

CONTENTS

Preface	page xiii
1 Equivalent Single-Degree-of-Freedom System and Free Vibration	1
1.1 Degrees of Freedom	3
1.2 Elements of a Vibratory System	5
1.2.1 <i>Mass and/or Mass-Moment of Inertia</i>	5
Pure Translational Motion	5
Pure Rotational Motion	6
Planar Motion (Combined Rotation and Translation) of a Rigid Body	6
Special Case: Pure Rotation about a Fixed Point	8
1.2.2 <i>Spring</i>	8
Pure Translational Motion	8
Pure Rotational Motion	9
1.2.3 <i>Damper</i>	10
Pure Translational Motion	10
Pure Rotational Motion	11
1.3 Equivalent Mass, Equivalent Stiffness, and Equivalent Damping Constant for an SDOF System	12
1.3.1 <i>A Rotor–Shaft System</i>	13
1.3.2 <i>Equivalent Mass of a Spring</i>	14
1.3.3 <i>Springs in Series and Parallel</i>	16
Springs in Series	16
Springs in Parallel	17
1.3.4 <i>An SDOF System with Two Springs and Combined Rotational and Translational Motion</i>	19
1.3.5 <i>Viscous Dampers in Series and Parallel</i>	22

Dampers in Series	22
Dampers in Parallel	23
1.4 Free Vibration of an Undamped SDOF System	25
1.4.1 <i>Differential Equation of Motion</i>	25
Energy Approach	27
1.4.2 <i>Solution of the Differential Equation of Motion</i>	
<i>Governing Free Vibration of an Undamped</i>	
<i>Spring–Mass System</i>	34
1.5 Free Vibration of a Viscously Damped SDOF System	40
1.5.1 <i>Differential Equation of Motion</i>	40
1.5.2 <i>Solution of the Differential Equation of Motion</i>	
<i>Governing Free Vibration of a Damped</i>	
<i>Spring–Mass System</i>	41
Case I: Underdamped ($0 < \xi < 1$ or $0 < c_{eq} < c_c$)	42
Case II: Critically Damped ($\xi = 1$ or $c_{eq} = c_c$)	45
Case III: Overdamped ($\xi > 1$ or $c_{eq} > c_c$)	46
1.5.3 <i>Logarithmic Decrement: Identification of Damping</i>	
<i>Ratio from Free Response of an Underdamped</i>	
<i>System ($0 < \xi < 1$)</i>	51
Solution	55
1.6 Stability of an SDOF Spring–Mass–Damper System	58
Exercise Problems	63
2 Vibration of a Single-Degree-of-Freedom System Under	
Constant and Purely Harmonic Excitation	72
2.1 Responses of Undamped and Damped SDOF Systems	
to a Constant Force	72
Case I: Undamped ($\xi = 0$) and Underdamped	
($0 < \xi < 1$)	74
Case II: Critically Damped ($\xi = 1$ or $c_{eq} = c_c$)	75
Case III: Overdamped ($\xi > 1$ or $c_{eq} > c_c$)	76
2.2 Response of an Undamped SDOF System	
to a Harmonic Excitation	82
Case I: $\omega \neq \omega_n$	83
Case II: $\omega = \omega_n$ (Resonance)	84
Case I: $\omega \neq \omega_n$	87
Case II: $\omega = \omega_n$	87
2.3 Response of a Damped SDOF System to a Harmonic	
Excitation	88
Particular Solution	89
Case I: Underdamped ($0 < \xi < 1$ or $0 < c_{eq} < c_c$)	92

Contents	ix
Case II: Critically Damped ($\xi = 1$ or $c_{eq} = c_c$)	92
Case III: Overdamped ($\xi > 1$ or $c_{eq} > c_c$)	94
2.3.1 <i>Steady State Response</i>	95
2.3.2 <i>Force Transmissibility</i>	101
2.3.3 <i>Quality Factor and Bandwidth</i>	106
Quality Factor	106
Bandwidth	107
2.4 Rotating Unbalance	109
2.5 Base Excitation	116
2.6 Vibration Measuring Instruments	121
2.6.1 <i>Vibrometer</i>	123
2.6.2 <i>Accelerometer</i>	126
2.7 Equivalent Viscous Damping for Nonviscous Energy	
Dissipation	128
Exercise Problems	132
3 Responses of an SDOF Spring–Mass–Damper System to Periodic and Arbitrary Forces	138
3.1 Response of an SDOF System to a Periodic Force	138
3.1.1 <i>Periodic Function and its Fourier Series Expansion</i>	139
3.1.2 <i>Even and Odd Periodic Functions</i>	142
Fourier Coefficients for Even Periodic Functions	143
Fourier Coefficients for Odd Periodic Functions	145
3.1.3 <i>Fourier Series Expansion of a Function with a Finite Duration</i>	147
3.1.4 <i>Particular Integral (Steady-State Response with Damping) Under Periodic Excitation</i>	151
3.2 Response to an Excitation with Arbitrary Nature	154
3.2.1 <i>Unit Impulse Function $\delta(t - a)$</i>	155
3.2.2 <i>Unit Impulse Response of an SDOF System with Zero Initial Conditions</i>	156
Case I: Undamped and Underdamped System ($0 \leq \xi < 1$)	158
Case II: Critically Damped ($\xi = 1$ or $c_{eq} = c_c$)	158
Case III: Overdamped ($\xi > 1$ or $c_{eq} > c_c$)	159
3.2.3 <i>Convolution Integral: Response to an Arbitrary Excitation with Zero Initial Conditions</i>	160
3.2.4 <i>Convolution Integral: Response to an Arbitrary Excitation with Nonzero Initial Conditions</i>	165
Case I: Undamped and Underdamped ($0 \leq \xi < 1$ or $0 \leq c_{eq} < c_c$)	166

Case II: Critically Damped ($\xi = 1$ or $c_{eq} = c_c$)	166
Case III: Overdamped ($\xi > 1$ or $c_{eq} > c_c$)	166
3.3 Laplace Transformation	168
3.3.1 <i>Properties of Laplace Transformation</i>	169
3.3.2 <i>Response of an SDOF System via Laplace Transformation</i>	170
3.3.3 <i>Transfer Function and Frequency Response Function</i>	173
Significance of Transfer Function	175
Poles and Zeros of Transfer Function	175
Frequency Response Function	176
Exercise Problems	179
4 Vibration of Two-Degree-of-Freedom-Systems	186
4.1 Mass, Stiffness, and Damping Matrices	187
4.2 Natural Frequencies and Mode Shapes	192
4.2.1 <i>Eigenvalue/Eigenvector Interpretation</i>	197
4.3 Free Response of an Undamped 2DOF System Solution	198
4.4 Forced Response of an Undamped 2DOF System Under Sinusoidal Excitation	201
4.5 Free Vibration of a Damped 2DOF System	203
4.6 Steady-State Response of a Damped 2DOF System Under Sinusoidal Excitation	209
4.7 Vibration Absorber	212
4.7.1 <i>Undamped Vibration Absorber</i>	212
4.7.2 <i>Damped Vibration Absorber</i>	220
Case I: Tuned Case ($f = 1$ or $\omega_{22} = \omega_{11}$)	224
Case II: No restriction on f (Absorber not tuned to main system)	224
4.8 Modal Decomposition of Response	227
Case I: Undamped System ($C = 0$)	228
Case II: Damped System ($C \neq 0$)	228
Exercise Problems	231
5 Finite and Infinite (Continuous) Dimensional Systems	237
5.1 Multi-Degree-of-Freedom Systems	237
5.1.1 <i>Natural Frequencies and Modal Vectors (Mode Shapes)</i>	239
5.1.2 <i>Orthogonality of Eigenvectors for Symmetric Mass and Symmetric Stiffness Matrices</i>	242

Contents	xi
5.1.3 <i>Modal Decomposition</i>	245
Case I: Undamped System ($C = 0$)	246
Case II: Proportional or Rayleigh Damping	249
5.2 Continuous Systems Governed by Wave Equations	250
5.2.1 <i>Transverse Vibration of a String</i>	250
Natural Frequencies and Mode Shapes	251
Computation of Response	255
5.2.2 <i>Longitudinal Vibration of a Bar</i>	258
5.2.3 <i>Torsional Vibration of a Circular Shaft</i>	261
5.3 Continuous Systems: Transverse Vibration of a Beam	265
5.3.1 <i>Governing Partial Differential Equation of Motion</i>	265
5.3.2 <i>Natural Frequencies and Mode Shapes</i>	267
Simply Supported Beam	269
Cantilever Beam	271
5.3.3 <i>Computation of Response</i>	273
5.4 Finite Element Analysis	279
5.4.1 <i>Longitudinal Vibration of a Bar</i>	279
Total Kinetic and Potential Energies of the Bar	283
5.4.2 <i>Transverse Vibration of a Beam</i>	286
Total Kinetic and Potential Energies of the Beam	291
Exercise Problems	295
APPENDIX A: EQUIVALENT STIFFNESSES (SPRING CONSTANTS) OF BEAMS, TORSIONAL SHAFT, AND LONGITUDINAL BAR.....	299
APPENDIX B: SOME MATHEMATICAL FORMULAE	302
APPENDIX C: LAPLACE TRANSFORM TABLE.....	304
References	305
Index	307