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Excerpt
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1 From Euler to Legendre

Our first six remarkable mathematicians were born in the forty-six years from 1707 to 1752. Four came from France, one from each of Italy and Switzerland.

Leonhard Euler (1707–1783)

In mathematics the eighteenth century was the age of Euler, but before we come to him it is necessary to say a few words about that remarkable family, the Bernoullis. Originally from Antwerp, they left the Netherlands in the late sixteenth century to escape the Spanish persecution of Protestants and settled in Basel, where they married into the merchant class. Generation after generation they produced remarkable mathematicians, beginning with the brothers Jakob and Johann, who were born too early to qualify for

inclusion here. Johann's quarrelsome son Daniel falls just within our period, but like Isaac Newton he was even more remarkable as a physicist than as a mathematician. In mathematics he was overshadowed by his uncle Jakob and eclipsed by his friend Euler.

Leonhard Euler was born in Basel on April 15, 1707. His forbears were artisans, for the most part, but his father Paul Euler was a minister of the Protestant Evangelical Reformed Church. Paul Euler knew the Bernoullis well, since he and Johann had lived at Jakob's house when they were studying mathematics under him. Paul Euler's wife, Margarete Brucker, was also the daughter of a minister. The year after their son was born the family moved to the nearby village of Riehen, where his father was pastor. The boy Leonhard grew up with two younger sisters, Anna Maria and Maria Magdalena. After some early education at home he was sent back to the city to live with his maternal grandmother and attend the old-fashioned Latin school of Basel, where no mathematics was taught. In 1720, at the age of thirteen, he matriculated into the faculty of philosophy at the university. At that time the education offered was only mediocre. Euler mastered all the available subjects and graduated in 1722, at the age of fifteen. In the same year he competed, without success, for professorships in logic and in law.

The next year Euler transferred to the faculty of theology, in accordance with his father's wishes, but in addition to divinity he began to study mathematics seriously. As he wrote in his autobiographical sketch: 'I soon found an opportunity to be introduced to a famous professor Johann Bernoulli. True he was very busy and so refused flatly to give me private lessons; but he gave me much more valuable advice to start reading more difficult mathematical books on my own and to study them as diligently as I could; if I came across some obstacle or difficulty, I was given permission to visit him freely every Saturday afternoon and he kindly explained to me everything I could not understand.' Although Johann Bernoulli was recognized as one of the leading mathematicians in Europe his ordinary teaching was at an elementary level.

Euler received his master's degree in 1724 at the age of seventeen after writing a thesis comparing the natural philosophy of Descartes with that of Newton. By this time he had got to know several of the younger members of the Bernoulli clan, including Johann's son Daniel, seven years his senior. Johann himself became increasingly aware that his student was a genius. When he wrote to Euler in later years the salutations show his growing respect: in 1728 he addressed him as 'The very learned and ingenious young man'. The next year this had become 'The highly renowned and learned man', and the year after 'The highly renowned and by far most sagacious mathematician' until by 1745 he was addressing him as 'The incomparable L. Euler, the prince among mathematicians'.

Although Euler was a devout Calvinist he succeeded, with the support of Johann Bernoulli, in persuading his father that his true vocation was in mathematics, not the church. During the next two years, while seeking employment, Euler wrote his first important memoir, on the theory of acoustics. Also he entered a prize competition sponsored by the Académie Royale des Sciences in Paris (hereafter usually called the Paris Academy) concerning the masting of sailing ships, and received an honorable mention.

Prize competitions were an important feature of scientific life at least until the end of the nineteenth century. Originally they were a way of seeking solutions to specific problems. They usually emanated from the royal academies, notably those in Berlin and Paris, and although they provided an opportunity for an unknown young researcher it was quite normal for the well established to enter. In the case of the Paris Academy, for example, prizes were awarded for memoirs addressing specific problems in the mathematical or physical sciences. Among the rules of procedure, each entry had to be under a pseudonym or motto, accompanied by a sealed envelope similarly inscribed containing the name of the author, although this could often be guessed by the judges. The Bernoullis, particularly Daniel, were often successful in these competitions.

With the support of Johann Bernoulli, Euler applied for the vacant professorship of physics at the University of Basel, but was turned down, partly as being too young. The prospects of a position in Switzerland did not seem hopeful. However Daniel Bernoulli had recently accepted the offer of a senior appointment at the newly established Imperial Russian Academy of Sciences in St Petersburg (hereafter usually called the St Petersburg Academy). In 1725 he moved there with his elder brother Nicholaus, and two years later arranged for his young compatriot to join him. Initially Euler was given a junior post in the medical section of the academy, which meant that he had to spend a few months studying anatomy and physiology, but before long he managed to get transferred to the mathematical section, and became a member of the permanent staff. Euler lodged with Daniel Bernoulli and often worked with him at a time when his research interests lay mainly in mechanics and physics, particularly hydrodynamics.

Euler learnt the Russian language, and soon settled in to enjoy the social life of the great city. Unfortunately his arrival coincided with the death of Catherine the First, who had tried to continue the progressive policies of her late husband, the formidable Tsar Peter the Great; this was followed by a period of reaction and intolerance. When Daniel Bernoulli returned to Basel after six years in the Russian capital, Euler succeeded him as the premier mathematician at the academy. In the same year, feeling financially secure, he married Catharina Gsell, the daughter of a Swiss artist then working in Russia. They lived in a comfortable house beside the River Neva.

During this first St Petersburg period, which lasted fourteen years, Euler wrote both elementary and advanced mathematical textbooks for use in Russian schools, and solved many practical problems put to him by the Russian government. As well as being professor of mathematics he was also in charge of the department of geography, where one of his duties was to prepare a map of the country. However he was mainly occupied with mathematical research. Perhaps his best-known work of this period is his formulation of and solution to the problem of the seven bridges of Königsberg, which dates from 1736. This marks the beginning of the branch of mathematics known today as graph theory; Euler uses Leibniz' term 'analysis situs' in this connection, perhaps having heard it from one of the Bernoullis. During this time he wrote a treatise *Mechanica*, in which he showed that mathematical analysis could be applied systematically to Newtonian dynamics. In fact during this period he wrote almost ninety works for publication, and made notes of various important ideas to be developed later. He entered for the annual prize offered by the Paris Academy and was the winner no less than twelve times, surpassing even Daniel Bernoulli's record. It was during this period that he became blind in the right eye; it seems probable that the cause was scrofula (glandular tuberculosis).

Unfortunately conditions in Russia again became oppressive after the death of the Regent Anna Leopoldovna, mother of the infant Tsar Ivan the Sixth, and so, in 1741, Euler accepted an invitation from the King of Prussia, Frederick the Great, to move to the Prussian Royal Academy of Sciences in Potsdam (hereafter usually called the Berlin Academy), which had just been reorganized after a period of decline. As he was still paid a pension by the St Petersburg Academy, for all the next twenty-five years when he was in Berlin, effectively he was working for both. At first the Queen could extract from Euler only monosyllables in response to her enquiries. She taxed him with timidity and reserve, despite the cordiality of his reception: 'Why, then, will you not talk to me?' she asked. 'Because, Madam, I have just come from a country where people are hanged if they talk.' The Eulers settled down first in a house in Behrenstrasse, which is still preserved, and then acquired an estate in Charlottenburg, just outside the city.

It was during this period that he completed his masterpiece, the memoir on the calculus of variations called, in the lingua franca, *Methodus inveniendi lineas curvas maxime minimive proprietate gaudentes*. Its publication in 1744 led to his election to the Royal Society of London and to its Paris Academy, among other honours. In 1750 he conjectured the famous formula relating the number of faces, edges, and vertices of a convex polyhedron, and attempted to prove it. In a more popular vein he published his celebrated *Lettres à une Princesse d'Allemagne* (Letters to a German Princess), consisting of lessons in science for the King's niece, the Princess of Anhalt-Dessau. These 234 letters, written in the period 1760–2, were among the most successful popularizations of science in the eighteenth century but they were much more: apart from their ostensible purpose they provide us with the most exhaustive and authoritative treatment of natural philosophy written by a leading scientist in the eighteenth century. They were translated from German into several languages. But these are just a few examples of his prodigious output.

Frederick the Great saw science as the servant of the state. Its importance for him lay in its ability to further technological progress. Above all he sought to strengthen his army and enrich his lands. Frederick himself had little knowledge of theoretical science or mathematics, so he gauged the importance of the achievements of the Academy's scientists by their practical and military applicability. Yet despite the king's limited conception of science a great deal of theoretical scientific research was carried out at the Academy, since the practical problems he assigned to its staff were not unduly laborious and so plenty of time remained for independent work. 'I can do just what I wish [in my research]', Euler told a friend. 'The King calls me his professor, and I think I am the happiest man in the world.' Unfortunately this situation was not to last.

Initially it was the great French scientist Pierre Maupertuis who presided over the Academy. Following his death in 1759, Euler took over the management but under the direct supervision of the King, who did not entirely trust him. In addition to various substantial administrative responsibilities, Euler was asked to undertake various practical tasks, including finance, ballistics, navigation, water supply, and so on. He was also expected to advise the King on such matters as the purchase of scientific instruments, the construction of watermills, the administration of lotteries, the improvement of canals, and even on the construction of a stone wall around the garden of the academy. Although mostly absent from Berlin during the Seven Years War the Prussian monarch took a close interest in the administration of the Academy, particularly in the making of professional appointments. Eventually this created an impasse in his relationship with Euler, who expected some degree of academic freedom. Increasingly he fell out of favour with the King, who tended to look down on him, referring to him as 'my cyclops' in allusion to his loss of the sight of one eye. So in 1766, at the age of fifty-nine, Euler left Berlin, much to the King's displeasure, and returned to St Petersburg.

Throughout this second period in St Petersburg, which lasted until his death seventeen years later, the Empress Catherine the Great, herself of German origin, was on the Russian throne. She provided for him generously, even lent him one of her cooks. 'I and all others who had the good fortune to be some time with the Russian Imperial Academy', he wrote later, 'cannot but acknowledge that we owe everything which we are and possess to the favourable conditions which we had there.' However Euler's return to the Russian capital was dogged by misfortune at first. His house was burnt down and he lost many of his possessions. In addition he became almost totally blind due to an unsuccessful operation to remove a cataract in his one good eye. Fortunately Euler was blessed with a prodigious memory. As a boy he had memorized the entire text of Virgil's Aeneid. His ability to perform complex calculations in his head was well known, and his memory undoubtedly helped him to cope with blindness in the latter part of his life, one of the most fruitful periods of his career.

Euler's first wife Catharina died in 1776. Of their thirteen children, only three sons and two daughters survived beyond their early years. Euler was especially fond of children, often writing mathematics with a child on his lap. In 1777 he was married again to Catharina's half-sister, Salome Abigail Gsell. On September 18, 1783 he suffered a stroke and died at the age of seventy-six. Earlier that day he had given a mathematics lesson to one of his grandchildren, carried out some calculations on the ascent of balloons, and held a discussion with his assistants concerning the orbit of the newly discovered planet Uranus.

Euler's energies seemed inexhaustible. In pure mathematics his major fields were calculus, differential equations, analytic and differential geometry of curves and surfaces, number theory, infinite series, and the calculus of variations. In applied mathematics he created analytical mechanics. He wrote eminently readable textbooks on mechanics, algebra, mathematical analysis, analytic geometry, differential geometry, and the calculus of variations that were standard works for a century or more. In mathematical physics he built on the work of Daniel Bernoulli. In hydrodynamics, for example, he discovered the fundamental differential equations for the motion of an ideal fluid; and he applied them to the flow of blood in the human body. In the theory of heat he followed Daniel Bernoulli in regarding heat as an oscillation of molecules. He was one of the few scientists of the eighteenth century to favour the wave as opposed to the particle theory of light. He studied the propagation of sound and he obtained many results on the refraction and dispersion of light

However Euler was also remarkable for the skill with which he applied mathematics to practical problems. For example, he investigated the bending of beams and calculated the safety load of columns. He calculated the perturbative effect of celestial bodies on the orbits of planets. He calculated the paths of projectiles in resisting media. His three volumes on optical instruments contributed to the design of telescopes and microscopes. His work on the design of ships aided navigation. He produced a theory of the tides. Nor were his interests confined to subjects closely related to mathematics; he wrote about chemistry, geography, cartography, and much else.

No other mathematician has published as much as Euler did. He wrote almost 900 papers, memoirs, books, and other works. Of these almost half date from the second St Petersburg period, when he was almost blind and everything had to be dictated to assistants. It is estimated that of all the pages published on mathematics, mathematical physics, astronomy, and the engineering sciences during the last three-quarters of the eighteenth century, one third were written by Euler. Some 560 titles were published, but much remained unpublished during in his lifetime. The *Opera Omnia* have been appearing at the rate of one large volume a year on average for the last seventy years (the original programme of publication is now almost complete). But apart from his voluminous published works Euler left a mass of correspondence, personal diaries, and other papers which will continue to occupy the attention of scholars for many years to come.

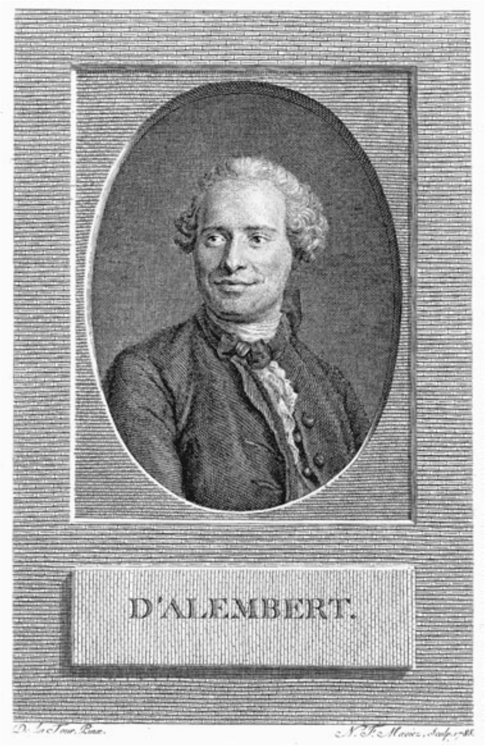
Euler was a simple man, not given to envy. As was said of Leibniz, he was glad to observe the flowering in other people's gardens of plants whose seeds he had provided. Euler had only a few immediate disciples, and none of them was a first-class scientist; yet, as said by Laplace, he was the teacher of all the mathematicians of his time. In mathematics the eighteenth century can fairly be labelled the age of Euler, but his influence on the development of mathematical sciences was not restricted to that period. The work of many outstanding nineteenth-century mathematicians arose directly from his.

Jean-le-Rond d'Alembert (1717–1783)

In the late seventeenth and early eighteenth centuries France had no mathematician to compare with the English Newton or the German Leibniz. However the relative mediocrity of French mathematics which marked the later part of the reign of Louis XIV was now followed by one of the brightest periods in all history, at a time when neither Britain nor Germany had any great mathematicians to show. D'Alembert, the first of the stars of French eighteenth-century mathematics, was often in dispute with Euler. Although Euler was the more powerful mathematician of the two he frequently exploited d'Alembert's ideas. However mathematics was only one of d'Alembert's many interests. He was a leading figure in the Enlightenment, the international movement which took on a special character in France.

Jean-le-Rond d'Alembert was born in Paris on November 17, 1717, so that he was ten years junior to Euler. He was the natural son of Claudine-Alexandrine Guérin, Marquise de Tencin, a well-known salon hostess and novelist of the period, and the Chevalier Louis-Camus Destouches-Canon, a cavalry officer. His mother, who was a lapsed nun, abandoned her new-born child on the steps of the church of Saint Jean-le-Rond, in the cloisters of the cathedral of Notre Dame, because she was afraid of being returned to her convent. However his father traced the child, who was being fostered in Picardy, and who had been given the Christian name of the church where he had been left. Destouches-Canon found him a home in Paris with an artisan named Rousseau and his wife. D'Alembert lived with them until he was forty-seven years old, and all his best scientific and literary work was done under their roof. Destouches-Canon also saw to the education of the boy, and when he died provided his son, who was then only nine years old, with a modest private income for the rest of his life. His mother, whose life was anything but virtuous, had nothing to do with him. The name d'Alembert may be a French version of the German Darenberg, but its significance is unknown.

D'Alembert attended the Collège de Quatre Nations (sometimes called after Mazarin, its founder), a Jansenist school offering a curriculum in the classics and rhetoric – and also offering more than the average amount of mathematics. In a break with tradition the mathematics lectures there were given in French, rather than Latin. The school possessed an excellent library,



of which the boy took full advantage. After obtaining the baccalaureate in 1735 he turned against a religious career, in spite of the efforts of his teachers to persuade him otherwise, and for a time studied both law and medicine before deciding on a career as a mathematician. Although he had received almost no formal scientific training, it is clear that on his own he had become familiar not only with Newton's works but also with those of l'Hôpital and other mathematicians of his day, particularly the Bernoullis.

In 1739, when he was twenty-two, d'Alembert sent his first communication to the Paris Academy. During the next two years he sent five more, dealing with methods of integrating differential equations and with the motion of bodies in resisting media. His communications were answered by Alexis Clairaut who, although only four years older than d'Alembert, was already an Academician. After several attempts to gain election, d'Alembert was successful in 1741.

For the next two years he worked on various problems in rational mechanics (what would nowadays be called theoretical mechanics) and published his *Traité de dynamique* (Treatise on dynamics). In this celebrated work he tried to formalize the new science of mechanics on Newtonian principles. In the first part of the treatise d'Alembert developed his own three laws of motion. Like others of his day he did not simply adopt those of Newton, which in any case were expressed in words, not symbols. He tried to show that the first two laws followed from basic ideas of space and time by reasoning which was mathematical rather than physical. It was not until he arrived at the third law that physical assumptions were involved; he implicitly assumed conservation of momentum, and eschewed the notion of force. The principle that bears his name can already be found in the work of Daniel Bernoulli. It is more like a convenient rule for using Newton's laws, and does not actually follow from them logically.

In 1744 d'Alembert published a companion volume, the *Traité de l'équilibre et du mouvement des fluides* (Treatise on the equilibrium and on the motion of fluids), treating the major problems of fluid mechanics then current. Clairaut published a competing work in the same year. D'Alembert's next work, *Réflexions sur la cause générale des vents* (Reflections on the general cause of the winds), although based on false assumptions, contained the first general use of partial differential equations in mathematical physics. This is just one of many instances where his methods were perfected by Euler. Another was his (incorrect) theory of vibrating strings, where we see the first appearance of a wave equation in physics. Then d'Alembert moved into celestial mechanics and this led him, in 1749, to publish his masterly *Recherches sur la précession des équinoxes et sur la nutation de la terre* (Investigations into the precession of the equinoxes and the nutation of the poles); nutation is a wobble of the earth's axis, due to lunar influences. During the next decade d'Alembert wrote one more major scientific work, his three-volume *Recherches sur différentes points importantes du système du monde* (Investigations into various important points concerning the solar system) of 1754/6. Devoted primarily to the motion of the moon it was written partially to guard his claims to originality against those of Clairaut. As often happened, d'Alembert's theory was the better one but Clairaut's methods were of more practical use to astronomers.

Having reached the top from humble beginnings d'Alembert did not want to lose his position. Once in the Academy, he had to struggle to stay ahead of his rivals. Whether by accident or inborn competitiveness, d'Alembert always seemed to be working on the same problems as other top mathematicians – initially Clairaut, later Daniel Bernoulli and Euler. He