Speakable and Unspeakable in Quantum Mechanics

John Bell FRS was one of the leading expositors and interpreters of modern quantum theory. He is particularly famous for his discovery of the crucial difference between the predictions of conventional quantum mechanics and the implications of local causality, a concept insisted on by Einstein. John Bell's work has played a major role in the development of our current understanding of the profound nature of quantum concepts and of the fundamental limitations they impose on the applicability of the classical ideas of space, time, and locality.

This book includes all of John Bell's published and unpublished papers on the conceptual and philosophical problems of quantum mechanics, including two papers that appeared after the first edition was published. All the papers have been reset, the references put in order and minor corrections made. The book includes a short preface written by the author for the first edition, and also an introduction by Alain Aspect that puts into context John Bell's enormous contribution to the quantum philosophy debate.

This collection will be of interest to graduate students and research workers in physics with an interest in the conceptual foundations of quantum theory. It will also be of value to philosophers of science working in this area.

Collected papers on quantum philosophy

Speakable and Unspeakable in Quantum Mechanics

J. S. BELL
CERN

With an Introduction by Alain Aspect

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To my Mother and Father

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On the hypothesis that the Schrödinger equation is exact. TH-1424-CERN October 27, 1971. Contribution to the International Colloquium on Issues in Contemporary Physics and Philosophy of Science, and their Relevance for our Society, Penn State University, September 1971. Reproduced in *Epistemological Letters*, July 1978, pp 1–28, and here in revised form as 15. Omitted.

Subject and Object. In *The Physicist's Conception of Nature* Dordrecht-Holland, D. Reidel (1973) pp 687–90.

On wave packet reduction in the Coleman–Hepp model. *Helvetica Physica Acta* **48** (1975) 93–8.

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The measurement theory of Everett and de Broglie's pilot wave. In *Quantum Mechanics, Determinism, Causality, and Particles*, edited by M. Flato *et al.* Dordrecht-Holland, D. Reidel, (1976) pp 11–17.

Free variables and local causality. *Epistemological Letters*, Feb. 1977.

Atomic-cascade photons and quantum-mechanical nonlocality. *Comments on Atomic and Molecular Physics* **9** (1980) pp 121–6. Invited talk at the Conference of the European Group for Atomic spectroscopy, Orsay-Paris, 10–13 July, 1979.

de Broglie-Bohm, delayed-choice double-slit experiment, and density matrix. *International Journal of Quantum Chemistry*: Quantum Chemistry Symposium 14 (1980) 155–9.

Quantum mechanics for cosmologists. In *Quantum Gravity* 2, editors C. Isham, R. Penrose, and D Sciama. Oxford, Clarendon Press (1981) pp 611–37. Revised version of 'On the hypothesis that the Schrödinger equation is exact' (see above).

Bertlmann's socks and the nature of reality. *Journal de Physique*, Colloque C2, suppl. au numero 3, Tome 42 (1981) pp C2 41–61.

On the impossible pilot wave. *Foundations of Physics* **12** (1982) pp 989–99.

Speakable and unspeakable in quantum mechanics. Introduction remarks at Naples–Amalfi meeting, May 7, 1984.

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Beables for quantum field theory. 1984 Aug 2, CERN-TH. 4035/84.

Six possible worlds of quantum mechanics. *Proceedings of the Nobel Symposium 65: Possible Worlds in Arts and Sciences*. Stockholm, August 11–15, 1986.

EPR correlations and EPW distributions. In *New Techniques and Ideas in Quantum Measurement Theory* (1986). New York Academy of Sciences.

Are there quantum jumps? In *Schrödinger. Centenary of a polymath* (1987). Cambridge University Press.

Against 'measurement'. In *62 Years of Uncertainty: Erice, 5–14 August* 1989. Plenum Publishers.

La nouvelle cuisine. In *Between Science and Technology*, editors A. Sarlemijn and P. Kroes. Elsevier Science Publishers (1990).

Preface to the first edition

Simon Capelin, of Cambridge University Press, suggested that I send him my papers on quantum philosophy and let him make them into a book. I have done so. The papers, from the years 1964–1986, are presented here in the order, as far as I now can tell, in which they were written. But of course that is not the order, if any, in which they should be read.

Papers 18 and 20, 'Speakable and unspeakable in quantum mechanics' and 'Six possible worlds of quantum mechanics', are nontechnical introductions to the subject. They are meant to be intelligible to nonphysicists. So also is most of paper 16, 'Bertlmann's socks and the nature of reality', which is concerned with the problem of apparent action at a distance.

For those who know something of quantum formalism, paper 3, 'The moral aspect of quantum mechanics', introduces the infamous 'measurement problem'. I thank Michael Nauenberg, who was co-author of that paper, for permission to include it here. At about the same level, paper 17, 'On the impossible pilot wave', begins the discussion of 'hidden variables', and of related 'impossibility' proofs.

More elaborate discussions of the 'measurement problem' are given in paper 6, 'On wavepacket reduction in the Coleman–Hepp model', and in 15, 'Quantum mechanics for cosmologists'. These show my conviction that, despite numerous solutions of the problem 'for all practical purposes', a problem of principle remains. It is that of locating precisely the boundary between what must be described by wavy quantum states on the one hand, and in Bohr's 'classical terms' on the other. The elimination of this shifty boundary has for me always been the main attraction of the 'pilot-wave' picture.

Of course, despite the unspeakable 'impossibility proofs', the pilot-wave picture of de Broglie and Bohm exists. Moreover, in my opinion, all students should be introduced to it, for it encourages flexibility and precision of thought. In particular, it illustrates very explicitly Bohr's insight that the result of a 'measurement' does not in general reveal some preexisting property of the 'system', but is a product of both 'system' and 'apparatus'. It seems to me that full appreciation of this would have aborted most of the 'impossibility proofs', and most of 'quantum logic'.. Papers 1 and 4, as well as 17, dispose of 'impossibility proofs'. More constructive expositions of various aspects of the pilot-wave picture are contained in papers 1, 4, 11, 14, 15, 17, and 19. Most of this is for nonrelativistic quantum mechanics, but the last paper, 19, 'Beables for quantum field theory', discusses relativistic extensions. While the usual predictions are obtained for experimental tests of special relativity, it is lamented that a preferred frame of reference is involved behind the phenomena. In this connection one paper, 9, 'How to teach special relativity', has been included although it has no particular reference to quantum mechanics. I think that it may be helpful as regards the preferred frame, at the fundamental level, in 19. Many students never realize, it seems to me, that this primitive attitude, admitting a special system of reference which is experimentally inaccessible, is consistent... if unsophisticated.

Any study of the pilot-wave theory, when more than one particle is considered, leads quickly to the question of action at a distance, or 'nonlocality', and the Einstein–Podolsky–Rosen correlations. This is considered briefly in several of the papers already mentioned, and is the main concern of most of the others. On this question I suggest that even quantum experts might begin with 16, 'Bertlmann's socks and the nature of reality', not skipping the slightly more technical material at the end. Seeing again what I have written on the locality business, I regret never having written up the version of the locality inequality theorem that I have been mostly using in talks on this subject in recent years. But the reader can easily reconstruct that. It begins by emphasizing the need for the concept 'local beable', along the lines of the introduction to 7. (If local causality in some theory is to be examined, then one must decide which of the many mathematical entities that appear are supposed to be real, and really here rather than there). Then the simpler locality condition appended to 21 is formulated (rather than the more elaborate condition of 7). With an argument modelled on that of 7 the factorization of the probability distribution again follows. The Clauser–Holt–Horne–Shimony inequality is then obtained as at the end of 16.

My attitude to the Everett–de Witt 'many world' interpretation, a rather negative one, is set out in paper 11, 'The measurement theory of Everett and de Broglie's pilot wave', and in 15, 'Quantum mechanics for cosmologists'. There are also some remarks in paper 20.

There is much overlap between the papers. But the fond author can see something distinctive in each. I could bring myself to omit only a couple which were used again later with slight modifications. The later versions are included as 15 and 19.

For reproduction here, some trivial slips have been corrected, and references to preprints have been replaced by references to publications where possible.

In the individual papers I have thanked many colleagues for their help. But I here renew very especially my warm thanks to Mary Bell. When I look through these papers again I see her everywhere.

J. S. Bell, Geneva, March, 1987.

Acknowledgements

1 On the problem of hidden variables in quantum theory. *Rev. Mod. Phys.* **38** (1966) 447–52. Reprinted by permission of The American Physical Society.

2 On the Einstein–Podolsky–Rosen paradox. *Physics* 1 (1964) 195–200. Reprinted by permission of The American Physical Society.

3 The moral aspect of quantum mechanics. (with M. Nauenberg) In *Preludes in Theoretical Physics*, edited by A. De Shalit, H. Feshbach, and L. Van Hove, North Holland, Amsterdam (1966) 279–86. Reprinted by permission of North-Holland Physics Publishing, Amsterdam.

4 Introduction to the hidden-variable question. *Proceedings of the International School of Physics 'Enrico Fermi', course IL: Foundations of Quantum Mechanics.* New York, Academic (1971) 171–81. Reprinted by permission of Societa Italiana di Fisica.

5 Subject and Object. In *The Physicist's Conception of Nature*, edited by J. Mehra. D. Reidel, Dordrecht, Holland, (1973) 687–90. Copyright © 1973 by D. Reidel Publishing Company, Dordrecht, Holland.

6 On wave packet reduction in the Coleman–Hepp model. *Helvetica Physica Acta*, **48** (1975) 93–8. Reprinted by permission of Birkhauser Verlag, Basel.

7 The theory of local beables. TH-2053-CERN, 1975 July 28. Presented at the sixth GIFT seminar, Jaca, 2–7 June 1975, and reproduced in Epistemological Letters March 1976. Reprinted by permission of the Association Ferdinand Gonseth. This article also appeared in Dialectica 39 (1985) 86.

8 Locality in quantum mechanics: reply to critics. Epistemological Letters, Nov. 1975, 2–6. Reprinted by permission of the Association Ferdinand Gonseth.

9 How to teach special relativity. *Progress in Scientific Culture*, Vol 1, No 2, summer 1976. Reprinted by permission of the Ettore Majorana Centre.

10 Einstein–Podolsky–Rosen experiments. *Proceedings of the symposium on Frontier Problems in High Energy Physics.* Pisa, June 1976, 33–45. Reprinted by permission of the Annali Della Schola Normale Superiore di Pisa.

11 The measurement theory of Everett and de Broglie's pilot wave. In *Quantum Mechanics, Determinism, Causality, and Particles*, edited by M. Flato et al. D. Reidel, Dordrecht, Holland, (1976) 11–17. Copyright © 1976 by D. Reidel Publishing Company, Dordrecht, Holland.

12 Free variables and local causality. Epistemological Letters, February 1977. Reprinted by permission of Association Ferdinand Gonseth. This article also appeared in Dialectica 39 (1985) 103.

13 Atomic-cascade photons and quantum-mechanical nonlocality. *Comments on atomic and Molecular Physics* **9** (1980) 121–26. Invited talk at the Conference of the European Group for Atomic spectroscopy, Orsay-Paris, 10–13 July, 1979. Reprinted by permission of the author and publishers. Copyright © Gordon and Breach Science Publishers, Inc.

14 de Broglie-Bohm, delayed-choice double-slit experiment, and density matrix. *Internationl Journal of Quantum Chemistry*: Quantum Chemistry Symposium 14 (1980) 155–59. Copyright © 1980 John Wiley and Sons. Reprinted by permission of John Wiley and Sons, Inc.

15 Quantum mechanics for cosmologists. In *Quantum Gravity* **2**, editors C. Isham, R. Penrose, and D. Sciama, Clarendon Press, Oxford (1981) 611–37. Reprinted by permission of Oxford University Press.

16 Bertlmann's socks and the nature of reality. *Journal de Physique*, Colloque C2, suppl. au numero 3, Tome 42 (1981) C2 41–61. Reprinted by permission of Les Editions de Physique.

17 On the impossible pilot wave. *Foundations of Physics* **12** (1982) 989–99. Reprinted by permission of Plenum Publishing Corporation.

18 Beables for quantum field theory. 1984 Aug 2, CERN-TH.4035/84. Reprinted by permission of Routledge & Kegan Paul.

19 Six possible worlds of quantum mechanics. *Proceedings of the Noble Symposium 65: Possible Worlds in Arts and Sciences*. Stockholm, August 11–15, 1986, edited by Sture Allén. Reprinted by permission of The Nobel Foundation.

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Introduction: John Bell and the second quantum revolution

Alain Aspect

1 The quantum revolutions: from concepts to technology

The development of quantum mechanics in the beginning of the twentieth century was a unique intellectual adventure, which obliged scientists and philosophers to change radically the concepts they used to describe the world[1]. After these heroic efforts, it became possible to understand the stability of matter, the mechanical and thermal properties of materials, the interaction between radiation and matter, and many other properties of the microscopic world that had been impossible to understand with classical physics. A few decades later, that *conceptual revolution* enabled a *technological revolution*, at the root of our information-based society. It is indeed with the quantum mechanical understanding of the structure and properties of matter that physicists and engineers were able to invent and develop the transistor and the laser – two key technologies that now permit the high-bandwidth circulation of information, as well as many other scientific and commercial applications.

After such an accumulation of conceptual – and eventually technological – successes, one might think that by 1960 all the interesting questions about quantum mechanics had been raised and answered. However, in his now-famous paper of 1964[2] – one of the most remarkable papers in the history of physics – John Bell drew the attention of physicists to the extraordinary features of entanglement: quantum mechanics describes a pair of entangled objects as a single global quantum system, impossible to be thought of as two individual objects, even if the two components are far apart. John Bell demonstrated that there is no way to understand entanglement in the framework of the usual ideas of a physical reality localized in space-time and obeying causality. This result was opposite to the expectations of Einstein, who had first pointed out, with his collaborators Podolsky and Rosen, the strong correlations between entangled particles, and analyzed these correlations in the framework of ideas of a local physical reality. The most remarkable feature of Bell's work was undoubtedly the possibility it offered to determine *experimentally* whether or not Einstein's ideas could be kept. The experimental tests of *Bell inequalities* gave an unambiguous answer: entanglement *cannot* be understood as usual correlations, whose interpretation relies on the existence of common properties, originating in a common preparation, and remaining attached to each individual object after separation, as components of their physical reality[a]. A few decades after the 1964 paper, the physics of entanglement is flourishing, and thousands of papers, theoretical and experimental, are found when one types 'Bell inequalities' on a search engine.

Starting in the 1970s, another concept has progressively become more and more important in quantum physics: the description of *single objects*, in contrast to the statistical use of quantum mechanics to describe only properties of large ensembles (for instance the fluorescence of an atomic vapor). That question had, like the EPR problem, been a subject of debate between Bohr and Einstein[3], but it was the development of experimental abilities to isolate and observe single microscopic objects like photons, electrons, ions and atoms that prompted physicists to take quantum mechanical dynamics of single objects, including 'quantum jumps', seriously. The experimental observation of quantum jumps (in the fluorescence light from a single ion) inspired new theoretical approaches, the so-called 'Quantum Monte-Carlo Wave Function' simulations, primarily used to describe 'elementary' microscopic objects like ions, atoms, and small molecules. Recently, progress in nanofabrication, as well as experimental breakthroughs, have allowed physicists to create mesoscopic systems (e.g., electric and magnetic devices, and gaseous Bose–Einstein condensates) which push the border of the quantum world to larger and larger systems that still need to be described as single quantum objects.

As a witness of that period I would like to argue that John Bell also played, indirectly, an important role in the emergence of the new theoretical approaches clarifying the quantum description of individual objects. Before the realization of the importance of Bell's theorem, which happened only in the 1970s, the conventional wisdom among physicists was that the 'founding fathers' of quantum mechanics had settled all the conceptual questions. Bell's work on entanglement did not

[a] An example of usual correlations is the identity of the eye colours of twin brothers, linked to their identical chromosome sets. Correlations in entangled twin photons are different in nature, as we explain later.

cast any doubt on the validity of quantum mechanics as a predictive tool. To the contrary, experiments found that nature definitely follows the quantum mechanical predictions even in those weird situations. But there was a lesson to be drawn: questioning the 'orthodox' views, including the famous 'Copenhagen interpretation', might lead to an improved understanding of the quantum mechanics formalism, even though that formalism remained impeccably accurate. It is my claim that Bell's example helped physicists to free themselves from the belief that the conceptual understanding that had been achieved by the 1940s was the end of the story.

I think it is not an exaggeration to say that the realization of the importance of entanglement and the clarification of the quantum description of single objects have been at the root of a *second quantum revolution*, and that John Bell was its prophet. And it may well be that this once purely intellectual pursuit will also lead to a *new technological revolution*. Indeed, we should have no doubt that the advances in the quantum concepts used to describe single objects will certainly rejoin and play a key role in the ongoing *revolution of nanotechnology*. Even more amazing, physicists have endeavoured to apply entanglement to '*quantum computation*', and most of the systems that are being experimentally tested as elementary quantum processors are entangled quantum systems, such as a few interacting ions. Whether or not the second revolution will have an impact on our societies is a premature question. But who would have imagined the ubiquitous presence of integrated circuits when the first transistor was invented?

2 The first quantum revolution

Searching for a consistent explanation of the black-body radiation spectrum at both high and low frequencies, M. Planck introduced in 1900 the quantization of energy exchange between light and matter[4]. A. Einstein took a step further in 1905 by proposing the quantization of light itself to understand the photoelectric effect[5]. The properties he deduced were then tested by R. A. Millikan in 1914[6]. At the same epoch convincing evidence of the existence of molecules – doubted until the beginning of the twentieth century – was provided by various observations, including Einstein's explanation of Brownian motion[7]. Together with many other experiments, these observations convinced physicists and philosophers to accept the granularity of matter and quantization of energy in the microscopic world and led to the development of quantum mechanics.

In addition to rendering an account of experimental data, the foundation of quantum mechanics resolved basic problems. For instance, N. Bohr's

1913 model of the atom explained both the absorption spectra of atomic gases and the stability of matter: without quantum mechanics, the Rutherford atom, composed of orbiting particles with opposite (i.e., attracting) charges, should radiate and collapse.

The first comprehensive paradigm of quantum mechanics centred about the Heisenberg and Schrödinger formalisms of 1925. The latter was a wave equation for matter, completing a beautiful duality: like light, matter can behave as either a particle or a wave. The wave–particle duality was originally L. de Broglie's 1924 proposition[8], and remains incomprehensible to the classical way of thinking. Within twenty years of its birth, the quantum mechanical formalism could explain chemical bonds, electrical properties, and thermal properties of matter at a microscopic level. Continuing progress in physics was pushing along different directions: towards the incredibly small, with particle physics, or into the domain of more exotic properties of matter, such as superconductivity (the absence of resistance in some conductors at low temperatures), or superfluidity (the absence of viscosity of liquid helium at low temperatures). Studies in light–matter interaction were refined by orders of magnitude, thanks to experimental breakthroughs made possible by advances in microwave technology[9]. All this progress took place perfectly within the quantum mechanical framework, which had been refined to be applied both in the elementary phenomenon (Quantum Electrodynamics) as well as in complex situations encountered in condensed matter. But in the early 1950s, quantum mechanics still appeared as a game to be played by physicists only for the sake of progress in knowledge, without any impact on everyday life.

The electronics and information age: quantum mechanics applied

Even if the public is not always aware, the applications of quantum physics are all around us in electronics and photonics. In technologies today, quantum mechanics is required to understand material properties (electrical, mechanical, optical, etc.) and the behaviour of elementary devices at the root of many technological achievements.

The transistor was invented in 1948 by a brilliant group of solid state physicists after fundamental reflection about the quantum nature of electrical conduction[10]. This invention and its descendents, microfabricated integrated circuits[11], clearly had a monumental impact. Like the steam engine over a century earlier, the transistor changed our lives and gave birth to a new era, the information age.

The second technological progeny of quantum mechanics is the laser, developed in the late 1950s[12]. Some of its applications are present in everyday life: bar code readers, CD readers and players, medical tools, etc. Less visible but perhaps more important is the use of laser light in telecommunications, where it dramatically boosts the flow of information: terabits (millions of millions of information units) per second can be transmitted across the oceans through a single optical fiber. These information highways connect us to the stored knowledge and active computation distributed around the world. Starting from the few bits per second of the first telegraph operators, we have come a long way.

The quantum mechanical understanding of atom–photon interactions has also continued to develop, and eventually led to applications. For example, in 1997 a Nobel prize was given to S. Chu, C. Cohen-Tannoudji, and W. D. Phillips, for the development of laser cooling and trapping of atoms. Here as well, fundamental research soon led to a spectacular application, cold atom clocks, which have already allowed the accuracy of time measurement to reach a level of 1 in 10^{15} (one second accuracy in thirty million years!). More is to come with cold atom or ion *optical* clocks. Atomic clocks are used in the global positioning system (GPS), and their replacement with cold atom clocks will eventually permit an improved positioning precision. Coming full circle, that improved technology of clocks can be applied to fundamental questions, such as tests of general relativity, or the search for slow variation in fundamental physical constants. The first quantum revolution, with its interplay between basic questions and applications, is still at work.

3 Entanglement and Bell's theorem

The Bohr–Einstein debate

Quantum mechanics was constructed at the price of several radical – and sometimes painful – revisions of classical concepts. For instance, to take into account particle–wave duality, quantum mechanics had to renounce the idea of a classical trajectory. This renunciation is best stated in the celebrated Heisenberg uncertainty principle, which describes quantitatively the impossibility of defining precisely and simultaneously the position and velocity of a particle. One can also illustrate this renunciation of classical trajectories by remarking that in an interference experiment the particle 'follows many paths at once'.

In fact, such renunciations were so radical that several, including Einstein and de Broglie, could not admit their inevitability, and differed with Bohr who had carved the Rosetta stone of interpretation of the new theory under the name 'the Copenhagen interpretation'. Einstein did not challenge the formalism of quantum mechanics and its predictions directly, but seemed to think that the renunciations put forward by Bohr could only signify the incompleteness of the quantum formalism. This position led to epic debates with Bohr, in particular the one that started in 1935 with the publication of the article by Einstein, Podolsky, and Rosen (EPR), whose title posed the question, 'Can Quantum-Mechanical description of physical reality be considered complete?'[13] In this article, Einstein and his co-authors showed that the quantum formalism permitted the existence of certain two-particle states for which one can predict strong correlations both in velocity and in position even when the two particles are widely separated and no longer interact. They showed that measurements of positions would always give values symmetric about the origin, so that a measurement on one particle would allow one to know with certainty the value of the position of the other one. Similarly measurements of the velocities of the two particles would always yield two opposite values so that a measurement on the first one would be enough to know with certainty the velocity of the other one. Of course one has to choose between an accurate position or velocity measurement on the first particle, because of the Heisenberg relations. But the measurement on the first particle does not disturb the (distant) second particle, so EPR concluded that the second particle must have had, even before measurement, well-determined values of position and velocity. And since the quantum formalism cannot give a simultaneous and precise value to these quantities, Einstein and his co-authors concluded that quantum mechanics was incomplete, and that physicists ought to devote themselves to try to complete it.

Niels Bohr was apparently bowled over by this argument, which rests on quantum mechanics itself to show its provisional character. His writings show his profound conviction that if the EPR reasoning were correct, it would not be just a matter of completing the formalism of quantum mechanics, but it would be all of quantum physics that would collapse. He immediately contested the EPR reasoning[14], claiming that in such a quantum state one could not speak about the individual properties of each of the particles, even if they were distant from one another. In contrast to Bohr, Schrödinger reacted positively to the EPR paper, and coined the term 'entanglement', to characterize the lack of factorability of an EPR state[15].

In fact, when the EPR paper appeared in 1935, quantum mechanics was being crowned with one success after another, so apart from Bohr and Schrödinger, most physicists ignored this debate, which seemed rather academic. It seemed that adhering to one or the other position was a matter of personal taste (or epistemological position), but did not have any practical consequence on how quantum mechanics was used. Einstein himself apparently didn't contest this attitude, and we had to wait thirty years to see a resounding counter-argument to this relatively universal position.

Bell's theorem

In 1964, a now famous short article[2] changed the situation dramatically. In this paper, John Bell takes the EPR argument seriously, and completes quantum mechanics, introducing supplementary parameters (also called 'hidden variables'[16]) given to the two particles at their initial preparation in an entangled state, and carried along by each particle after separation. In the original EPR situation, the hidden variables would be the initial positions of the two particles, taken as identical, and their velocities, taken as equal and opposite. Reasoning on entangled states of two spin-$\frac{1}{2}$ particles (a simpler version of the EPR situation, introduced by Bohm[17]) Bell shows that one can easily explain the existence of correlations between the results of measurements on the two particles by allowing the result of a measurement on one particle to depend only on the supplementary parameters carried by that particle and on the setting of the apparatus making that measurement. But then, a few lines of calculation suffice to show a contradiction with the predictions of quantum mechanics. More precisely, even if such a 'hidden variable' theory can reproduce some of the predicted quantum correlations, it cannot mimic the quantum mechanical predictions for *all* possible settings of the measuring apparatus. Thus it is not possible, in general, to understand EPR-type correlations by 'complementing' the quantum theory along the lines proposed by Einstein. This result, known as *Bell's theorem*, continues to surprise us even today, since we are used to explaining all types of correlations by a scheme akin to hidden variables[b]. For instance, if we have a pair of identical twins we do not know what

[b] A famous example is Bertlmann's socks (paper 16 of this volume). Physicists are as surprised as non-physicists, maybe more so: see D. Mermin, 'Is the moon there?', *Physics Today* **38**, 11 (1985).

their blood type is before testing them, but if we determine the type of one, we know for sure that the other is the same type. We easily explain this by the fact that they were born with, and still carry along, the same specific chromosomes that determine their blood type. What Bell's paper shows us is that if we try to describe the correlations between the entangled particles in the way we understand the correlations between the twins, we will be making a serious error.

A crucial hypothesis in Bell's reasoning is the '*locality hypothesis*' which needs to be fulfilled by the supplementary parameter models to lead to a conflict with quantum mechanics. This very natural assumption states that there is no direct non-local, interaction between the two measuring apparatuses far from each other. In other words, the conflict arises only if the result of a measurement on the first particle does not depend on the setting of the second measuring apparatus[c]. As stressed by Bell in his 1964 paper[2], this very natural hypothesis would become a direct consequence of Einstein's views that no influence can travel faster than light, in an experimental scheme where the settings of the measuring apparatus are rapidly changed while the particles are flying between the source and the measuring apparatus[18].

To establish the incompatibility between quantum mechanics and the local hidden variable theories, Bell showed that the correlations predicted by any local hidden variable model are limited by inequalities – today called 'Bell inequalities' – that are violated by certain quantum predictions. The choice between the positions of Einstein and Bohr was then no longer a question of personal taste. Instead, it became possible to settle the question experimentally, by looking carefully at measurements of correlations between entangled particles. Surprisingly, in 1964, there was no experimental result that permitted such a quantitative test. Experimentalists began then to think about how to construct an experiment to create and measure states for which quantum mechanics would predict a violation of Bell inequalities. In 1969 a version of inequalities was published that was well adapted to real experiments, where apparatus have some inefficiencies[19]: it became clear that conclusive experiments were possible, provided that the experimental imperfections remain small enough. Some experiments were carried out with γ-ray photons

[c] The importance of a locality hypothesis had already been suggested in Bell's discussion of the 'impossibility proofs' of hidden variables[16], and the locality assumption was clearly stated in the first paper presenting inequalities[2]. Actually, the locality (or separability) question appears in most of the papers of this volume.

emitted in positronium annihilation, or with protons, but the most convincing were realized with visible light photon pairs. After a first series of pioneering experiments[20], a new generation of experiments in the early 1980s[21,22] obtained a collection of clear-cut results in agreement with quantum theory that violated Bell inequalities even with rapidly switched polarizers. A third generation of experiments, undertaken since the beginning of the 1990s has definitely confirmed these results[23,24,25]. There is no doubt[d], then, that, in contrast with twin brothers, two entangled photons are *not* two distinct systems carrying identical copies of the same parameters. A pair of entangled photons must instead by considered as a single, inseparable system, described by a global wavefunction that cannot be factorized into single-photon states.

The inseparability of an entangled photon state has been shown to hold even if the photons are far apart – including a 'space-like' separation in the relativistic sense – that is, a separation such that no signal traveling at a velocity less than or equal to the velocity of light can connect the two measurements. This was already the case in the 1982 experiments, where the photons were separated by 12 meters at the time of the measurements[21]. In addition, it was possible to change the setting of the measuring polarizers during the twenty-nanosecond flight of the photons between the source and the detector[22], in order to implement Bell's ideal scheme. In more recent experiments, where new sources have permitted the injection of entangled photons into two optical fibers, a violation of Bell inequalities was observed at separations of hundreds of meters[24], and even more[25], and it has even been possible to *randomly* change the setting of the polarizers during the propagation of the photons in the fibers[24]. 'Timing experiments' with variable polarizers emphasize that everything happens as if the two entangled photons were still in contact, and as if the measurement of one photon would affect

[d] Actually, there remains a loophole for advocates of local hidden variable theories, when the efficiency of the detectors used in real experiments is small compared to unity, so that many photons remain undetected[20]. However, as stressed by John Bell [in paper 13: 'Atomic cascade photons and quantum mechanical nonlocality', *Comments on Atomic and Molecular Physics* **9** (1980) 121] 'it is difficult for me to believe that quantum mechanics, working very well for currently practical set-ups, will nevertheless fail badly with improvements in counter efficiency…'. A first experiment with a detection efficiency close to 1 [M. A. Rowe *et al.*, 'Experimental violation of a Bell's Inequality with efficient detection' *Nature* **409** (2001) 791] has confirmed a clear violation of Bell's inequalities. In that experiment however, the measurements were not space-like separated. An experiment in which the detection efficiency is large *and* the locality condition is enforced by relativistic separation and use of variable measuring apparatus, is still to be done.

the other instantaneously. This seems to contradict the principle of relativistic causality that specifies that no interaction can propagate faster than the speed of light.

It should be stressed, however, that there is no violation of causality in the operational sense, and that one cannot use this non-separability to send a signal or usable information faster than the speed of light[26]. To illustrate this impossibility, one can note that in order to exploit the EPR correlations for sending information, it would be necessary to transmit a kind of 'decoding grid', which can be done only via a classical channel, which does not communicate at super-luminal speed[27]. Thus, as troubling as it may be, quantum entanglement does not allow us to build the super-luminal telegraph at work in science-fiction novels. Nevertheless, as we will describe in Section 5 of this introduction, fascinating applications of entanglement are developing in a new field: 'quantum information'.

4 Quantum mechanics and single objects

The experimental evidence supporting quantum mechanics typically comes from large ensembles. For instance, atomic spectra are taken from clouds of myriad atoms; semiconductors are bulk materials; and laser beams contain a tremendous number of photons, produced by an optical amplifier that contains a huge number of atoms. In these situations, we can apply without any problem the formalism of quantum mechanics, which usually yields probabilistic predictions. Since our observations bear on large ensembles, we realize statistical measurements, to which the quantum probabilities can be directly compared. These concepts are fully at work in the density matrix formalism of quantum mechanics, easily interpreted in the framework of large ensembles, even though the Copenhagen school has always claimed that the standard formalism can also be applied to individual objects. That issue was also deeply discussed between Einstein and Bohr[3], but it remained a matter of principle until experimentalists were able to deal with single microscopic objects.

From the ensemble to the single quantum system

Beginning in the 1970s, physicists invented methods to manipulate and observe single elementary objects – such as a single photon[28], electron or ion. They became able to trap charged particles for hours (or even days or months) with electric and magnetic fields that held the particle in a vacuum chamber, far from any material wall. In the decade that followed, atomic-resolution scanning microscopy techniques allowed the observation and manipulation of single atoms deposited on a surface. These experimental advances, crowned by several Nobel prizes[29], had major consequences in fundamental physics. The achievement of trapping individual elementary objects led to remarkable improvements in the knowledge of certain microscopic quantities, whose values provide tests of fundamental theories. For instance, spectroscopy of a single trapped electron yields a measurement of the 'g-factor' of the electron to thirteen significant figures – a precision equivalent to measuring the distance between the earth and the moon to better than the diameter of a human hair! The g-factor of the electron is a fundamental quantity that can also be calculated using quantum electrodynamics, the refined theory of quantum mechanics applied to elementary electric charges and photons. The essentially perfect agreement between experiment and theory shows the incredible accuracy of the predictions of this theory. The trapping of elementary objects has also permitted crucial tests of the symmetry between matter and antimatter: one can verify with spectacular precision proton–antiproton or electron–positron symmetry[30]. It is also possible to verify that two electrons, or two atoms of the same chemical element, have exactly the same properties. Indiscernibility has little sense in classical physics, where two beads, however identical they may seem, can always be distinguished by small defects or marks. On the other hand, indistinguishability is in fact at the core of quantum physics[31].

In parallel with experimental achievements, the observation of individual microscopic objects obliged physicists to think more carefully about the significance of quantum mechanics when applied to an individual object. We know that, in general, quantum mechanics gives probabilistic predictions. For example, one may calculate that an atom illuminated by a certain laser beam has a certain probability P_B to be in a 'bright' state; and the complementary probability $P_D = 1 - P_B$ to be in a 'dark' state[32]. By dark or bright, we mean that when illuminated by an auxiliary probe laser, an atom in a dark state radiates no photon, while in a bright state it emits many fluorescence photons, easy to observe with a photodetector, or even the naked eye. If we have a vapor with a large number of atoms in this situation, the interpretation of the probabilistic quantum prediction is undemanding: it suffices to admit that a fraction P_B of the atoms are in the bright state (and thus scatter photons when probed with the auxiliary laser), while the remaining fraction are in the dark state (do not scatter photons). But what would happen for a single atom placed in the same situation? Asked to respond to this question, the 'Copenhagen school' would respond that the atom is in a 'superposition' of the dark and bright states. In such a superposition, the atom is simultaneously in both the bright and the dark states, and it is impossible to know in advance what will happen when we apply the probe laser: the atom may be found in the bright state or it may be found in the dark state. Of course, the Copenhagen spokes-person would add, after repeating the measurement many times, one would observe a bright state for a fraction P_B of the cases, and the dark state for a fraction P_D.

In fact, this answer is not complete, since it tells us only about averaged results of repeated measurements. But how does the state of a *single* atom evolve with time if we observe it continuously? Or, more precisely, what would we observe if we had left the weak probe laser on all the time? This question was academic in the 1930s, when experimentalists could not even imagine observing individual, isolated particles. However, the Copenhagen physicists had an answer that invoked the postulate of 'wave packet reduction'. When first illuminated by the probe laser, the atom in a superposition of the dark and bright states would collapse into one of the two basic states, say for instance the bright state, where fluorescence photons can be seen. A further evolution can put the atom again in a superposition state, and eventually lead it to collapse into the dark state, and the fluorescence would suddenly stop. Thus, one would predict that the atom would, at random moments, switch from the dark state to the bright state.

The existence of such 'quantum jumps', implying a discontinuous evolution of the system, was strongly opposed by a number of physicists – including Schrödinger – who saw a convenient trick with a pedagogical value, but who contended that quantum mechanics inherently applies only to large ensembles, and not to single quantum objects. The experimental progress discussed above allowed the debate to be resolved experimentally, in 1986, by the direct observation of quantum jumps in the fluorescence of a single trapped ion. In this type of experiment, one indeed observes[32] that the ion evolves randomly between periods where it is invisible, and periods where it fluoresces intensely! This result was very striking, and resolved beyond all doubt that quantum jumps do exist, and that quantum theory can describe the behavior of a single object.

Quantum jumps in action: new clocks and new theoretical methods

The – experimentally enforced – conceptual acceptance of quantum jumps led to surprising developments in both the theoretical and experimental arenas. Experimentally, one can use the phenomenon of alternation between dark and bright states for the most precise ever spectroscopic measurements of ion spectral lines[33]. These weak lines are candidates for new atomic clocks, even more precise than the ones currently in use. Quantum jumps have also inspired a new theoretical method called 'Quantum Monte-Carlo Wave Function' in which a possible history of the system is simulated by drawing random quantum jumps, whose probability is governed by quantum mechanical laws[34]. By generating a large number of these 'possible histories', one can build the probability distribution of the results. In the large-number limit, these distributions coincide with the density matrix predictions. There are some situations in which this calculation method is remarkably more efficient than traditional methods. Furthermore, quantum Monte-Carlo methods have allowed the discovery that certain quantum processes obey unusual statistics – Lévy statistics, which are also encountered in domains as disparate as biology and the stock market. The use of these 'exotic' statistical methods provide new and efficient methods to address quantum problems[35].

From microscopic to mesoscopic

Having established that quantum mechanics can describe the dynamics of a single system, one naturally asks how big that system can be. Certainly we do not need quantum mechanics for mecroscopic objects, which are well described by classical physics – this is the reason why quantum mechanics seems so foreign to our everyday existence. Of course we need quantum mechanics to understand properties of the bulk material of which the macroscopic object is made, but not for the behaviour of the object as a whole. But between the scale of a single atom and the macroscopic world, one finds the mesoscopic scale, where it is the object itself, and not only the material of which it is made, that needs to be described by quantum mechanics.

For instance, nanofabricated conducting rings demonstrate effects that can only be understood by treating their electrons with a global wavefunction[36]. Another famous example, for which the 2001 Nobel prize was awarded to E. Cornell, W. Ketterle, and C. Wieman, is a gaseous Bose–Einstein condensate, which must also be treated as a single 'large quantum' object, with a number of atoms usually ranging from a few thousands to tens of millions[37], or more[38].

Even if these mesoscopic quantum objects are, for the moment, merely curiosities in research laboratories, the uninterrupted miniaturization of microelectronics may soon oblige engineers to understand their circuits using the laws of quantum mechanics. In Section 5, we will give the example of a would-be 'quantum computer', but even ordinary transistors will exhibit new quantum properties if downscaled to the size of a few thousands of atoms.

From mesoscopic to macroscopic: decoherence

What then separates the microscopic and mesoscopic quantum world from the macroscopic classical world? John Bell was deeply concerned by that question, which was a major reason of his discomfort with the standard interpretation of quantum mechanics where this frontier plays a crucial role[39].

One of the most important features of quantum physics is to allow the existence of superpositions of states: if a system has several possible quantum states, it can not only be in any one of these states, but it can also be in a hybrid state, or 'coherent superposition', composed of several basic states. Entanglement is a sophisticated case of state superposition, but even single systems can be put in superposition states, and we have discussed above the case of an atom in a superposition of a dark state and a bright state. The situation becomes quite troubling when the two states involved are obviously incompatible. For instance, consider an atom arriving at an atomic beam splitter. The atom can either be transmitted or reflected, two options that can lead to well-separated paths. But the atom can also come out in a superposition of the reflected and transmitted states, i.e. simultaneously present in two clearly separated regions of space. One can show experimentally that this superposition state does exist, by recombining the two paths and observing interference fringes, which can only be explained by admitting that both paths were followed simultaneously. Such behaviour has been observed both with microscopic objects (electrons, photons, neutrons, atoms, molecules as large as C_{60} fullerenes), and with mesoscopic objects (electric currents in nanocircuits), but never with macroscopic objects, even though this is not *a priori* forbidden by the quantum formalism. The problem has attracted the attention of many physicists, starting with Schrödinger who gave an amusing ('burlesque' in his own words), illustration involving his famous cat[40]. In the proposed scenario, the cat's life rests upon a quantum event that could be in a superposition of states. Why, then, don't we find Schrödinger's cat in a coherent superposition of dead and alive?

To explain the non-existence of superposition states of macroscopic objects, quantum physicists invoke quantum decoherence[41]. Decoherence comes from the interaction of the quantum system with the 'outside world'. For instance, in our example of the atom following two simultaneous paths in an interferometer, one can illuminate[42] the atomic trajectory by laser light, which allows one to see the atom's position and reveal which path is taken: this measurement reduces the superposition to the classical situation where the atom has followed either one path or the other, and destroys the interference. As objects become larger and larger, they become more sensitive to external perturbations, which can destroy (partially or completely) coherent superpositions. This argument gives a plausible explanation for the different behaviors of the classical and the quantum world. Nobody knows, however, whether there is a hypothetical limit beyond which decoherence would be inevitable, or whether we always can, at least in principle, take sufficient precautions to protect the system against perturbations, no matter how large it is. A clear answer to that question would have immense consequences, both conceptually and for future quantum technologies.

5 The second quantum revolution in action: quantum information

The existence of Bell inequalities, which establish a clear frontier between classical and quantum behaviour, and their experimental violation, are important conceptual results, which force us to recognize the extraordinary character of quantum entanglement. But in an unexpected way, it has been discovered that entanglement also offers completely new possibilities in the domain of information treatment and transmission. A new field has emerged, broadly called Quantum Information, which aims to implement radically new concepts that promise surprising applications. So far, there are two primary examples: quantum cryptography[43], already operational, and quantum computing[44,45], still a nascent enterprise.

Quantum cryptography

Cryptography is the science of encoding and/or transmitting a secret message without its being read/understood by a third party. Much of classical cryptography involves secure transmission on a public channel. As the field has progressed, methods have become more and more refined, involving sophisticated algorithms and driven by equally clever methods of 'code breaking'. Both encoding and code breaking have progressed due to advances in mathematics and to the ever-increasing power of computers. When contemplating this continuing progress of coding and code breaking, it seems clear that the security of a transmission can be assured only on the hypothesis that the adversary (who is trying to break the code) has neither more advanced mathematics nor more powerful computers than the sender and intended receiver. In classical cryptography, the only absolutely sure transmission scheme uses the one time pad method, where the emitter and the receiver have two identical copies of a coding key, not shorter than the secret message to be transmitted, and that is used only once. The (preliminary to secure communication) distribution of the two copies of the key then becomes the critical stage, that involves secret channels which might be intercepted by an 'eavesdropper' with technologies more advanced than those of the sender and intended receiver.

By contrast, in quantum cryptography, the security of a transmission rests on the fundamental physical laws at work in quantum mechanics. There, it is possible to detect an eavesdropper by using the trace that is *necessarily* left by such efforts[46], since in quantum physics all measurements perturb the system in some way. In the absence of such a trace, one can be certain that the message has passed without having been read by a spy.

A particular topic in quantum cryptography is especially spectacular: the use of EPR pairs to distribute securely the two copies of the random key that two distant partners will use later, for a one time pad coded transmission. How can they be sure that no one has read either copy of the key during the transmission? The use of pairs of entangled particles offers an elegant solution: the two partners (Alice and Bob) effecting measurements on the two particles of a single entangled pair will find random but perfectly correlated results. By repetition of such measurements, they generate two identical copies of a random sequence. And what we have learned from the violation of Bell inequalities is that, as long as the measurements have not been made, their results are not predictable, which means that the key does not yet exist. A non-existent key cannot be read by any eavesdropper (Eve)! It is only at the moment of the measurement that the two identical keys appear in the apparatuses of the two partners. Bell inequalities play a crucial role in that scheme: their violation permits one to be sure that the particles received by Alice and Bob have not been fraudulently prepared by Eve in a state known by her, which would allow her to decipher messages between them. It has already been possible to demonstrate that this principle can work in practice[43].

Quantum computing[44,45]

In the early 1980s, the fundamental assumptions of information theory started to be challenged by several physicists who suggested that if one had a quantum computer, one could implement radically new algorithms to perform certain tasks. The names of Landauer, Feynman, Deutsch, and others can be quoted, and an important breakthrough happened in 1994 when P. Shor[47] showed that a quantum computer should allow one to factor large numbers, in times much shorter than with conventional methods. Factorization belongs to a class of problems (complexity class) whose solution (with classical computers) requires a time superpolynomial in the size of the problem (that is, the time needed grows faster than any power of the number of digits in the number to be factored). With a quantum computer, on the other hand, the computation time would only grow as a power of the size of the number[e]. This discovery had considerable conceptual implications, since it showed that, contrary to what had been thought previously, the complexity class of a problem was not independent of the type of machine used. In addition to that conceptual revolution, a quantum computer would certainly have applications beyond present imagination.

Several groups have started to develop the basic elements of a quantum computer: quantum bits and quantum gates. A quantum logic gate performs basic operations on quantum bits – or 'qubits' – just as an electronic logic gate manipulates ordinary bits. However, in contrast to normal bits, which can take only one of two values, 0 or 1, quantum bits can be put in a superposition of two states. A quantum logic gate must thus be capable of combining two quantum bits to produce an entangled state. It is the possibility of working with such entangled states that opens up new and powerful possibilities compared to the classical algorithms.

Will the quantum computer exist someday? It would be presumptuous to answer, but experimental research on quantum gates is extremely active, and has already obtained important results. Many approaches are being explored, with a diversity of physical realizations of qubits, including atoms, ions, photons, nuclear spins, Josephson junctions[48],... .

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[e] It may make a significant difference: see for instance the example in ref. 44, where the factorization time of a 400-digit number can be reduced from the age of the universe to a few years.

For all these systems there are large unknowns. Quantum calculation relies on the ability to entangle dozens or even hundreds and thousands of quantum bits, and perform thousands of operations before decoherence disrupts the quantum register. Decoherence results from the interaction with the outside world (see Section 4), and its effect is to wash out entanglement, putting previously entangled objects into a state where they behave as separate objects. The scaling up to a large number of entangled qubits may turn out to be overwhelmingly difficult, since it is generally observed that decoherence dramatically increases when the number of the entangled particles increases. Here again, nobody knows whether there is a maximum size beyond which entanglement destruction by decoherence is definitely unavoidable, or whether it is only a matter of increasing experimental difficulty (or of finding special situations where the problem might not be as dramatic). An entire community of experimentalists and theorists are engaged in that quest. Understanding and reducing the effects of decoherence may well be the key question facing quantum computation as a technological revolution. But even in the absence of an efficient quantum computer, the idea of quantum computation is certainly a milestone in computation science.

6 John Bell's legacy: questioning quantum mechanics is fruitful

Quantum mechanics was, and continues to be, revolutionary, primarily because it demands the introduction of radically new concepts to better describe the world. In addition we have argued that *conceptual* quantum revolutions in turn enable *technological* quantum revolutions.

John Bell started his activity in physics at a time when the first quantum revolution had been so successful that nobody would 'waste time' in considering questions about the very basic concepts at work in quantum mechanics. It took him a decade to have his questions taken seriously. For somebody who has observed reactions to his work on the EPR situation and entanglement, in the early 1970s, it is certainly amusing to see that an entry of the Physics and Astronomy Classification Scheme is now assigned to 'Bell inequalities'[49]. With his questions about entanglement, John Bell was able to clarify the Einstein–Bohr debate in an unanticipated manner, offering the opportunity to settle the question experimentally. His work, without a doubt, triggered the second quantum revolution, primarily based on the recognition of the extraordinary features of entanglement, and pursued with efforts to use entanglement for quantum information. In fact, it not only triggered the