motivated economics students who found nonlinear dynamics inspiring and wanted to know more. This book began as an attempt to organise our own ideas on the subject and give the students a fairly comprehensive but reasonably short introduction to the relevant theory. Cambridge University Press thought that the results of our efforts might have a more general audience.

The theory of nonlinear dynamical systems is technically difficult and includes complementary ideas and methods from many different fields of mathematics. Moreover, as is often the case for a relatively new and fast growing area of research, coordination between the different parts of the theory is still incomplete, in spite of several excellent monographs on the subject. Certain books focus on the geometrical or topological aspects of dynamical systems, others emphasise their ergodic or probabilistic properties. Even a cursory perusal of some of these books will show very significant differences not only in the choice of content, but also in the characterisations of some fundamental concepts. (This is notoriously the case for the concept of attractor.)

For all these reasons, any introduction to this beautiful and intellectually challenging subject encounters substantial difficulties, especially for non-mathematicians, as are the authors and the intended readers of this book. We shall be satisfied if the book were to serve as an access to the basic concepts of nonlinear dynamics and thereby stimulate interest on the part of students and researchers, in the physical as well as the social sciences, with a basic mathematical background and a good deal of intellectual curiosity.

The book includes those results in dynamical system theory that we deemed most relevant for applications, often accompanied by a common-sense interpretation. We have also tried, when necessary, to eliminate the confusion arising from the lack of consistent and universally accepted definitions of some concepts. Full mathematical proofs are usually omitted and the reader is referred either to the original sources or to some more recent version of the proofs (with the exception of some 'canonical' theorems whose discussion can be found virtually in any textbook on the subject).

We devote an unusually large space to the discussion of stability, a subject that in the past played a central role in the theory of differential equations and related applied research. The fundamental monographs on stability were published in the 1960s or early 1970s yet there is surprisingly little reference to them in modern contemporary research on dynamical systems. We have tried to establish a connection between the classical theory of stability and the more recent discussions of attracting sets and attractors.

Although the word 'chaos' does not appear in the title, we have dedicated substantial attention to chaotic sets and attractors as well as to 'routes to chaos'. Moreover, the geometric or topological properties of chaotic dynamics are compared to their measure-theoretic counterparts.

We provide precise definitions of some basic notions such as neighbourhood, boundary, closure, interior, dense set and so on, which mathematicians might find superfluous but, we hope, will be appreciated by students from other fields.

At an early stage in the preparation of this book, we came to the conclusion that, within the page limit agreed upon with the publisher, we could not cover both theory and applications. We squarely opted for the former. The few applications discussed in detail belong to economics where our comparative advantages lie, but we emphasised the multi-purpose techniques rather than the specificities of the selected models.

The book includes about one hundred exercises, most of them easy and requiring only a short time to solve.

In 1992, Cambridge University Press published a book on Chaotic Dynamics by the first author, which contained the basic concepts of chaos theory necessary to perform and understand numerical simulations of difference and differential equations. That book included a user-friendly software program called DMC (Dynamical Models Cruncher). A refurbished, enlarged and Windows-compatible version of the program is available, at no cost, from the webpage

<http://uk.cambridge.org/economics/catalogue/0521558743>

along with a workbook of computer exercises coordinated with the 'paper and pencil' exercises found in the book. The webpage will also be used to circulate extra exercises, selected solutions and, we hope, comments and criticisms by readers.

We take this opportunity to give our warm thanks to those who, in different capacities and at different times, helped us complete this book.

Laura Gardini, Hans-Walter Lorenz, Ami Radunskaya, Marcellino Gaudenzi, Gian Italo Bischi, Andrea Sgarro, Sergio Invernizzi and Gabriella Caristi, commented on preliminary versions of parts of the book or gave their advice on specific questions and difficulties. We did not always follow their suggestions, and, at any rate, the responsibility for all remaining errors and misunderstandings remains entirely with us. Thanks also to Eric Kostelich, Giancarlo Benettin and Luigi Galgani who, in various conversations, helped us clarify specific issues.

At Cambridge University Press we would like to thank Patrick McCartan, for suggesting the idea; Ashwin Rattan, Economics Editor, for his support and patience; Alison Woollatt for her TeX advice. Thanks also to Barbara Docherty for her excellent editing.

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Alfredo Medio and Marji Lines
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