1

Options

OUTLINE

- European call and put options
- payoff diagrams
- how and why options are traded

1.1 What are options?

Throughout the book we use the term *asset* to describe any financial object whose value is known at present but is liable to change in the future. Typical examples are

- shares in a company,
- commodities such as gold, oil or electricity,
- currencies, for example, the value of US $100 in euros.

We will have much to say about assets in subsequent chapters, but let us get straight to the point and define an *option*.

Definition A *European call option* gives its *holder* the right (but not the obligation) to purchase from the *writer* a prescribed asset for a prescribed price at a prescribed time in the future. ◇

The prescribed purchase price is known as the *exercise price* or *strike price*, and the prescribed time in the future is known as the *expiry date*.

To illustrate the idea, suppose that, today, your friend Professor Smart (the writer) writes a European call option that gives you (the holder) the right to buy 100 shares in the International Business Machines (IBM) Corporation for $1000 three months from now. After those three months have elapsed, you would then take one of two actions:

(a) if the actual value of 100 IBM shares turns out to be more than $1000 you would exercise your right to buy the shares from Professor Smart – because you could immediately sell them for a profit.

(b) if the actual value of 100 IBM shares turns out to be less than \$1000 you would not exercise your right to buy the shares from Professor Smart – the deal would not be worthwhile.

Because you are not obliged to purchase the shares, you do not lose money (in case (a) you gain money and in case (b) you neither gain nor lose). Professor Smart, on the other hand, will not gain any money on the expiry date, and may lose an unlimited amount. To compensate for this imbalance, when the option is agreed (today) you would be expected to pay Professor Smart an amount of money known as the *value* of the option.

The direct opposite of a European call option is a European put option.

Definition A *European put option* gives its *holder* the right (but not the obligation) to sell to the *writer* a prescribed asset for a prescribed price at a prescribed time in the future. ◇

The key question that we address in this book is: how much should the holder pay for the privilege of holding an option? In other words, how do we compute a fair option value?

To answer this question we have to devise a *mathematical model* for the behaviour of the asset price, come up with a precise interpretation of 'fairness' and do some analysis. These steps, which take up the next seven chapters, will lead us to the celebrated Black–Scholes formula. Looking at practical issues and more exotic options will then draw us into *computational algorithms*, which take up the bulk of the remainder of the book.

The rest of this chapter is spent on a brief review of how and why options are traded.

1.2 Why do we study options?

Options have become extremely popular; so popular that in many cases more money is invested in them than in the underlying assets. Why do they get so much attention? There are two good reasons.

(1) Options are extremely attractive to investors, both for *speculation* and for *hedging*.
(2) There is a systematic way to determine how much they are worth, and hence they can be bought and sold with some confidence.

Point (2) is the main subject of this book. To illustrate point (1), if you believe that Microsoft Corporation shares are due to increase then you may speculate by becoming the holder of a suitable call option. Typically, you can make a greater profit relative to your original payout than you would do by simply purchasing the shares. On the other hand, if you are the owner of an American company that is committed to purchasing a factory in Germany for an agreed price in euros in three months' time, then you may wish to hedge some risk by taking out an option that makes some profit in the event that the US dollar drops in value against the euro.

A further attraction is that by combining different types of option, an investor can take a position that reaps benefits from various types of asset behaviour. To understand this, it is useful to visualize options in terms of *payoff diagrams*.

We let E denote the exercise price and $S(T)$ denote the asset price at the expiry date. (Of course, $S(T)$ is not known at the time when the option is taken out.) In later chapters, $S(t)$ will be used to denote the asset price at a general time t, and T will denote the expiry date. At expiry, if $S(T) > E$ then the holder of a European call option may buy the asset for E and sell it in the market for $S(T)$, gaining an amount $S(T) - E$. On the other hand, if $E \geq S(T)$ then the holder gains nothing. Hence, we say that the *value* of the European call option at the expiry date, denoted by C, is

$$C = \max(S(T) - E, 0). \tag{1.1}$$

Plotting $S(T)$ on the x-axis and C on the y-axis gives the payoff diagram in Figure 1.1. Consider now a European put option. If, at expiry, $E > S(T)$ then the holder may buy the asset at $S(T)$ in the market and exercise the option by selling it at E, gaining an amount $E - S(T)$. On the other hand, if $S(T) \geq E$ then the holder should do nothing. Hence, the *value* of the European put option at the expiry date, denoted by P, is

$$P = \max(E - S(T), 0). \tag{1.2}$$

The corresponding payoff diagram is plotted in Figure 1.2. Because of their shape, the piecewise linear payoff curves in Figures 1.1 and 1.2 are sometimes referred to as (ice) *hockey sticks*.

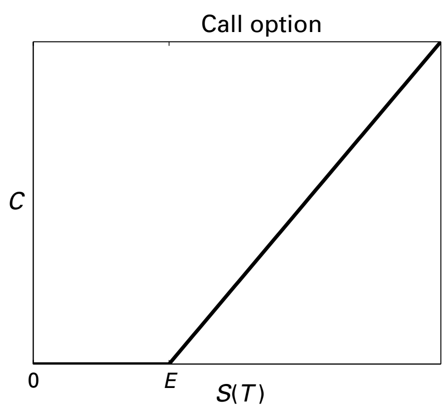

Fig. 1.1. Payoff diagram for a European call. Formula is $C = \max(S(T) - E, 0)$.

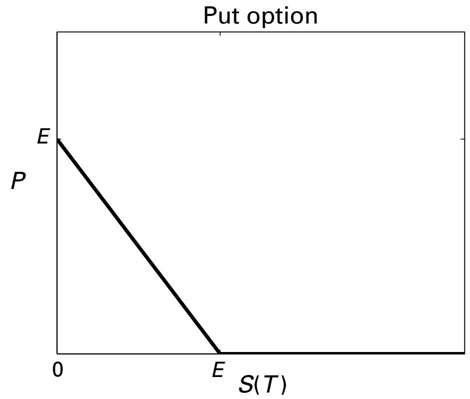

Fig. 1.2. Payoff diagram for a European put. Formula is $P = \max(E - S(T), 0)$.

Now we may plot payoff diagrams for combinations of options. For example, suppose you hold a call option and a put option on the same asset with the same expiry date and the same strike price, E. Then the overall value at expiry is the sum of $\max(S(T) - E, 0)$ and $\max(E - S(T), 0)$, which is equivalent to $|S(T) - E|$, see Exercise 1.2. This combination goes under the unfortunate name of a *bottom straddle*. The holder of a bottom straddle benefits when the asset price at expiry is *far away from the strike price* – it does not matter whether the asset finishes above or below the strike.

Another possibility is to hold a call option with exercise price E_1 and, for the same asset and expiry date, to write a call option with exercise price E_2, where $E_2 > E_1$. At the expiry date, the value of the first option is $\max(S(T) - E_1, 0)$ and the value of the second is $-\max(S(T) - E_2, 0)$. Hence, the overall value at expiry is $\max(S(T) - E_1, 0) - \max(S(T) - E_2, 0)$. The corresponding payoff diagram is plotted in Figure 1.3. This combination gives an example of a *bull spread*. We see from the figure that the holder of such a spread benefits when the asset price finishes above E_1, but gets no extra benefit if it is above E_2.

1.3 How are options traded?

Options can be traded on a number of official exchanges. The first of these, the Chicago Board Options Exchange (CBOE), started in 1973 and there are more than 50 throughout the world in 2004. Most exchanges operate through the use of *market makers*, individuals who are obliged to buy or sell options whenever asked to do so. On request, the market maker will quote a price for the option. More precisely, two prices will be quoted, the *bid* and the *ask*. The bid is the price at which the market maker will buy the option from you and the ask is the

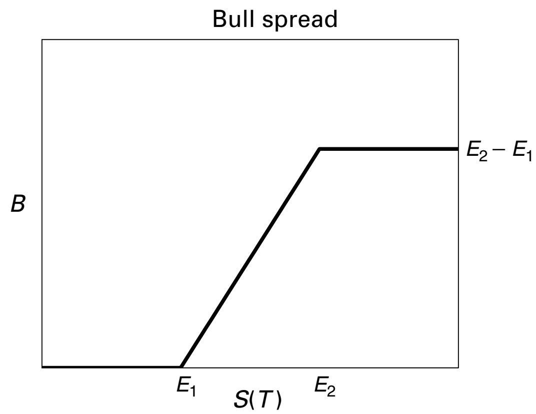

Fig. 1.3. Payoff diagram for a bull spread. Formula is $B = \max(S(T) - E_1, 0) - \max(S(T) - E_2, 0)$.

price at which the market maker will sell it to you. The bid is lower than the ask, because the market maker needs to make a living. The difference between the ask and the bid is known as the *bid–ask spread*. Typically, market makers aim to make their profits from the bid–ask spread and do not wish to speculate on the market; they seek to hedge away their risks using the type of technique that is covered in Chapters 8 and 9.

Options are also traded directly between large financial institutions – so called *over-the-counter* or OTC deals. These options often have nonstandard features that are tailored to the particular needs of the parties involved.

The *Financial Times* newspaper tabulates the prices of some options that may be traded on the London International Financial Futures & Options Exchange (LIFFE). For example, the issue from Friday, 19 September 2003 included the information

		Calls			Puts		
Option		Oct	Nov	Dec	Oct	Nov	Dec
Royal Bk Scot.	1600	67.0	92.5	109.5	29.0	49.0	62.5
(1634.0)	1700	19.5	43.5	59.0	82.0	100.0	112.5

The number 1634.0 is the closing price of The Royal Bank of Scotland's shares from the previous day. The numbers 1600 and 1700 are two exercise prices, in pence. (The *Financial Times* lists information for these exercise prices only, but the exchange offers options for many other exercise prices.) The numbers 67.0, 92.5, 109.5 are the prices of the call options with exercise price 1600 and expiry dates in

Oct, Nov and Dec, respectively (more precisely, for 18:00 on the third Wednesday of each month). Similarly, 19.5, 43.5, 59.0 are the prices of call options with exercise price 1700 for those expiry dates. The numbers 29.0, 49.0, 62.5 give the prices of put options with exercise price 1600 and expiry dates in Oct, Nov and Dec, and 82.0, 100.0, 112.5 are the corresponding put option prices for exercise price 1700. The numbers quoted lie somewhere between the bid and the ask.

The *Wall Street Journal* publishes option data in a similar form. Many providers offer electronic data access, with some basic information being available in the public domain; see Section 5.5 for some pointers.

1.4 Typical option prices

Figure 1.4 shows some prices for call and put options on IBM shares that were available on the New York Stock Exchange on 13 October 2002. Some of the data from Figure 1.4 is repeated in a slightly different format in Figure 1.5. The prevailing asset price, more precisely the price paid at the most recent trade, was 74.25, marked 'Now' in Figure 1.4. Option prices were available for a range of strike prices and expiry times. These prices relate to American, rather than European, options. Americans are introduced in Chapter 18. For the moment we note that an American call has the same value as a European call (assuming that no dividends are paid), and an American put has a higher value than a European put.

In this example, for a given expiry time, the call option price decreases as the strike price increases. This is perfectly reasonable. Increasing the strike price has a negative effect on the payoff and hence reduces the call option's worth. Similarly, the put price increases with increasing strike price. It can also be observed from the figures that, for a given strike price, both the call and the put option prices

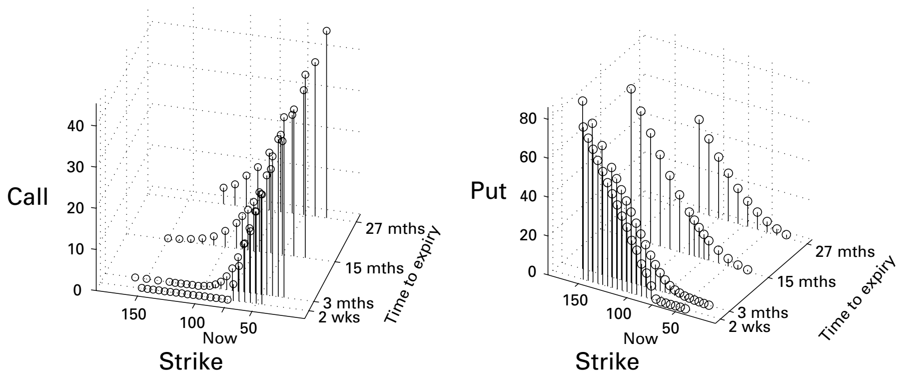

Fig. 1.4. Market values for IBM call and put options, for a range of strike prices and times to expiry.

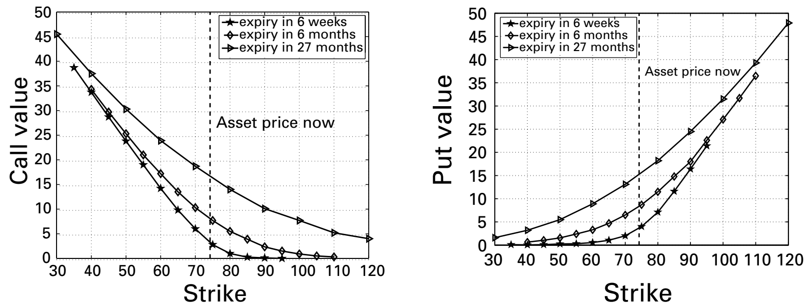

Fig. 1.5. Market values for IBM call (left) and put (right) options, for a range of strike prices and times to expiry. This displays a subset of the data in Figure 1.4.

increase when the time to expiry increases. This behaviour is generic for European call options, as we will see in Section 2.6.

1.5 Other financial derivatives

European call and put options are the classic examples of *financial derivatives*. The term derivative indicates that their value is *derived* from the underlying asset – it has nothing to do with the mathematical meaning of a derivative. This book focuses exclusively on options. We will develop our mathematical analysis with European options in mind, and in later chapters we will introduce American and other more exotic options.

1.6 Notes and references

There are many introductory texts that explain how stock markets operate; see, for example, Dalton (2001); Walker (1991). Chapter 6 of Hull (2000) is also a good source of basic practical information about option trading, including

- what range of expiry dates and exercise prices are typically offered,
- how dividends and stock splits are dealt with, and
- how money and products actually change hands.

Section 5.5 gives the web pages of some stock exchanges.

EXERCISES

1.1. ⋆ Insert the word ‘rise’ or ‘fall’ to complete the following sentences:

The holder of a European call option hopes the asset price will . . .
The writer of a European call option hopes the asset price will . . .

The holder of a European put option hopes the asset price will ...
The writer of a European put option hopes the asset price will ...

1.2. ⋆ Convince yourself that $\max(S(T) - E, 0) + \max(E - S(T), 0)$ is equivalent to $|S(T) - E|$ and draw the payoff diagram for this bottom straddle.

1.3. ⋆⋆ Suppose that for the same asset and expiry date, you hold a European call option with exercise price E_1 and another with exercise price E_3, where $E_3 > E_1$ and also write two calls with exercise price $E_2 := (E_1 + E_3)/2$. This is an example of a *butterfly spread*.[1] Derive a formula for the value of this butterfly spread at expiry and draw the corresponding payoff diagram.

1.4. ⋆ The holder of the bull spread with payoff diagram in Figure 1.3 would like the asset price on the expiry date to be at least as high as E_2, but, if it is, the holder does not care how much it exceeds E_2. Make similar statements about the holders of the bottom straddle in Exercise 1.2 and the butterfly spread in Exercise 1.3.

1.7 Program of Chapter 1 and walkthrough

Our first MATLAB program uses basic plotting commands to draw a bull spread payoff diagram, as shown in Figure 1.3, for particular parameters E_1 and E_2. The program is called `ch01` and is stored in the file `ch01.m`. It is listed in Figure 1.6. The program is run by typing `ch01` at the MATLAB prompt. The first three lines begin with the symbol `%` and hence are *comment lines*. These lines are ignored by MATLAB, they are used to provide information to humans who are reading through the code. Comment lines may be inserted anywhere, but those at the start of a code have a special property – typing `help ch01` causes the information

```
CH01    Program for chapter 1
Plots a simple payoff diagram
```

to be echoed to the user. It is customary for the first comment line to begin with the name of the file in capital letters, even though the file itself has a lower case name.

The first command, `clf`, clears the current figure window, so that any previous graphical output is removed. The lines `E1 = 2;` and `E2 = 4;` are *assignment statements*. Variables `E1` and `E2` are automatically created and given those values. The semi-colon at the end of each line causes output to be suppressed. Without those semi-colons, the information

```
E1 = 2
E2 = 4
```

would be displayed on your screen. The line `S = linspace(0,6,100)` sets up a one-dimensional array `S` with 100 components, equally spaced between 0 and 6. This could be confirmed after running the program by typing `S` at the MATLAB prompt. The command `max(S-E1,0)` creates a one-dimensional array whose ith entry is the maximum of `S(i)-E1` and `0`. Note that MATLAB is happy to mix arrays and scalars, and will apply the `max` function in a componentwise manner. Overall

[1] Serve with warm toast.

the line `B = max(S-E1,0) - max(S-E2,0);` creates a one-dimensional array `B` of payoff values corresponding to `S`.

```
%CH01     Program for chapter 1
%
% Plots a simple payoff diagram

clf
E1 = 2;
E2 = 4;

S = linspace(0,6,100);
B = max(S-E1,0)-max(S-E2,0);
plot(S,B)
ylim([0,3])

xlabel('S')
ylabel('B')
title('Bull Spread Payoff')
grid on
```

Fig. 1.6. Program of Chapter 1: `ch01.m`.

We then plot the payoff diagram with `plot(S,B)`. By default, MATLAB chooses the range for the axes, the location of the axis tick marks, the colour and type of the line, and many other features. These may be altered with extra commands or via the menu-driven toolbars in the figure window. We have specified `ylim([0,3])`, which overrides the y-axis limits that MATLAB would otherwise choose automatically. Axis labels and a title are produced by `xlabel('S')`, `ylabel('B')` and `title('Bull Spread Payoff')`. The final command, `grid on`, causes horizontal and vertical dotted reference lines to appear in the plot. Running the program, that is, typing `ch01` at the prompt, puts a picture similar to Figure 1.3 in a pop-up figure window.

Typing `help linspace`, `help max`, `help plot`, etc., at the command line gives more information about those functions, and MATLAB's online documentation, roused by typing `doc`, forms a hypertext style manual.

PROGRAMMING EXERCISES

P1.1. Use the `input` command to produce a variant of `ch01` that allows `E1` and `E2` to be specified by the user.

P1.2. Create a program that plots the payoff diagram for a butterfly spread, as described in Exercise 1.3.

Quotes

Because the action is faster and the margins thinner – five percent down
will buy you a futures contract on the DAX 30 in Frankfurt,

the CAC 40 in Paris, the FTSE 100 in London, the Nikkei 225 in Tokyo,
or the Standard & Poor's 500 in New York – trading in derivatives now swamps
the markets on which they depend.

THOMAS A. BASS (Bass, 1999)

If you believe an asset will rise in price,
then you may buy a call option to capture a very large potential gain
with a small investment;
however, if your belief is wrong
then you may very easily lose your entire option investment.

ROBERT ALMGREN (Almgren, 2002)

Imagine visiting your local used-car dealer to sell him your old Ford.
He kicks the tires, points to a dent in the fender,
and offers you a hundred bucks.
Suppose the following day you are tempted to go back
and buy your Ford off the lot.
The dealer will point to the low mileage and tell you
that he can't let the car go for less than two hundred dollars.
This is the difference between the *bid*, buying price, and the *ask*, selling price.

THOMAS A. BASS (Bass, 1999)

The Pterodactyl is very rarely encountered in real trading.
The technicians may, however, wish to know
that it consists of a spread of traditional butterflies. . . .
The use of this position is not recommended unless the author needs a new car.

A. L. H. SMITH (Smith, 1986)

Recent history is replete with examples of derivatives trading gone awry.

PHILIP MCBRIDE JOHNSON (Johnson, 1999)

Winter, spring, summer or fall,
all you have to do is call. . . .

CAROL KING, *You've Got a Friend*, EMI Music Inc.