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On the Transient Response of Distributed Structures Interacting with Discrete Components

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Abstract

Distributed structures are often coupled to external components in engineering applications: The recording head in disk and tape drives, the guide bearing in circular and band saws, and the payload in cable transport systems are a few examples. Because the transient response of a distributed structure is characterized by complex multiple wave scattering at an external component, this behavior has rarely been explored. Classical transient analyses by finite difference, finite element, and modal expansion approaches use spatial discretization. As a result, some discontinuities in the interaction force between a distributed structure and an external component, a characteristic of multiple wave scattering, are not predicted in the normal application of these methods. A new transient analysis is developed in this chapter for the response of distributed structures interacting with discrete components. The transient response of a time-varying, cable transport system is analyzed first. The transient and steady-state responses of constrained translating strings are obtained next. Transient behaviors of cables transporting dynamic payloads and translating, magnetic tape-head systems are thoroughly investigated. Application of the method to the classical piano string response under a hammer strike avoids key limitations of the existing standing and traveling wave methods.

1.1 Introduction

Distributed structures are usually coupled to external dynamic components in engineering applications. In magnetic tape and disk drives data transfer is accomplished through contact of recording media with a recording head. The guide bearing used in wood machining to minimize the circular and band saw vibration is another example. In cable transport systems, such as tramways and ski lifts, the payload is attached to a cable for transport across a span.

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In all these cases, a wave propagating along a distributed structure will partly transmit through, and partly reflect from, an attached external system. Hence the transient response of a distributed structure coupled to an external component is characterized by complex, multiple wave scattering.

Although research interest in the dynamic analysis of constrained distributed structures has increased in recent years, most studies (Butkovskiy, 1983; Bergman and Nicholson, 1985; Yang, 1992; Tan and Chung, 1993) investigate natural frequencies, modes of vibration, and steady harmonically forced response. The transient behavior has rarely been explored. Developing an understanding of the transient behavior is important for a number of reasons. First, some constrained distributed systems do not have a steady-state response. For instance, the vibration of a cable transport system is inherently a transient response problem. Its eigensolutions and steady-state response do not exist because of the time-varying inertia and stiffness of the system as the payload moves along the span (Zhu and Mote, 1994a). Second, a steady-state analysis usually seeks response to a sinusoidal, or periodic, excitation. Naturally, many other loading conditions require response solution too. The wave motion generated in a magnetic tape when a fast moving recording head suddenly contacts or leaves the tape is of interest in magnetic recording because it is desirable to minimize transient effects (Lacey and Talke, 1990; Sundaram and Benson, 1990). A transient analysis, however, can solve for the response to arbitrary loadings and initial conditions. Third, even for the case of a sinusoidal excitation, the transient response of a constrained distributed structure is characterized by multiple wave scattering and differs significantly from its steady-state response. For a lightly damped system, the transient wave motion can dominate the response within the time interval of interest. Hence in machines that operate satisfactorily at steady state, problems can arise during transients. The design of high-precision and high-productivity machines benefits from a fundamental understanding of the transient dynamic behavior.

Transient analyses commonly use spatial discretization of the governing partial differential equation for a distributed structure through finite difference, finite element, or modal expansion approaches. Stahl et al. (1974) used finite differences in both the temporal and spatial domains to calculate the transient air pressure between the moving tape and a stationary recording head. By Galerkin discretization, Ulsoy and Mote (1982) determined the transient response of a traveling plate under transverse load. When a distributed system is approximated by spatially discretized models, some dynamic characteristics associated with the distributed model are lost. In control of a translating string vibration, use of a finite number of modes leads to control and observation spill-overs (Ulsoy, 1984). For transient analysis of a distributed structure coupled to external

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dynamic components, the principal task is to determine the force of interaction between the structure and the external component (Wickert and Mote, 1991). Once the force is known, the response of each subsystem can then be predicted. Characteristic of the multiple wave scattering phenomenon, the time history of the interaction force between the structure and an external component can exhibit discontinuities when the propagating wavefronts in the structure have nonzero slopes (Zhu and Mote, 1994a, 1995), and these discontinuities are not predicted by the spatially discretized models. This behavior is illustrated in the following simple system.

Consider a pulse of length l propagating in an infinite, stationary, taut string with a discrete, rigid, or flexible constraint at the origin $x = 0$, as shown in Fig. 1.1. The slopes of the leading and trailing edges of pulse are given by $\Theta'(0)$ and $\Theta'(l)$ respectively (see Appendix A). The pulse is reflected from the rigid support in Fig. 1.1(a) and is partly reflected from, and partly transmitted through, the flexible constraint in Fig. 1.1(b,c). In the first two cases, when the leading and trailing edges of the pulse collide with the constraint, the dimensionless interaction force between the string and the constraint is discontinuous, with magnitudes $-2\Theta'(0)$ and $2\Theta'(l)$, respectively. For the flexible massless constraint in Fig. 1.1(c), the discontinuities in the dimensionless interaction force are $-2c/(c+2)\Theta'(0)$ and $2c/(c+2)\Theta'(l)$ when the pulse enters and

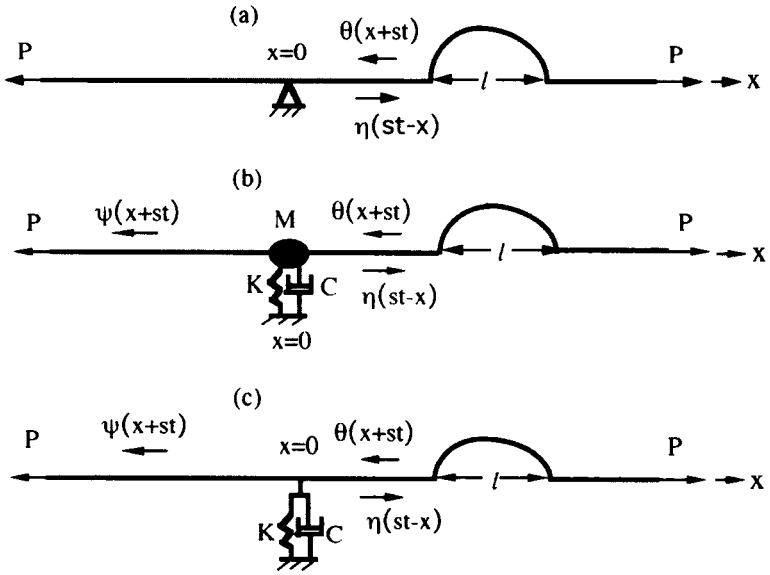

Fig. 1.1. Propagating pulses in constrained stationary strings.

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exits the constraint. Here c is the dimensionless viscous damping coefficient. Hence if the pulse has nonzero slopes at either the leading or trailing edges, the interaction force will be discontinuous. The interaction force is always continuous only when the constraint is modeled by a stiffness element.

In a previous investigation, Wickert and Mote (1990) derived the general response of an axially moving material to arbitrary initial conditions and excitations in terms of the Green's function. The transient response of a traveling string with an attached inertia was subsequently studied (Wickert and Mote, 1991). Though abrupt variations in the force of interaction between the string and inertia occur at the instants when propagating wavefronts coincide with the particle, the interaction force is continuous in this case because the wavefront has vanishing slope. For steady-state response of a constrained distributed structure subject to a harmonic excitation, discontinuities can occur in the constraint force during the initial transient. When distributed or lumped damping exists in the structure, as shown in an example in Section 1.3.4, the discontinuous constraint force during the initial transient approaches, at the steady state, the sinusoidal force predicted by the steady forced analysis.

Existence of the discontinuity in the interaction force has implications to both practice and theory. Discontinuities in the tape-head contact force cause signal strength variations in magnetic recording. Discontinuities in the interaction force between the cable and payload cause discontinuous acceleration of the payload. Discontinuities in the contact force between the piano string and hammer affect the duration of their contact and alter the harmonic spectra of the string after a hammer strike (Hall, 1987a, 1987b). The discontinuities in the contact force between the string and a hard hammer have also been experimentally observed (Yanagisawa et al., 1981).

A particular limitation of the classical spatial discretization approaches is their failure to predict the dynamic behavior caused by the discontinuous force of interaction in the distributed model. No analysis is available in the literature resolving this difficulty.

A new transient analysis is developed in the present chapter predicting the response of distributed structures interacting with discrete dynamic components. Unlike the finite element and other discretization approaches, the method is exact in the spatial domain and retains the distributed property of the structure. It captures all discontinuities in the interaction force between the structure and an external component, and it allows for treatment of nonlinearity in the external components.

In Section 1.2, the free and forced responses of a cable transport system, modeled as a translating string transporting a damped, linear oscillator, are predicted for arbitrary initial conditions, external forces, and boundary disturbances.

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Analogous to the mechanism for the simple case depicted in Fig. 1.1, the time history of the force of interaction between the cable and the suspension is in general discontinuous for two reasons: 1. The presence of the suspension damping causes discontinuity in the force of interaction and 2. for all initial conditions, except those satisfying the special equation (1.45), all propagating wavefronts in the string will have nonzero slopes and lead to discontinuity in the force of interaction.

The delay-integral equation describing the force of interaction between the cable and the suspension is derived using the Green's function formulation, following Wickert and Mote (1991). A difficulty arises in the analysis of the governing integral equation in which both the kernel and the dependent variable are discontinuous. The solution technique is developed to eliminate the discontinuities in the kernel using the theory of distributions. The forcing integrals are evaluated by both the distribution and series expansion methods. The limitation for the use of the series expansion of the Green's function in evaluation of the forcing integrals is indicated. Exact in the spatial domain, a finite difference algorithm in the temporal domain is presented for the solution of the resulting delay-integro-differential equation with a discontinuous dependent variable. The interaction force for the special case of harmonic support excitation at the entrance to the span is calculated in the example in Section 1.2.4.

The transient and steady-state responses of a translating string in contact with discrete, rigid, and flexible constraints are determined in Section 1.3. A translating magnetic tape-head system or a guided band saw system are modeled. The contact force between the string and the one-sided constraint is composed of an equilibrium preload component and a dynamic component.

The transient analysis developed in Section 1.3.2 predicts the response to arbitrary initial conditions, external forces, and boundary disturbances. The exact expression describing the dynamic contact force component is derived using the Green's function formulation. The eigensolutions and steady harmonic response of the constrained translating string are determined in closed form in Section 1.3.3. The transient and steady-state analyses of a magnetic tape-head system model are presented in Section 1.3.4.

Section 1.4 addresses the application of the discontinuous contact force methodology to the classical problem of the piano string response under a hammer strike. The method of solution applies to linear as well as nonlinear hammer models and handles nicely the multiple contacts occurring between the string and hammer. Particular sets of parameters were selected in the examples in Section 1.4.3 to compare the predictions with available results found in the literature and to obtain results that could not have been obtained by other existing methods.

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1.2 Free and Forced Response of a Time-varying, Cable Transport System

1.2.1 Model and Equations of Motion

Consider first a cable transport system modeled as a translating string of length L and mass per unit length ρ, transporting a suspended mass M with suspension stiffness and damping constants K and C, respectively (see Fig. 1.2). A constant, subcritical transport speed V is prescribed. The string is subjected to gravitational force ρg, distributed external force $F(X, T)$, and a prescribed transverse motion $E(T)$ at the left, span entrance, end. The boundary excitation at the right, span exit, end can be included in a similar manner. The payload mass is subjected to an external force $Q(T)$.

The natural response of the above system is not adequately modeled by the solution of an eigenvalue problem, because the inertia and stiffness properties in the model depend on the position of the payload along the span. During the free transit of the payload through the span, the transient response problem under the gravity and initial conditions is referred to as the *free vibration problem*. Under external excitation $F(X, T)$, $Q(T)$, and $E(T)$, the transient response problem is the *forced vibration problem*. The procedure developed in this section determines the forced response, and the free response is obtained as a special case of it.

Application of the model is limited by the following conditions. The string transverse displacement $U(X, T)$ is small and planar. The initial tension P in

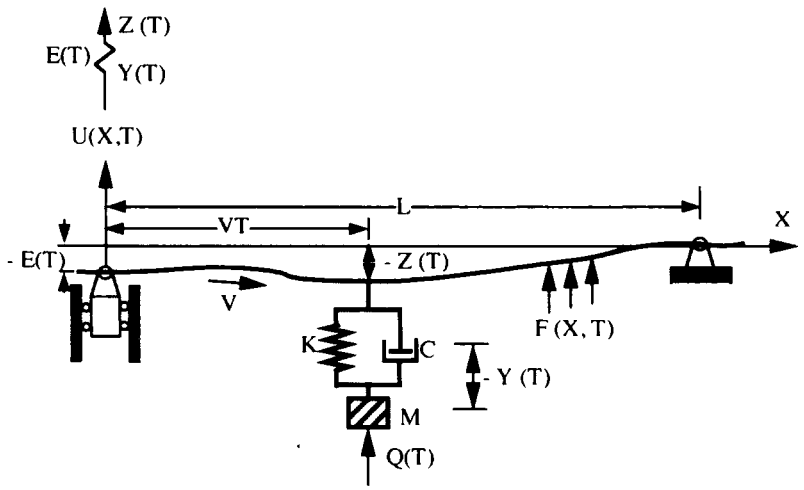

Fig. 1.2. Schematic of the cable trasport system under external forces $F(X, T)$ and $Q(T)$, and boundary excitation $E(T)$ at $x = 0$. All displacement variables are positive in the upward direction.

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the string is sufficiently large that changes in tension caused by the payload have a negligible effect on the system response. The payload mass undergoes transverse vibration relative to the string without rotation.

Let $U(X, T)$ be the displacement of the string relative to the horizontal X axis. Assuming that the payload is positioned at $X = 0$ at time $T = 0$, $Z(T) = U(VT, T)$ is the displacement of its attachment point to the string. Also let $Y(T)$ be the displacement of the payload mass measured from the unstretched length of the spring at displacement $Z(T) = 0$. The interaction force $R(T)$ between the string and the payload suspension is the variable to be determined. Introduce the following dimensionless variables:

$$\begin{gathered} x = X/L, \quad u = U/L, \quad y = Y/L, \quad z = Z/L, \quad e = E/L, \\ t = T(P/\rho L^2)^{1/2}, \quad m = M/\rho L, \quad k = KL/P, \\ c = C/(P\rho)^{1/2}, \quad v = V(\rho/P)^{1/2}, \quad w = \rho g L/P, \\ f = FL/P, \quad q = Q/P, \quad r = R/P. \end{gathered} \tag{1.1}$$

The dimensionless equation of motion for a uniform traveling string is

$$u_{tt}(x, t) + 2vu_{xt}(x, t) + (v^2 - 1)u_{xx}(x, t) = r(t)\delta(x - vt) + f(x, t) - w \tag{1.2}$$

with the boundary and initial conditions given by

$$u(0, t) = e(t), \qquad u(1, t) = 0, \tag{1.3}$$

$$u(x, 0) = a(x) - \frac{wx(1 - x)}{2(1 - v^2)}, \qquad u_t(x, 0) = b(x). \tag{1.4}$$

In Eq. (1.4) $a(x)$ is the initial displacement from the static deflection $-wx(1 - x)/2(1 - v^2)$ of the traveling string under weight w, and $b(x)$ is the initial transverse velocity. The terms $a(x)$ and $b(x)$ are dimensionless. The dimensionless equation of motion and initial conditions for the payload are

$$m\ddot{y}(t) = -r(t) + q(t) - mw \tag{1.5}$$

and

$$y(0) = y_0, \qquad \dot{y}(0) = \dot{y}_0, \tag{1.6}$$

where y_0 and $\dot{y}_0$ are specified. The interaction force $r(t)$ between the suspension and the string is

$$r(t) = k(y(t) - z(t)) + c(\dot{y}(t) - \dot{z}(t)) \tag{1.7}$$

and the attachment point displacement and velocity are

$$z(t) = u(vt, t), \qquad \dot{z}(t) = u_t(vt, t) + vu_x(vt, t). \tag{1.8}$$

Equations (1.2)–(1.8) can be reduced to a single integral equation governing the interaction force $r(t)$.

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1.2.2 Integral Formulation

1.2.2.1 Delay-integral Equation of Volterra Type

The solution to Eqs. (1.2)–(1.4) proceeds by first applying the transformation (Meirovitch, 1967)

$$u(x,t) = \hat{u}(x,t) + (1-x)e(t) \tag{1.9}$$

to render the boundary condition (1.3) homogeneous. The transformed equations for $\hat{u}(x,t)$ are

$$\begin{aligned}\hat{u}_{tt}(x,t) + 2v\hat{u}_{xt}(x,t) + (v^2-1)\hat{u}_{xx}(x,t)\\ = r(t)\delta(x-vt) + f(x,t) - w - (1-x)\ddot{e}(t) + 2v\dot{e}(t),\end{aligned} \tag{1.10}$$

$$\hat{u}(0,t) = \hat{u}(1,t) = 0, \tag{1.11}$$

$$\begin{aligned}\hat{u}(x,0) &= a(x) - \frac{wx(1-x)}{2(1-v^2)} - (1-x)e(0),\\ \hat{u}_t(x,0) &= b(x) - (1-x)\dot{e}(0).\end{aligned} \tag{1.12}$$

The Green's function (Wickert and Mote, 1990) can then be used to determine $\hat{u}(x,t)$. With inclusion of the parabolic static deflection of the string under its own weight, the transverse displacement of the string becomes

$$\begin{aligned}\hat{u}(x,t) = -\frac{wx(1-x)}{2(1-v^2)} + \int_0^t\int_0^1 [f(\xi,\tau) - (1-\xi)\ddot{e}(\tau) + 2v\dot{e}(\tau)]\\ \times g(x,\xi;t-\tau)\,d\xi\,d\tau + \int_0^t r(\tau)g(x,v\tau;t-\tau)\,d\tau\\ + \int_0^1 [b(\xi) - (1-\xi)\dot{e}(0)]g(x,\xi;t)\,d\xi + \int_0^1 [a(\xi) - (1-\xi)e(0)]\\ \times [g_t(x,\xi;t) - 2vg_\xi(x,\xi;t)]\,d\xi,\end{aligned} \tag{1.13}$$

where the Green's function for a fixed-fixed traveling string (Wickert and Mote, 1991),

$$\begin{aligned}g(x,\xi;t-\tau) &= \sum_{n=1}^{\infty}\frac{2}{n\pi}\sin[n\pi(1-v^2)(t-\tau)\\ &\quad + n\pi v(x-\xi)]\sin n\pi x \sin n\pi\xi\\ &= \phi[(1-v^2)(t-\tau) - (1-v)(x-\xi)]\\ &\quad + \phi[(1-v^2)(t-\tau) + (1+v)(x-\xi)]\\ &\quad - \phi[(1-v^2)(t-\tau) - (1-v)x - (1+v)\xi]\\ &\quad - \phi[(1-v^2)(t-\tau) + (1+v)x + (1-v)\xi],\end{aligned} \tag{1.14}$$

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is piecewise continuous, because $\phi(\cdot)$ is a Fourier sine series whose sum is the piecewise continuous sawtooth wave:

$$\phi(s) = \sum_{n=1}^{\infty} \frac{\sin(n\pi s)}{2n\pi} = \begin{cases} \sum_{n=-\infty}^{\infty} [H(s-2n) - H(s-2(n+1))] \frac{2n+1-s}{4}, & s \neq 0, \pm 2, \pm 4, \ldots \\ 0, & s = 0, \pm 2, \pm 4, \ldots \end{cases} \tag{1.15}$$

where $H(\cdot)$ is the Heaviside function. The first two derivatives of $\phi(s)$, which can be differentiated distributionally (Stakgold, 1979), are

$$\phi'(s) = \frac{1}{2} \sum_{n=1}^{\infty} \cos(n\pi s) = -\frac{1}{4} + \frac{1}{2} \sum_{n=-\infty}^{\infty} \delta(s-2n) \tag{1.16}$$

and

$$\phi''(s) = -\frac{1}{2} \sum_{n=1}^{\infty} n\pi \sin(n\pi s) = \frac{1}{2} \sum_{n=-\infty}^{\infty} \delta'(s-2n), \tag{1.17}$$

where $\delta(\cdot)$ is the Dirac delta function. The series in Eqs. (1.16) and (1.17), which are divergent in the classical sense, converge to the generalized functions. The first partial derivatives of the Green's function are thus obtained from Eqs. (1.14) and (1.16):

$$\begin{aligned} g_x(x, \xi; t-\tau) = -\frac{1-v}{2} \sum_{n=-\infty}^{\infty} & \{\delta[(1-v^2)(t-\tau) - (1-v)(x-\xi) - 2n] \\ & + \delta[(1-v^2)(t-\tau) - (1-v)x - (1+v)\xi - 2n]\} \\ & + \frac{1+v}{2} \sum_{n=-\infty}^{\infty} \{\delta[(1-v^2)(t-\tau) + (1+v)(x-\xi) - 2n] \\ & - \delta[(1-v^2)(t-\tau) + (1+v)x + (1-v)\xi - 2n]\}, \end{aligned} \tag{1.18}$$

$$\begin{aligned} g_t(x, \xi; t-\tau) = \frac{1-v^2}{2} \sum_{n=-\infty}^{\infty} & \{\delta[(1-v^2)(t-\tau) - (1-v)(x-\xi) - 2n] \\ & + \delta[(1-v^2)(t-\tau) + (1+v)(x-\xi) - 2n] \\ & - \delta[(1-v^2)(t-\tau) - (1-v)x - (1+v)\xi - 2n] \\ & - \delta[(1-v^2)(t-\tau) + (1+v)x + (1-v)\xi - 2n]\}. \end{aligned} \tag{1.19}$$

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Through use of Leibnitz's rule, $g(x,\xi;0)=0$, and Eq. (1.13), Eq. (1.9) is differentiated to give

$$u_x(x,t) = -e(t) - \frac{w(1-2x)}{2(1-v^2)} + \int_0^t\int_0^1 [f(\xi,\tau) - (1-\xi)\ddot{e}(\tau) + 2v\dot{e}(\tau)] \times g_x(x,\xi;t-\tau)\,d\xi\,d\tau + \int_0^t r(\tau)g_x(x,v\tau;t-\tau)\,d\tau + \int_0^1 [b(\xi) - (1-\xi)\dot{e}(0)]g_x(x,\xi;t)\,d\xi + \int_0^1 [a(\xi) - (1-\xi)e(0)][g_{tx}(x,\xi;t) - 2vg_{\xi x}(x,\xi;t)]\,d\xi, \tag{1.20}$$

$$u_t(x,t) = (1-x)\dot{e}(t) + \int_0^t\int_0^1 [f(\xi,\tau) - (1-\xi)\ddot{e}(\tau) + 2v\dot{e}(\tau)] \times g_t(x,\xi;t-\tau)\,d\xi\,d\tau + \int_0^1 r(\tau)g_t(x,v\tau;t-\tau)\,d\tau + \int_0^1 [b(\xi) - (1-\xi)\dot{e}(0)]g_t(x,\xi;t)\,d\xi + \int_0^1 [a(\xi) - (1-\xi)e(0)][g_{tt}(x,\xi;t) - 2vg_{\xi t}(x,\xi;t)]\,d\xi. \tag{1.21}$$

The solutions of Eqs. (1.5) and (1.6) are expressed in the integral forms

$$y(t) = y_0 + \dot{y}_0 t - \frac{1}{2}wt^2 + \frac{1}{m}\int_0^t (t-\tau)q(\tau)\,d\tau - \frac{1}{m}\int_0^t (t-\tau)r(\tau)\,d\tau \tag{1.22}$$

and

$$\dot{y}(t) = \dot{y}_0 - wt + \frac{1}{m}\int_0^t q(\tau)\,d\tau - \frac{1}{m}\int_0^t r(\tau)\,d\tau. \tag{1.23}$$

Substitution of Eqs. (1.20) and (1.21) into Eq. (1.8) gives expressions for $z(t)$ and $\dot{z}(t)$. These results, along with Eqs. (1.22) and (1.23), are substituted into Eq. (1.7) to yield a Volterra integral equation of the second kind describing the interaction force $r(t)$:

$$r(t) = p(t) + \int_0^t \kappa(t,\tau)r(\tau)\,d\tau, \tag{1.24}$$

where

$$\kappa(t,\tau) = -\frac{c}{m} - \frac{k}{m}(t-\tau) - kg(vt,v\tau;t-\tau) - c\frac{\partial}{\partial t}g(vt,v\tau;t-\tau), \tag{1.25}$$

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