1 An overview of computational electromagnetics for RF and microwave applications

Even if we do discover a complete unified theory, it would not mean that we would be able to predict events in general . . . even if we do find a complete set of basic laws, there will still be in the years ahead the intellectually challenging task of developing better approximation methods, so that we can make *useful predictions of the probable outcomes in complicated and realistic situations*.

From [1, pp. 168–169] (the present author's emphasis)

Computations: no-one believes them, except the person who made them.
Measurements: everyone believes them, except the person who made them . . .

Attributed to the late Professor B. Munk, Ohio State University

1.1 Introduction

Electromagnetics, the study of electrical and magnetic fields and their interaction, has been one of the core technologies of the twentieth century, and shows every sign of continuing this into the twenty-first. Whilst there are many useful ways of subdividing the field, power frequency versus radio frequency, or alternatively quasi-static versus full-wave, is one of the most insightful here. This book focusses exclusively on radio-frequency, full-wave electromagnetic modelling, as typically encountered in communication systems.

The core of modern electromagnetic engineering is of course Maxwell's equations. Written in modern form,[1] they are:

$$\nabla \times \boldsymbol{E} = -\frac{\partial}{\partial t}\boldsymbol{B} \tag{1.1}$$

$$\nabla \times \boldsymbol{H} = \boldsymbol{J} + \frac{\partial}{\partial t}\boldsymbol{D} \tag{1.2}$$

$$\nabla \cdot \boldsymbol{D} = \rho \tag{1.3}$$

$$\nabla \cdot \boldsymbol{B} = 0 \tag{1.4}$$

with the associated constitutive equations

$$\boldsymbol{B} = \mu \boldsymbol{H} \tag{1.5}$$

$$\boldsymbol{D} = \varepsilon \boldsymbol{E} \tag{1.6}$$

[1] Maxwell did not actually write his equations in this form; vector analysis was a late nineteenth-century development.

The actual *solution* of the Maxwell equations is complex, and for realistic problems, approximations are usually required – as indicated by the introductory quote from Hawking, although he had in mind an altogether more ambitious theory (of everything!). The numerical approximation of Maxwell's equations, the subject of this book, is known as *computational electromagnetics* (CEM).

CEM techniques have been available for close on five decades now. These techniques have gestated, grown and matured to the point where they form an invaluable part of current RF and microwave engineering practice [2]. However, the widespread adoption of computational methods to complement the traditional tools of analysis and measurement has attracted criticism, summarized with more than a grain of truth by the second quote at the beginning of the chapter. Ironically, the availability of powerful, commercial codes may well have made the situation worse, not better, since more and more frequently, codes are being applied by users unfamiliar with the basic formulations underlying the codes, and not infrequently to problems for which the codes were not designed. One of the major aims of this book is to make RF computational electromagnetics comprehensible and accessible to a far wider group of RF engineers than has been the case in the past.

CEM is a multi-disciplinary field. Its core disciplines are electromagnetic theory and numerical methods, but for useful implementations, geometric modelling and visualization, computer science and algorithms all have important roles to play. In this book, the focus falls on the core disciplines.

The applications of CEM are legion, and include antennas, biological EM effects, medical diagnosis and treatment, electronic packing and high-speed circuitry, superconductivity, microwave devices, monolithic microwave integrated circuits, law enforcement, environmental issues, materials, avionics, communications, energy generation and conservation, low observable vehicles (stealth), radars and imaging, surveillance and intelligence gathering. In this book, we focus primarily on applications in antennas, wireless communications, radar, and (passive) microwave devices, although an example will be given of a biological EM effect study.

An historical aside – a brief history of electromagnetics

Interest in static electricity and magnetism, of course, dates back to ancient times. The Ancient Greeks circa 400 BC noted that rubbing amber attracted bits of straw, and the Chinese reportedly found lodestones (natural magnets) circa 2600 BC, first using them for burial purposes, and later for navigation. The modern study of electromagnetic phenomena dates to the late eighteenth century, with the great progress in experimental methods by Alessandro Volta (1745–1827), Hans Christian Oersted (1777–1851) and Michael Faraday (1791–1867) on the one hand, and the more mathematical modelling approach of Charles Augustin de Coloumb (1736–1806) and André-Marie Ampère (1775–1836) on the other. Amongst these, the following milestones stand out: the development of the battery by Volta provided a continuous source of electricity for the first time; Coloumb's careful measurements of the electric force resulted in the famous inverse square law; Oersted's 1820 discovery

showed that (direct) current deflected a magnet; Ampère developed mathematical laws describing this and the force between current carrying wires; and finally, Faraday's crucial contribution in 1831 showed that a changing magnetic field sets up an alternating current (i.e. an electric field), and for the first time connected two forces of nature which until then had been thought quite independent.

James Clark Maxwell (1831–1879), the most brilliant physicist of the nineteenth century,[a] combined the work of his predecessors in elegant theoretical fashion and postulated that changing electric fields should generate magnetic fields; he then showed that this implied *wave motion*. Hermann Ludwig-Ferdinand Helmholtz (1821–1894) was one of the first to recognize the significance of Maxwell's predictions in this regard; in 1888, his student Heinrich Rudolph Hertz (1857–1894) showed experimentally that electromagnetic fields indeed propagate, and at the speed of light. Oliver Heaviside (1850–1925) also made contributions in this regard, although his work is not widely recognized nowadays [3]. In what we would now describe as the first commercial spin-off of this work, Guglielmo Marconi (1874–1937) was the first to profit financially from the emerging field of wireless.

Electromagnetics was also to have a profound influence on the outstanding physicist of the twentieth century, Albert Einstein (1879–1955). Perhaps less well known than some of his results – certainly amongst the general public – Einstein showed that the magnetic field is the relativistic correction of the electric field, confirming the unified field theoretic nature of Maxwell's electromagnetic theory.

The above is the conventional Western history of electromagnetics. Contributions to the theory of light, intimately connected to electromagnetics, were made by many over an extremely long period of time, including contributions from Arabic scholars. An exceptionally erudite historical perspective may be found in [4].

[a] Maxwell not only unified electricity and magnetism in 1864, he also developed the kinetic theory of gases, before his life was cut tragically short by illness.

1.2 Full-wave CEM techniques

Full-wave CEM methods approximate the Maxwell equations numerically, without any initial physical approximations being made. These are also sometimes called *low-frequency* methods, to distinguish them from asymptotic *high-frequency* methods, but this can be confusing for several reasons.[2] The full-wave techniques which will be studied in this book are the finite difference time domain (FDTD) method; the method of moments (MoM); and the finite element method (FEM). Whilst there are other methods available, these are the most widely used, and all have been implemented in powerful

[2] Firstly, the high-frequency radio band is specifically the spectrum from 3–30 MHz; secondly, the meaning of low and high are entirely relative, and the same methods may be, and are, useful from power frequencies up to the visible spectrum and beyond; and finally, "high-frequency" as a general term in electronic engineering is widely used to distinguish from "power frequency," with the latter usually using quasi-static approaches.

computer codes. These techniques are frequently classified further by whether they are based on integral or differential equations, and by whether they operate in the time or frequency domain. We will discuss this in the context of each method subsequently.

Sometimes, the expressions "static" or "quasi-static" will be used. The former applies obviously to the situation where one is dealing with either steady-state charges (and the associated electric fields) or currents (and the associated magnetic fields). The latter applies to situations where the time rate of change is low enough that the fields still satisfy the static equations to a very good approximation – or put differently, the $(\partial/\partial t)\boldsymbol{B}$ term in Eq. (1.1) is negligible (in which case one obtains electroquasistatics) or similarly for the $(\partial/\partial t)\boldsymbol{D}$ term in Eq. (1.2) (which yields magnetoquasistatics). A very detailed discussion of these approximations and their use may be found in [5]. However, we will not pursue this far in this book, which deals almost entirely with full-wave methods.

There is another class of numerical method for solving the Maxwell equations, generally called the *asymptotic* techniques. These methods require fundamental approximations in the Maxwell equations, the validity of which increases asymptotically with frequency. Examples are physical optics (PO), geometrical optics (GO) and the uniform theory of diffraction (UTD). This is a field of study in its own right. For suitable problems, these methods are very powerful, but the underlying approximations of the physics limits their use for general problems. Furthermore, unlike the full-wave methods, where Moore's law and the resulting increase in computer speed and memory continually extend the limits of applicability, the asymptotic methods have fundamental limits. Hence, in this book, only full-wave methods are considered. However, a hybridization with an asymptotic technique will be discussed as an example of an advanced application.

The full-wave techniques are potentially very accurate. Central to *all* these methods is the idea of *discretizing* some unknown electromagnetic property, typically the surface current for the MoM, and the $\boldsymbol{E}$ field for the FEM and FDTD method. (For the latter, the $\boldsymbol{H}$ field is also discretized.) This process of discretization is also known as *meshing*. It entails subdividing the geometry into a (large) number of small *elements*. These may be one-dimensional segments, two-dimensional surface "patches" (often triangles), three-dimensional tetrahedral elements or a regular three-dimensional "staggered" grid, depending on the problem at hand and the method used. Within each element, a simple functional dependence is assumed for the spatial variation of the unknown – for instance, a linear approximation – but the *amplitude* (and possibly phase) of the unknown is determined by application of the method to the patchwork of elements which approximates the original geometry. This functional dependence is also known as the basis (or expansion) function.[3]

Generally, the accuracy of the methods is related to the discretization (i.e. mesh size). The finer is the mesh, the better is the accuracy of the methods.[4] The largest mesh size (alternatively, the finest geometrical resolution) is limited by the available

[3] With the FDTD method as usually introduced, the fields are sampled at points; it is however possible to define basis functions for the FDTD, a topic we discuss briefly in Chapter 12.

[4] This is not invariably true: limitations imposed by approximations in the formulations may place some lower bound on element size. A classic example is a thin-wire MoM formulation, where using too many segments may violate the underlying thin-wire assumptions. This is discussed in detail in Section 4.3.

Table 1.1 Strengths and weaknesses of CEM methods as widely implemented for open region problems

Formulation	Equation type	Domain	Radiation condition	PEC only	Homogeneous penetrable	Inhomogeneous penetrable
MoM	Integral	Frequency	Yes	⌣	⌣	⌢
FEM	Differential	Frequency	No	⌢	⌣	⌣
FDTD	Differential	Time	No	⌢	⌣	⌣

Key: ⌣ good; ⌢ not optimal.

computational resources. In other fields such as structural mechanics, the mesh fineness is usually determined by the requirement to resolve the *structural geometry* adequately; in radio-frequency electromagnetics, the requirement on the mesh is usually to sample the *phase* adequately. For many years, the CEM community has worked with a rule of thumb of ten segments per wavelength. This was originally derived for wire antenna problems, where the mesh is one-dimensional; for surfaces, this guideline becomes 100 segments per square wavelength (and a similar extension for volumetric meshes to 1000 per cubic wavelength). Much work on better elements has been done to reduce this requirement – it will readily be appreciated that as the dimensionality of the problem goes up, so this becomes progressively more crucial. It should also be noted that when very accurate field data are required – for example, when computing antenna input impedance – a *finer* mesh may be required, at least locally around the feed point of the antenna. Furthermore, this guideline ignores the problem of dispersion in differential equation based solvers, which effectively requires *denser* meshes for electromagnetically larger problems.

Although the full-wave methods share the basic idea of discretization, and indeed have been viewed within a very general framework as simply different implementations of one overarching theoretical formulation, in practice, the methods have quite different challenges for theoreticians, code developers and users, as well as different optimal areas of application, and as such, they will be considered separately in this overview chapter. In Chapter 12, some of the underlying mathematical connections between the methods will emerge.

In the rest of this overview chapter, the MoM, FEM and FDTD method will be reviewed *qualitatively*, emphasizing basic principles such as the underlying formulation (integral/differential equation based, frequency or time domain) and areas of application (perfectly or highly conducting materials versus homogeneous or inhomogeneous penetrable structures; microwave devices versus radiation or scattering analysis). This review is especially designed for readers who have a particular problem to solve, but are not sure which is the best method to use. Details of each method will be found in the subsequent chapters of the book. Key references only are given; a far more extensive list of references will be found at the end of each chapter.

By way of introduction, some of the most important characteristics of the MoM, FEM and FDTD method are presented in Tables 1.1 and 1.2. Table 1.1 provides a comparison of the methods for open region (radiation and scattering) problems. It is important to

Table 1.2 Strengths and weaknesses of CEM methods for guided wave problems

Formulation	Equation type	Domain	Wideband	PEC only	Homogeneous penetrable	Inhomogeneous penetrable
MoM	Integral	Frequency	∼	⌣	⌣	⌢
FEM	Differential	Frequency	∼	⌣	⌣	⌣
FDTD	Differential	Time	⌣	⌣	⌣	⌣

Key: ⌣ good; ∼ satisfactory, but not necessarily the best; ⌢ not optimal.

note that what is presented in this table are the key characteristics of the method *as widely implemented and understood in the CEM community*. As will be seen in the description of each method in the following sections, a number of simplifications have been made in this table: the MoM, for instance, can be seen in a more general sense as including the FEM, although this is not normal usage; and to give another example, the FEM can also operate in the time domain, but there are no commercial implementations of this at present. For the MoM, homogeneous penetrable materials (dielectrics, for instance) can either be modelled using equivalent surface currents or, if the problem consists of layered materials, using a Sommerfeld formulation. This has not been noted in the table, since it depends on the details of the problem. Table 1.2 provides a similar comparison of the methods for guided wave problems.[5] Again, the details of the precise implementation have not been commented on.

This can be further illustrated by studying one of the most significant differences between the methods (as usually deployed), namely meshing requirements. In Figs. 1.1, 1.2 and 1.3, the meshes required by typical MoM, FDTD and FEM codes to handle a problem involving scattering from a homogeneous sphere are compared. MoM codes often use triangular meshes to approximate surfaces, an approach which provides accurate modelling of general geometries. The reduction in dimension afforded by the MoM (here, from a 3D volume to a 2D surface) is clear – the price of this is that every element on the surface now interacts with every other. FDTD codes use a regular "brick," or cuboidal, mesh. This approach is core to the speed of the method, but clearly requires a fine mesh (as here) to model curved geometries with reasonable fidelity. The interaction between elements in an FDTD mesh is local, and the volume need not be materially homogeneous. FEM codes generally use unstructured tetrahedral meshes; similar to triangular surface meshes, tetrahedral meshes offer accurate modelling of fine geometrical details. Again, similiar to the FDTD, the interaction between elements is local, and the volume can be inhomogeneous. (Unlike the FDTD, the irregular, or unstructured, mesh means that geometrically local elements may not be local in the matrix describing the problem[6].) The FEM model as shown in Fig. 1.3 also includes an outer spherical shell,

[5] It is tempting to use the term "closed problems" here, but a number of important guiding structures, such as microstrip, are partially open. It is assumed in this table that FEM and FDTD codes have an appropriate method of terminating this region. Since the energy decays rapidly away from the guiding structure, and this radiation is a secondary effect in most applications, the open boundary is usually less problematic here than in the case of the radiation and scattering problems.

[6] The FDTD is actually a matrix-free method.

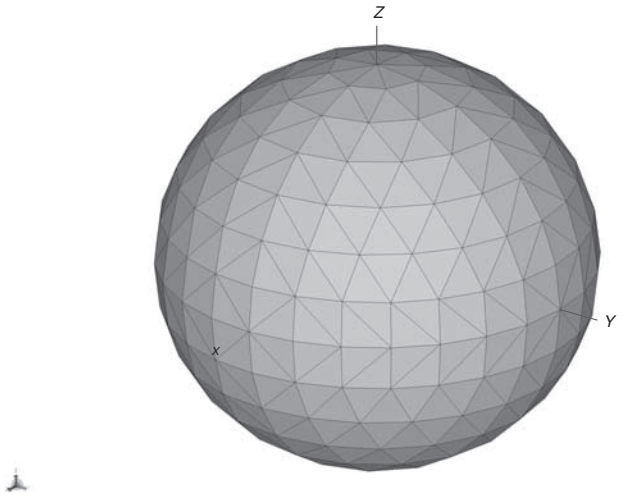

Figure 1.1 A triangular surface mesh of a sphere, as used by typical MoM code.

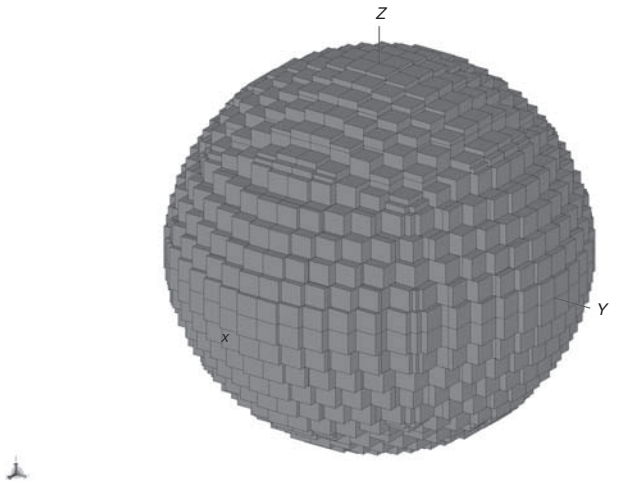

Figure 1.2 A cuboidal volumetric mesh of a sphere, as used by typical FDTD code.

which would usually be free space, providing a "buffer" between the scatterer and whatever mesh closure scheme is applied on the outer boundary to approximate an infinitely large mesh (also known as the "radiation condition"); the FDTD also needs this, but it is not shown in Fig. 1.2. The MoM formulation, which incorporates the radiation condition at formulation level, does not need this. However, and importantly, the MoM surface formulation can only be applied to *homogeneous* scatterers; when dealing with an inhomogeneous scatterer, the MoM also requires a volumetric mesh such as that in Figs. 1.2 or 1.3.

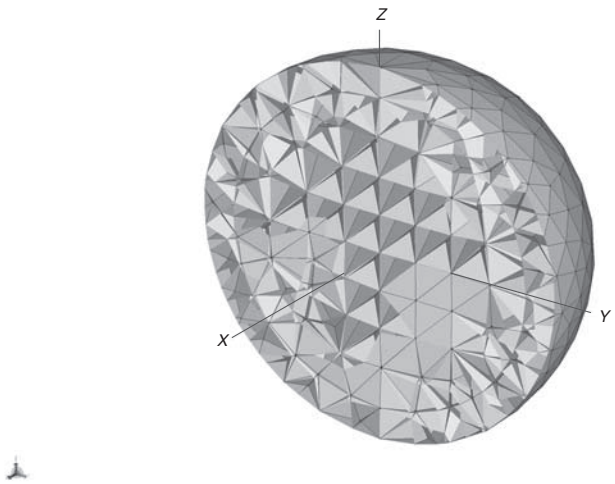

Figure 1.3 A tetrahedral volumetric mesh of a sphere and spherical shell, as used by typical FEM code.

1.3 The method of moments (MoM)

The MoM is probably the most widely used numerical technique in RF CEM, and has a long history in the field; some of this is presented in Chapter 4. For antenna engineering, the MoM has been the most widely used CEM method.[7] In the method of moments, the radiating/scattering structure is replaced by equivalent currents. These are normally surface currents. (Volumetric currents can be used for inhomogeneous dielectric bodies. This is however very expensive computationally.) This surface current is discretized into wire segments and/or surface patches. A matrix equation is then derived, representing the effect of every segment/patch on every other segment/patch. This interaction is computed using the Green function for the problem. (Green functions will be discussed later in this book – indeed, an entire chapter, Chapter 7, is devoted to one such function.) Most MoM codes use the free-space Green function. The relevant boundary condition is then applied to all the interactions, yielding a set of linear equations. The solution of this linear system yields the (approximate) current on each segment/patch. The resulting matrix which must be factored (or used in an iterative solution scheme) is fully populated, with complex valued entries. Typical matrix dimensions range from some hundreds for small antenna problems to several thousand – the upper limit is imposed by computational limitations, either limited memory and/or excessive runtime.

Traditionally, the MoM has been applied in the frequency domain, i.e. single frequency, or monochromatic, sinusoidal excitation, with an $e^{j\omega t}$ convention assumed. The working

[7] The name "method of moments" is peculiar to the CEM community. Perhaps the most descriptive alternative name is the "method of weighted residuals." The term "boundary element method" is frequently used synonymously with MoM, and for surface formulations this is correct, but there are some moment method formulations which use volume, not boundary, elements. We discuss this further in Chapter 4.

variables (unknowns) are thus complex valued, with a magnitude and phase, as for any phasor analysis. Time domain integral equation (TDIE) formulations have been used on occasions, but stability and other issues have proven difficult, and TDIE codes are rare.

The use of the MoM for antenna analysis was given a major boost by the US government's de facto decision during the late 1980s to release the numerical electromagnetic code – method of moments (widely known as NEC-2) into the public domain. NEC-2 is a powerful, general-purpose antenna modelling program, but with no graphical abilities whatsoever and very limited meshing abilities. NEC-2 is discussed in Chapter 5. A later version, NEC-4, added some specialized functionality. At present, there are some excellent commercial codes which offer all the functionality of NEC-2, but with proper graphical user tools and frequently greatly enhanced abilities; examples are FEKO (which will be used quite extensively in this book), SuperNEC, Ensemble and IE3D. (Only SuperNEC is a direct descendant of NEC, the others are independent implementations.) There are also some semi-commercial packages such as GEMACS which are limited to US Department of Defense contractors, and hence not generally available for commercial use world-wide.

The strong points of the MoM (as usually applied) are the following:

- Efficient treatment of perfectly or highly conducting surfaces. Only the surface is meshed; no "air region" around the antenna need be meshed. For wire antennas, the treatment is even more efficient, since only a one-dimensional discretization of the wire is undertaken.
- The MoM automatically incorporates the "radiation condition" – i.e. the correct behavior of the field far from the source (proportional to $1/r$ in free space). This is very important when dealing with radiation or scattering problems.
- The working variable is the current density, from which many important antenna parameters (impedance, gain, radiation patterns, etc.) may be derived, some directly and some via straightforward numerical integration.
- Via the Sommerfeld potentials, efficient formulations may be derived for stratified (layered) media. Important examples are printed antennas, components and feed networks (e.g. microstrip technology) and antenna-above-real-earth calculations.
- The availability of NEC-2 in the public domain – this powerful code has served as the basis for much MoM-based antenna design, and due to the open source nature, has lent itself to all manner of numerical experimentation and improvement.

The weak points of the MoM may be summarized as follows:

- The MoM does not handle electromagnetically penetrable materials as well as differential equation formulations. This is especially true if the material is inhomogeneous. (If the materials are homogeneous, a reasonably efficient fictitious, equivalent surface current formulation may be used, but inhomogeneous materials require fictitious equivalent volumetric currents, and become very expensive computationally.)
- The MoM does not scale gracefully with frequency – for typical applications requiring a surface mesh, the scaling is $\mathcal{O}((kd)^6)$ where kd is the electromagnetic size of the

structure.[8] (This assumes a cubic structure, for simplicity.) Note that this is implies an $\mathcal{O}(f^6)$ scaling – doubling the frequency can result in a runtime 64 times as long! We will see that this is a major problem with *all* the full-wave computational methods, although the details do vary slightly from method to method. For an MoM volumetric mesh, required by an inhomogeneous structure, the scaling is $\mathcal{O}((kd)^9)$; this is so large that such methods are usually very limited in application.

- Some MoM formulations, in particular those based on the magnetic field integral equation (MFIE), require the surface to be *closed*. This is frequently impractical.

In conclusion, the MoM is the preferred method for frequency domain radiation and scattering problems involving perfectly or highly conducting wires and/or surfaces. If the problem involves inhomogeneous dielectric materials, it is unlikely to be the best formulation, but if hybridized with the FEM a very efficient formulation can result.

1.4 The finite difference time domain (FDTD) method

The finite difference time domain (FDTD) method is of a similar vintage to the MoM and FEM in electromagnetics, dating back to the 1960s. Like the FEM, it is partial differential equation based, and one does *not* need a Green function. *Unlike* the FEM, the FDTD method does not use variational functionals or weighted residuals – it directly approximates the differential operators in the Maxwell curl equations, on a grid staggered in time and space. $\boldsymbol{E}$ and $\boldsymbol{H}$ fields are computed on a regular grid, with a marching-on-in-time discretization of time, with field components being offset by $\Delta s/2$ relative to each other and the $\boldsymbol{E}$ and $\boldsymbol{H}$ fields evaluated $\Delta t/2$ apart in time, where Δs and Δt are the spatial and temporal discretizations respectively. This permits a scheme which uses first-order numerical differentiation to provide second-order accuracy. It is also the only widely used CEM scheme to operate in the time domain. (Time domain MoM and FEM formulations have been used, but usually for a rather specialized application. Frequency domain finite difference formulations are also available, but again have never become very popular for general problems.)

Some history of the FDTD method may be found in Chapter 2. For various reasons, the method languished in relative obscurity throughout most of the 1960s and 1970s, but sprang to prominence in the 1980s. There were both technological driving factors behind this – on the one hand, increasing interest in the modelling of inhomogeneous materials, in particular for the assessment of human exposure to RF fields, and on the other, the development of low-observable "stealth" technology – and enabling technology in the shape of the enormous growth in computer power – in particular, memory, for which the FDTD method has a voracious appetite in three dimensions. The development by Berenger of the perfectly matched layer in 1994 solved the previously problematic issue

[8] The notation $\mathcal{O}(x)^p$ means *of the* (*asymptotic*) *order of* and indicates to the highest power (p) present in the variable (x); note that it says nothing of the constants. This can be important, since CEM analysis is quite often undertaken in the "pre-asymptotic" region, where lower powers in x may dominate especially runtime.