

Introduction to Optical Quantum Information Processing



Quantum information processing offers fundamental improvements over classical information processing, such as computing power, secure communication, and high-precision measurements. However, the best way to create practical devices is not yet known. This textbook describes the techniques that are likely to be used in implementing optical quantum information processors.

After developing the fundamental concepts in quantum optics and quantum information theory, this book shows how optical systems can be used to build quantum computers according to the most recent ideas. It discusses implementations based on single photons and linear optics, optically controlled atoms and solid-state systems, atomic ensembles, and optical continuous variables.

This book is ideal for graduate students beginning research in optical quantum information processing. It presents the most important techniques of the field using worked examples and over 120 exercises.

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Shaftesbury Road, Cambridge CB2 8EA, United Kingdom

One Liberty Plaza, 20th Floor, New York, NY 10006, USA

477 Williamstown Road, Port Melbourne, VIC 3207, Australia

314–321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre, New Delhi – 110025, India

103 Penang Road, #05–06/07, Visioncrest Commercial, Singapore 238467

Cambridge University Press is part of Cambridge University Press & Assessment,
a department of the University of Cambridge.

We share the University's mission to contribute to society through the pursuit of
education, learning and research at the highest international levels of excellence.

www.cambridge.org

Information on this title: www.cambridge.org/9780521519144

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First published 2010

A catalogue record for this publication is available from the British Library

ISBN 978-0-521-51914-4 Hardback

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To Rose, Xander and Janet

Contents

Preface page xi

Part I Quantum optics and quantum information

1	The quantum theory of light	3
1.1	The classical electromagnetic field	3
1.2	Quantization of the electromagnetic field	6
1.3	Mode functions and polarization	16
1.4	Evolution of the field operators	25
1.5	Quantum states of the electromagnetic field	37
1.6	References and further reading	46
2	Quantum information processing	48
2.1	Quantum information	48
2.2	Quantum communication	57
2.3	Quantum computation with qubits	62
2.4	Quantum computation with continuous variables	80
2.5	References and further reading	89
3	Figures of merit	90
3.1	Density operators and superoperators	90
3.2	The fidelity	100
3.3	Entropy, information, and entanglement measures	101
3.4	Correlation functions and interference of light	105
3.5	Photon correlation measurements	108
3.6	References and further reading	110

Part II Quantum information in photons and atoms

4	Photon sources and detectors	113
4.1	A mathematical model of photodetectors	113
4.2	Physical implementations of photodetectors	121
4.3	Single-photon sources	129
4.4	Entangled photon sources	139
4.5	Quantum non-demolition photon detectors	142
4.6	References and further reading	144

5	Quantum communication with single photons	145
5.1	Photons as information carriers	145
5.2	Quantum teleportation and entanglement swapping	162
5.3	Decoherence-free subspaces for communication	170
5.4	Quantum cryptography	172
5.5	References and further reading	177
6	Quantum computation with single photons	179
6.1	Optical N -port interferometers and scalability	179
6.2	Post-selection and feed-forward gates	181
6.3	Building quantum computers with probabilistic gates	192
6.4	Photon counting and quantum memories	202
6.5	Threshold theorem for linear-optical quantum computing	207
6.6	References and further reading	209
7	Atomic quantum information carriers	210
7.1	Atomic systems as qubits	210
7.2	The Jaynes–Cummings Hamiltonian	222
7.3	The optical master equation and quantum jumps	227
7.4	Entangling operations via path erasure	236
7.5	Other entangling gates	245
7.6	References and further reading	251
Part III Quantum information in many-body systems		
8	Quantum communication with continuous variables	255
8.1	Phase space in quantum optics	255
8.2	Continuous-variable entanglement	267
8.3	Teleportation and entanglement swapping	272
8.4	Entanglement distillation	280
8.5	Quantum cryptography	281
8.6	References and further reading	293
9	Quantum computation with continuous variables	294
9.1	Single-mode optical qunat gates	294
9.2	Two-mode Gaussian qunat operations	299
9.3	The Gottesman–Knill theorem for qunats	303
9.4	Nonlinear optical qunat gates	307
9.5	The one-way model for qunats	309
9.6	Quantum error correction for qunats	318
9.7	References and further reading	326
10	Atomic ensembles in quantum information processing	327
10.1	An ensemble of identical two-level atoms	327
10.2	Electromagnetically induced transparency	337

10.3	Quantum memories and quantum repeaters	344
10.4	The atomic ensemble as a single qubit	352
10.5	Photon–photon interactions via atomic ensembles	355
10.6	References and further reading	360
11	Solid-state quantum information carriers	361
11.1	Basic concepts of solid-state systems	361
11.2	Definition and optical manipulation of solid-state qubits	375
11.3	Interactions in solid-state qubit systems	381
11.4	Entangling two-qubit operations	384
11.5	Scalability of solid-state devices	393
11.6	References and further reading	395
12	Decoherence of solid-state qubits	397
12.1	Phonons	397
12.2	Electron–phonon coupling	400
12.3	The master equation for electrons and phonons	403
12.4	Overcoming decoherence	406
12.5	Strong coupling effects	412
12.6	References and further reading	419
13	Quantum metrology	421
13.1	Parameter estimation and Fisher information	421
13.2	The statistical distance	425
13.3	The dynamical evolution of states	433
13.4	Entanglement-assisted parameter estimation	437
13.5	Optical quantum metrology	440
13.6	References and further reading	452
Appendix A Baker–Campbell–Hausdorff relations		454
Appendix B The Knill–Laflamme–Milburn protocol		457
Appendix C Cross–Kerr nonlinearities for single photons		462
<i>References</i>		465
<i>Index</i>		477

Preface

The field of quantum information processing has reached a level of maturity, and spans such a wide variety of topics, that it merits further specialization. In this book, we consider quantum information processing with optical systems, including quantum communication, quantum computation, and quantum metrology. Optical systems are the obvious choice for quantum communication, since photons are excellent carriers of quantum information due to their relatively slow decoherence. Indeed, many aspects of quantum communication have been demonstrated to the extent that commercial products are now available. The importance of optical systems for quantum communication leads us to ask whether we can construct integrated systems for communication and computation in which all processing takes place in optical systems. Recent developments indicate that while full-scale quantum computing is still extremely challenging, optical systems are one of the most promising approaches to a fully functional quantum computer.

This book is aimed at beginning graduate students who are starting their research career in optical quantum information processing, and it can be used as a textbook for an advanced master's course. The reader is assumed to have a background knowledge in classical electrodynamics and quantum mechanics at the level of an undergraduate physics course. The nature of the topic requires familiarity with quantized fields, and since this is not always a core topic in undergraduate physics, we derive the quantum mechanical formulation of the free electromagnetic field from first principles. Similarly, we aim to present the topics in quantum information theory in a self-contained manner.

The book is organized as follows: in Part I, we develop the quantum theory of light, give an introduction to quantum communication and computation, and we present a number of advanced quantum mechanical techniques that are essential for the understanding of optical quantum information processing. In Part II, we consider quantum information processing using single photons and atoms. We first develop the theory of photodetection and explore what we mean by photon sources, followed by an exposition of quantum communication with single photons, quantum computation with single photons and linear optics, and quantum computing where the information carriers, the qubits, are encoded in atoms. In Part III, we explore quantum information processing in many-body systems. We revisit linear optical quantum communication and computation, but now in the context of quantum continuous variables, rather than qubits. We discuss how atomic ensembles can be used as quantum memories and repeaters, and we study in detail how to define robust qubits in solid-state systems such as quantum dots and crystal defects. The last chapter of the book deals with quantum metrology, where we explore how quantum states of light can be exploited to attain a measurement precision that outperforms classical metrology. As is inevitable in a book of this nature, a number of important topics have been omitted due

to length restrictions. We have not included quantum information processing in ion traps, photonic band-gap materials, optical lattices, and Bose–Einstein condensates. We have also omitted the topic of quantum imaging.

We wish to thank a number of colleagues who have made valuable comments, and suggested many improvements: Charles Adams, Simon Benjamin, Samuel Braunstein, Earl Campbell, Jim Franson, Erik Gauger, Dominic Hosler, Nick Lambert, Peter van Loock, Janet Lovett, Ahsan Nazir, Todd Pittman, Nusrat Rafiq, Andrew Ramsay, Marshall Stoneham, Joachim Wabnig, David Whittaker, and Marcin Zwierz. We thank Joost Kok for suggesting the artist Victor Vasarely for the cover image. BWL thanks the Royal Society for financial support. Finally, we would like to thank Andrew Briggs and the Quantum Information Processing Interdisciplinary Research Collaboration (QIP IRC) for continued support.