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Discrete and Continuous Nonlinear Schrödinger Systems

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M. J. Ablowitz, B. Prinari and A. D. Trubatch
Frontmatter
More information

Appendices

Preface

Nonlinear systems are generic in the mathematical representation of physical phenomena. It is unusual for one to be able to find solutions to most nonlinear equations. However, a certain physically significant subclass of problems admits deep mathematical structure that further allows one to find classes of exact solutions. Solitons are a particularly important subclass of such solutions. Solitons are localized waves that, in an appropriate sense, interact elastically with each other. They have proved to be extremely interesting to physicists and engineers due, in part, to their localized and stable nature.

This broad field of study is sometimes called "soliton theory" or "integrable systems." This field has witnessed numerous important developments, which have been studied intensively worldwide over the past 30 years. Some of the directions that researchers have pursued include the following: direct methods to find solutions; studies of the underlying analytic structure of the equations; associated Painlevè-type solutions and relevant generalizations; tests to locate integrable systems; studies of the underlying geometric structures inherent in integrable systems; Bäcklund and Darboux transformations, which can be used to produce new classes of solutions; and so on.

In principle, one would like to be able to solve the general initial-value problem associated with these special nonlinear soliton systems. Depending on the boundary conditions under consideration, sometimes this is feasible. Moreover, in some cases, the mathematical representation is constructive, useful, and indeed elegant. The case for which the Cauchy problem admits a complete solution and the qualitative properties of the solution are well understood corresponds to initial data decaying sufficiently rapidly at infinity. Because of the constructive and relatively explicit nature of the solution, this case has received considerable attention. The method of solution is usually referred to as the inverse scattering transform (IST). The IST applies to many interesting nonlinear soliton systems, including nonlinear partial differential equations in $1 + 1$

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(one space–one time) and 2 + 1 (two space–one time) dimensions, nonlinear discrete (difference) evolution equations, and singular integro-differential equations. A discussion of the IST as it applies to many of these cases can be found in various monographs (cf. [6]).

Nevertheless, there are particular problems that, because of their wide applicability, deserve special attention. It is the purpose of this book to investigate one such important set of equations: nonlinear Schrödinger (NLS) systems. NLS systems in continuous media have been studied heavily since the mid-1960s. The continuous scalar NLS equation arises in the prototypical situation governing slowly varying waves of small amplitude (cf. [30]). In the early 1970s, it was shown [91] that the NLS equation governs the long-distance pulse propagation in optical fibers. In the late 1980s and 1990s, it was discovered [131], [122] that vector NLS systems govern the propagation of polarized waves in optical fibers. Today, these NLS equations, with suitable modifications and additional terms, are used routinely to make predictions about the transmission of information in fibers. Without doubt, wave transmission in optical fibers, used for communication purposes, is an application of critical importance.

In recent years there has developed significant interest in the study of nonlinear waves in media that are governed by nonlinear semi-discrete evolution systems (discrete in space–continuous time). Once again there is a class of physically interesting nonlinear systems that includes the so-called discrete NLS equations. These discrete NLS equations reduce to the continuous NLS equation when the discretization parameter vanishes.

For certain scalar- and vector-continuous and discrete NLS systems, the IST method can be applied in an effective and complete way. The initial-value problem, for given data decaying sufficiently rapidly at infinity, can be linearized in terms of integral equations. Multisoliton solutions can be obtained in all cases. This is described in detail in this book. In an appendix we also describe the IST associated with another class of important discrete equations: the Toda lattice [165] and the so-called nonlinear ladder network [121], [98]. While the research literature has many (but not all) of the results obtained here, it requires researchers to access numerous papers. Early fundamental research on NLS systems appears in the papers of Zakharov and Shabat [192], who first analyzed the scalar-continuous NLS equation; Manakov [120], regarding the continuous-vector NLS equation; and by Ablowitz and Ladik [11, 12], who introduced the scalar integrable discrete NLS equation. The vector extension of the discrete NLS system is more recent (cf. [18]). The unified description of these physically interesting integrable discrete and continuous NLS systems within the context of the IST methodology does not appear anywhere else. One of our motivations in writing this book was to develop the direct and inverse scattering formalism based on the Riemann–Hilbert approach. From a pedagogical point of view, readers will find that the structure follows the one laid out for the Korteweg–de Vries equation in the monograph of Ablowitz and Clarkson [6]. Here we have also attempted to include many of the mathematical details, to make the book suitable for students as well as researchers who wish to study this topic.

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