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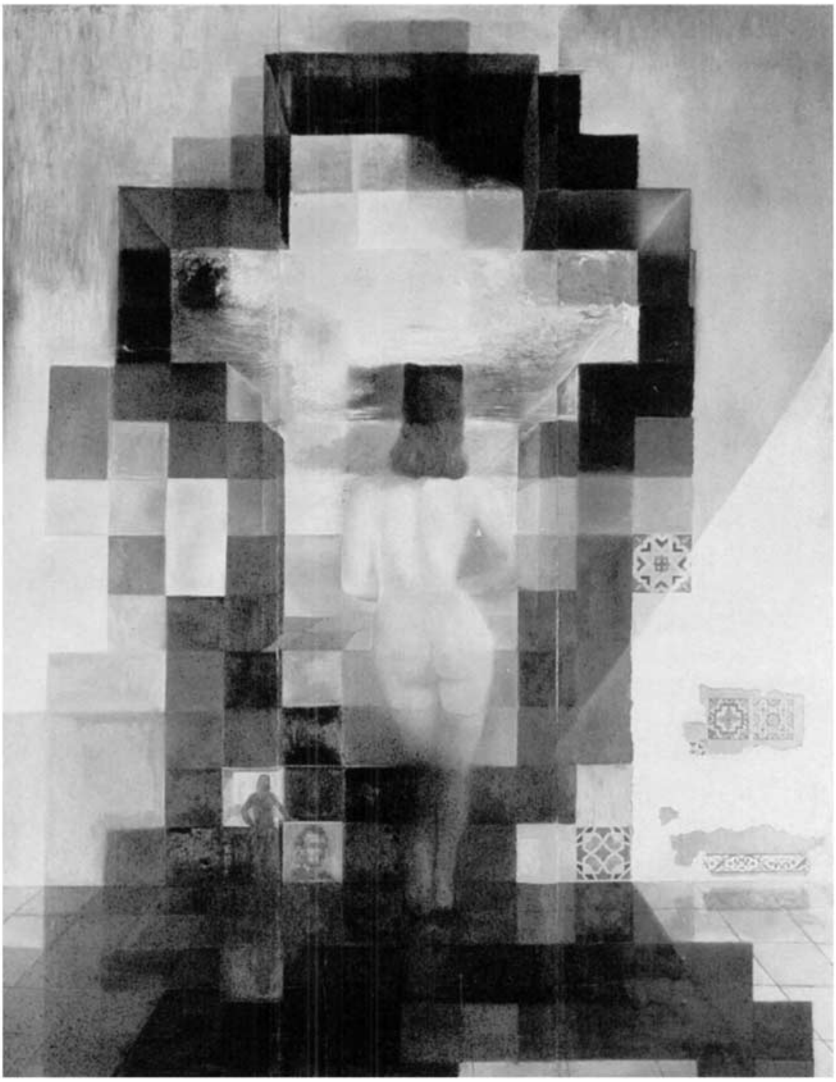

Gala Contemplating the Mediterranean Sea (detail). © Salvador Dali, Gala-Salvador Dali Foundation, DACS, London, 2003. Image supplied by Bridgeman Art Gallery. One of the most important concepts presented in this book is that of intermediate asymptotics. It is illustrated in chapter 2, Figure 2.3, by a tiled version of the photograph of Abraham Lincoln on a $5 bill (Harmon 1973). The paper by Harmon, and, in particular, this tiled picture inspired Salvador Dali to create in 1976 the painting presented here, where some tiles are themselves pictures: of his wife Gala entering the sea, Harmon's original tiled picture of Lincoln, and others. This painting is in fact an excellent example of multiscale intermediate asymptotics.

SCALING

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In grateful memory of my dear friends
Yakov Borisovich Zeldovich and Alexandr Solomonovich Kompaneets

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Foreword

For the past seven years students and faculty at the University of California at Berkeley have had the privilege of attending lectures by Professor G.I. Barenblatt on mechanics and related topics; the present book, which grew out of some of these lectures, extends the privilege to a wider audience. Professor Barenblatt explains here how to construct and understand self-similar solutions of various physical problems, i.e. solutions whose structure recurs over differing length or time scales and different parameter ranges. Such solutions are often the key to understanding complex phenomena; there is no universal recipe for finding them, but the tools that can be useful, including dimensional analysis and nonlinear eigenvalue problems, are explained here with admirable conciseness and clarity, together with some of the multifarious uses of self-similarity in intermediate asymptotics and their connection with wave propagation and the renormalization group. Whenever possible, Professor Barenblatt shuns dry and distant abstraction in favor of the telling example from his incomparable stock of such examples; with the appearance of this book, there is no longer any excuse for any scientist not to master these simple, elegant, crucial and sometimes surprising ideas.

This book is also very timely. Dimensional analysis and simple similarity arguments (what is called here complete similarity) are quite familiar to most scientists, with the possible exception of many mathematicians, yet the deeper, more beautiful and exceptionally useful idea of incomplete similarity, with its extraordinary ramifications, is not yet part of everyone's scientific culture. Maybe part of the reason is the absence of a book that is both sound and accessible. After all, the original papers by Barenblatt and Zeldovich and by others were addressed to the expert; the previous books by Professor Barenblatt are rich in theory and examples and therefore not always easy to read; the very interesting book by Goldenfeld on the renormalization group, where the connection with incomplete similarity is carefully explained, assumes a wider

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knowledge of modern theoretical physics than can be expected from experts in other fields. If this were the only reason for ignorance then the current book would solve the problem: it is accessible and direct and can be read with profit even by undergraduates.

I suspect however that the difficulty in assimilating the notion of incomplete similarity had deeper sources as well: here is a simple mathematical procedure which makes it possible to contemplate, and indeed often rationally analyze, the disquieting possibility that small parameters may have persistent large-scale effects, not confined to the margins of a physical domain as in most textbook examples of singular perturbations, and not safely relegated to the exotic realm of phase transitions and critical phenomena, but observable in simple physical situations. It is natural to resist ideas which fly in the face of comfortable habits of thought, but this would be a big mistake: the possibility is real, and its understanding requires full attention because it is important. Indeed, the major pedagogical outcome of the example involving turbulent boundary layers in Chapter 8 is to show how admitting the possibility of incomplete similarity can lead to conclusions that are innovative, striking and subversive of long-accepted beliefs.

The importance of this range of ideas is now growing fast. We are at the beginning of the age of multiscale science and multiscale computation, with a growing need to understand not only phenomena on each of many scales but also the interaction between phenomena at very different scales; such interactions abound in fields such as materials science and biology and by definition occur when the impact of the parameters that describe small scales propagates across the full range of scales in a problem. Incomplete similarity is the basic paradigm of how such an impact propagates and is a major tool in the analysis and understanding of new classes of problems and in the emerging art of solving them on a computer.

What we have before us is a clear, masterful and uniquely timely book by one of the great applied mathematicians, who brings us his own great knowledge and experience of a key topic and, furthermore, some of the accumulated experience of the great Soviet school of applied mathematics in which he grew up and of which he is the most distinguished living embodiment.

Alexandre Chorin
University Professor
University of California

Preface

Applied mathematics is the art of constructing mathematical models of phenomena in nature, engineering and society. In constructing models it is impossible to take into account all the factors which influence the phenomenon; therefore some of the factors should be neglected, and only those factors which are of crucial importance should be left. So we say that every model is based on a certain *idealization* of the phenomenon. In constructing the idealizations the phenomena under study should be considered at 'intermediate' times and distances (think of the impressionists!). These distances and times should be sufficiently large for details and features which are of secondary importance to the phenomenon to disappear. At the same time they should be sufficiently small to reveal features of the phenomena which are of basic value. We say therefore that every mathematical model is based on '*intermediate asymptotics*'.[1]

The construction of an appropriate idealization is the most difficult stage in mathematical modelling. It is always performed in steps. Trial and error, and comparison with experiments, physical or computational, play a basic role. The reader can find a truly remarkable and very exact description of this process in Maurice Maeterlinck's *Blue Bird* – it was not by accident that this play won Maeterlinck a Nobel Prize.

Mathematics is considered to be the language of science, because its role in constructing mathematical models is similar to that of language in human communications. All people use language. However, among users of language a particularly important group can be distinguished. These are the *authors*:

[1] The writing of *War and Peace* by Lev Tolstoy is a remarkable example of such an intermediate vision. The novel was extemely successful from the very beginning because lesser details of the Napoleonic war had decayed in people's memories whereas gigantic historic events and their influence on human destinies appeared unshadowed both by small details in the past and current events in the life of society. I am afraid that the literature has missed such an opportunity for the 'Thirty Years War', 1914–1945.

poets, novelists, playwrights, essayists etc., who create fictional images and paradigms – idealized models of people and social phenomena around them. The greatest of these paradigms continue to live for centuries and even millenia. They transform human culture and, sometimes, language itself.

To a certain extent a similar role is played by applied mathematicians. Using the language of mathematics, developing and transforming it when necessary,[2] applied mathematicians create their paradigms – models of phenomena. These idealized models should be sufficiently complete images of phenomena, and at the same time they should enable further mathematical study – analytic and experimental, computational and physical. The right to existence of these models is determined by one thing only: they have to work, i.e. they must predict the behavior of the systems under study in interesting but as yet unexplored ranges of external conditions. When this goal is achieved, it leads to practical applications.

Of special importance is the following fact: the construction of models, like any genuine art, cannot be taught by reading books and/or journal articles. the reason is that in articles and especially in books the 'scaffolding' is removed, and the presentation of results is shown not in the way that they were actually obtained but in a different, perhaps more elegant way. Therefore it is very difficult, if not impossible, to understand the real 'strings' of the work: how the author really came to certain results and how to learn to obtain results on your own.

Therefore, just like in every art an appropriate way to become an applied mathematician is to become part of a good school, i.e. to work for some time in a team under the guidance of a genuine master. Knowledge, experience and inspiration come from constant discussions on scientific and non-scientific matters, not only with the master gender but also with colleagues and other members of the team. Conversations about music, literature, visual art etc. are as important in the process of education as are scientific debates. To be part of a school means to live in a unique environment where an intensive flow of ideas is customary. The present author was fortunate to belong to the school of A.N. Kolmogorov, and to work closely for a long time with Ya.B. Zeldovich, after being taught and strongly influenced by his first teacher, the eminent analyst B.M. Levitan. I want to mention here also a remarkable physicist and teacher of physics, A.S. Kompaneets. I owe him so much for my understanding of physics. My gratitude to these outstanding people is immense.

[2] I emphasize *when necessary*. To reproach applied mathematicians for using well-known tool of analysis to construct models is ridiculous: it is the same as reproaching Rafael for not inventing new brushes and paints. Leonardo did, and without any improvement to his paintings this made great trouble for future restorers.

As with every art, constructing intermediate asymptotics and models has many practical devices and tricks. They should be assimilated. Moreover, they should enter the conscience and subconscience of a researcher who has decided to become an applied mathematician. One of them is the ability to extract from available evidence *scaling laws*. One may ask, why is it that scaling laws are of such distinguished importance? The answer is that *scaling laws never appear by accident*. They always manifest a property of a phenomenon of basic importance, 'self-similar' intermediate asymptotic behavior: the phenomenon, so to speak, repeats itself on changing scales. This behavior should be discovered, if it exists, and its absence should also be recognized. The discovery of scaling laws very often allows an increase, sometimes even a drastic change, in the understanding of not only a single phenomenon but a wide branch of science. The history of science of the last two centuries knows many such examples.

It is this subject that the present book is about. In writing this book I have followed the rule which I learned from my great mentor: never start teaching or research in a new field of applied mathematics from general concepts, statements, theories and theorems. Consider some instructive examples and the general theory will come and be cast naturally.

In Berkeley I delivered a course of 30 double lectures which closely follows the present book. I expect that this book can be considered as a textbook for graduate and advanced undergraduate courses. Some parts can also be used in courses such as the strength of materials, the theory of elasticity, fracture, electrodynamics, heat and mass transfer, fluid mechanics, the flow of non-Newtonian fluids and many others. However, if the course is to be taken as a whole then my recommendation is to consider the first six-and-a-half chapters (up to the biological example) as mandatory. It will take approximately three-quarters of a semester. The remaining time can be used for the detailed presentation of a particular topic appropriate for the audience. I myself have selected turbulence. To many people the subject presented in the chapter on turbulence, based on our joint work with A.J. Chorin and V.M. Prostokishin, may seem rather controversial, although not to me. This example gives a unique possibility of presenting together general pricincples and the use of freshly obtained large experimental databases.

I have previously written several books about the subject presented here. (I remember with deep gratitude the publisher from 'Gidrometeoizdat', Mrs O.V. Vlasova, Mrs T.G. Nedoshivina, and Mrs L.L. Belen'kaya. They published my first book in Russian in spite of the serious risk of losing their jobs.) Naturally, some material from my earlier books will find its place in the present book too, particularly material regarding dimensional analysis and physical similarity, in only slightly modified form. However, the central part of this book is entirely new: in particular I have replaced some complicated and difficult basic examples with simpler ones.

I want to express my thanks to Cambridge University Press (Dr D. Tranah and Dr A. Harvey). In fact, the very idea that I should write such an 'intermediate' book matching my inaugural lecture (Barenblatt 1994) and the large book (Barenblatt 1996) belongs with these gentlemen.

I want to express my gratitude to Professor V.M. Prostokishin, who attended all my lectures and gave me important advice both about the lectures and the present book. I am grateful to Professor L.C. Evans and Professor M. Brenner for reading the manuscript and for valuable comments. I want to thank Professors S. Kamin, R. Dal Passo, M. Bertsch, N. Goldenfeld, D.D. Joseph, L.A. Peletier, G.I. Sivashinsky and J.L. Vazquez for the stimulating and friendly exchange of thoughts concerning the subjects presented in this book over many years. I thank Mrs Deborah Craig for processing the manuscript.

To my friend Alexandre Chorin I want to express special thanks for our remarkable time in Berkeley. I have learned from him a lot, in particular his basic paradigm of computational science: this is a different, independent and very productive way of mathematical modelling. I hope to be able to use this knowledge in my future work.