Thermodynamic Formalism

The Mathematical Structures of Equilibrium Statistical Mechanics

Second Edition

Reissued in the Cambridge Mathematical Library this classic book outlines the theory of thermodynamic formalism which was developed to describe the properties of certain physical systems consisting of a large number of subunits. It is aimed at mathematicians interested in ergodic theory, topological dynamics, constructive quantum field theory, and the study of certain differentiable dynamical systems, notably Anosov diffeomorphisms and flows. It is also of interest to theoretical physicists concerned with the computational basis of equilibrium statistical mechanics. The level of the presentation is generally advanced, the objective being to provide an efficient research tool and a text for use in graduate teaching. Background material on physics has been collected in appendices to help the reader. Extra material is given in the form of updates of problems that were open at the original time of writing and as a new preface specially written for this edition by the author.

DAVID RUELLE is a Professor Emeritus at the Institut des Hautes Etudes Scientifiques, Bures-sur-Yvette, Paris.

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DAVID RUELLE
Institut des Hautes Etudes Scientifiques

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We haven't seen everything yet,
but when we do
it won't be for the first time
or the last, either.
You know us.

J. VINOGRAD

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Foreword to the first edition

Thermodynamics is still, as it always was, at the center of physics, the standard-bearer of successful science. As happens with many a theory, rich in applications, its foundations have been murky from the start and have provided a traditional challenge on which physicists and mathematicians alike have tested their latest skills.

Ruelle's book is perhaps the first entirely rigorous account of the foundations of thermodynamics. It makes heavier demands on the reader's mathematical background than any volume published so far. It is hoped that ancillary volumes in time will be published which will ease the ascent onto this beautiful and deep theory; at present, much of the background material can be gleaned from standard texts in mathematical analysis. In any case, the timeliness of the content shall be ample reward for the austerity of the text.

GIOVANNI GALLAVOTTI
General Editor, Section on Statistical Mechanics
and
GIAN-CARLO ROTA
Editor, Encyclopedia of Mathematics and its Applications.

Preface to the first edition

The present monograph is based on lectures given in the mathematics departments of Berkeley (1973) and of Orsay (1974–1975). My aim has been to describe the mathematical structures underlying the thermodynamic formalism of equilibrium statistical mechanics, in the simplest case of classical lattice spin systems.

The thermodynamic formalism has its origins in physics, but it has now invaded topological dynamics and differentiable dynamical systems, with applications to questions as diverse as the study of invariant measures for an Anosov diffeomorphism (Sinai [3]), or the meromorphy of Selberg's zeta function (Ruelle [7]). The present text is an introduction to such questions, as well as to more traditional problems of statistical mechanics, like that of phase transitions. I have developed the general theory – which has considerable unity – in some detail. I have, however, left aside particular techniques (like that of correlation inequalities) which are important in discussing examples of phase transitions, but should be the object of a special study.

Statistical mechanics extends to systems vastly more general than the classical lattice spin systems discussed here (in particular to quantum systems). One can therefore predict that the theory discussed in this monograph will extend to vastly more general mathematical setups (in particular to non-commutative situations). I hope that the present text may contribute some inspiration to the construction of the more general theories, as well as clarifying the conceptual structure of the existing formalism.

Preface to the second edition

Twenty-five years have elapsed since the first printing of *Thermodynamic Formalism*, and in the meantime a number of significant developments have taken place in the area indicated by the subtitle *The Mathematical Structures of Equilibrium Statistical Mechanics*. Fortunately, our monograph was concerned with basics, which have remained relatively unchanged, so that *Thermodynamic Formalism* remains frequently quoted. In the present re-issue, some misprints have been corrected, and an update on the open problems of Appendix B has been added. We shall now outline briefly some new developments and indicate unsystematically some source material for these developments. The mathematical aspects of the statistical mechanics of lattice systems, including phase transitions, are covered in the monographs of Sinai [a], and Simon [b]. It may be mentioned that research in this important domain has become less active than it was in the 1960s, '70s, and '80s (but a really good idea might reverse this evolution again). The relation between Gibbs and equilibrium states has been extended to more general topological situations (see Haydn and Ruelle [c]). For a connection of Gibbs states with non-commutative algebras and K-theory, see for instance [d] and the references given there, in particular to the work of Putnam. Particularly fruitful developments have taken place which use the concepts of transfer operators and dynamical zeta functions. In the present monograph these concepts are introduced (in Chapters 5 and 7) in a situation corresponding to uniformly hyperbolic smooth dynamics (Anosov and Axiom A systems, here presented in the topological setting of Smale spaces). The hyperbolic orientation has led to very interesting results concerning the distribution of periods of periodic orbits for hyperbolic flows (in particular the lengths of geodesics on a manifold of negative curvature). These results have been beautifully presented in the monograph of Parry and Pollicott [e]. More recent results of Dolgopyat on exponential decay of correlation for hyperbolic flows [f,g] may be mentioned at this point. It was realized by Baladi and Keller that the ideas of transfer operators

and dynamical zeta functions work well also for piecewise monotone maps of the interval (which are not uniformly hyperbolic dynamical systems). This new development can in particular be related to the kneading theory of Thurston and Milnor. We have thus now a much more general theory of transfer operators, very usefully presented in the monograph of Baladi [h], which has in particular an extensive bibliography of the subject. For a general presentation of dynamical zeta functions see also Ruelle [i]. As we have seen, the ideas of statistical mechanics, of a rather algebraic nature, have found geometric applications in smooth dynamics, and particularly the study of hyperbolic systems. Extensions to nonuniformly hyperbolic dynamical systems are currently an active domain of research, with SRB states playing an important role for Sinai, Ruelle, Bowen, Strelcyn, Ledrappier, Young, Viana, . . .). This, however, is another story.

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