Fourier and Laplace Transforms

This book presents in a unified manner the fundamentals of both continuous and discrete versions of the Fourier and Laplace transforms. These transforms play an important role in the analysis of all kinds of physical phenomena. As a link between the various applications of these transforms the authors use the theory of signals and systems, as well as the theory of ordinary and partial differential equations.

The book is divided into four major parts: periodic functions and Fourier series, non-periodic functions and the Fourier integral, switched-on signals and the Laplace transform, and finally the discrete versions of these transforms, in particular the Discrete Fourier Transform together with its fast implementation, and the z-transform. Each part closes with a separate chapter on the applications of the specific transform to signals, systems, and differential equations. The book includes a preliminary part which develops the relevant concepts in signal and systems theory and also contains a review of mathematical prerequisites.

This textbook is designed for self-study. It includes many worked examples, together with more than 450 exercises, and will be of great value to undergraduates and graduate students in applied mathematics, electrical engineering, physics and computer science.

Fourier and Laplace Transforms

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Preface

This book arose from the development of a course on Fourier and Laplace transforms for the Open University of the Netherlands. Originally it was the intention to get a suitable course by revising part of the book *Analysis and numerical analysis*, part 3 in the series *Mathematics for higher education* by R. van Asselt et al. (in Dutch). However, the revision turned out to be so thorough that in fact a completely new book was created. We are grateful that Educaboek was willing to publish the original Dutch edition of the book besides the existing series.

In writing this book, the authors were led by a twofold objective:
- the ‘didactical structure’ should be such that the book is suitable for those who want to learn this material through self-study or distance teaching, without damaging its usefulness for classroom use;
- the material should be of interest to those who want to apply the Fourier and Laplace transforms as well as to those who appreciate a mathematically sound treatment of the theory.

We assume that the reader has a mathematical background comparable to an undergraduate student in one of the technical sciences. In particular we assume a basic understanding and skill in differential and integral calculus. Some familiarity with complex numbers and series is also presumed, although chapter 2 provides an opportunity to refresh this subject.

The material in this book is subdivided into parts. Each part consists of a number of coherent chapters covering a specific part of the field of Fourier and Laplace transforms. In each chapter we accurately state all the *learning objectives*, so that the reader will know what we expect from him or her when studying that particular chapter. Besides this, we start each chapter with an introduction and we close each chapter with a summary and a selftest. The *selftest* consists of a series of exercises that readers can use to test their own knowledge and skills. For selected exercises, answers and extensive hints will be available on the CUP website.

Sections contain such items as definitions, theorems, examples, and so on. These are clearly marked in the left margin, often with a number attached to them. In the remainder of the text we then refer to these numbered items.

For almost all theorems proofs are given following the heading *Proof*. The end of a proof is indicated by a right-aligned black square: ■. In some cases it may be wise to skip the proof of a theorem in a first reading, in order not to lose the main line of argument. The proof can be studied later on.

Examples are sometimes included in the running text, but often they are presented separately. In the latter case they are again clearly marked in the left margin (with possibly a number, if this is needed as a reference later on). The end of an example is indicated by a right-aligned black triangle: ◄.

Mathematical formulas that are displayed on a separate line may or may not be numbered. Only formulas referred to later on in the text have a number (right-aligned and in brackets).

Some parts of the book have been marked with an asterisk: *. This concerns elements such as sections, parts of sections, or exercises which are considerably more difficult than the rest of the text. In those parts we go deeper into the material or we present more detailed background material. The book is written in such a way that these parts can be omitted.

The major part of this book has been written by Dr R.J. Beerends and Dr H.G. ter Morsche. Smaller parts have been written by Drs J.C. van den Berg and Ir E.M. van de Vrie. In writing this book we gratefully used the comments made by Prof. Dr J. Boersma and the valuable remarks of Ir G. Verkroost, Ir R. de Roo and Ir F.J. Oosterhof.

Finally we would like to thank Drs A.H.D.M. van Gijsel, E.D.S. van den Heuvel, H.M. Welte and P.N. Truijen for their unremitting efforts to get this book to the highest editorial level possible.