

## Vibration of Mechanical Systems

This is a textbook for a first course in mechanical vibrations. There are many books in this area that try to include everything, thus they have become exhaustive compendiums that are overwhelming for an undergraduate. In this book, all the basic concepts in mechanical vibrations are clearly identified and presented in a concise and simple manner with illustrative and practical examples. Vibration concepts include a review of selected topics in mechanics; a description of single-degree-of-freedom (SDOF) systems in terms of equivalent mass, equivalent stiffness, and equivalent damping; a unified treatment of various forced response problems (base excitation and rotating balance); an introduction to systems thinking, highlighting the fact that SDOF analysis is a building block for multi-degree-of-freedom (MDOF) and continuous system analyses via modal analysis; and a simple introduction to finite element analysis to connect continuous system and MDOF analyses. There are more than 60 exercise problems and a complete solutions manual. The use of MATLAB<sup>®</sup> software is emphasized.

Alok Sinha is a Professor of Mechanical Engineering at The Pennsylvania State University (PSU), University Park. He received his PhD degree in mechanical engineering from Carnegie Mellon University. He has been a PSU faculty member since August 1983. His areas of teaching and research are vibration, control systems, jet engines, robotics, neural networks, and nanotechnology. He is the author of *Linear Systems: Optimal and Robust Control*.

He has served as a Visiting Associate Professor of Aeronautics and Astronautics at MIT, Cambridge, MA, and as a researcher at Pratt & Whitney, East Hartford, CT. He has also been an associate editor of *ASME Journal of Dynamic Systems, Measurement, and Control*. At present, he serves as an associate editor of *ASME Journal of Turbomachinery* and *AIAA Journal*.

Alok Sinha is a Fellow of ASME. He has received the NASA certificate of recognition for significant contributions to the Space Shuttle Microgravity Mission.

Cambridge University Press  
978-0-521-51873-4 - Vibration of Mechanical Systems  
Alok Sinha  
Frontmatter  
[More information](#)

---

# VIBRATION OF MECHANICAL SYSTEMS

Alok Sinha

The Pennsylvania State University



**CAMBRIDGE**  
UNIVERSITY PRESS

Cambridge University Press  
978-0-521-51873-4 - Vibration of Mechanical Systems  
Alok Sinha  
Frontmatter  
[More information](#)

---

CAMBRIDGE UNIVERSITY PRESS  
Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore,  
São Paulo, Delhi, Dubai, Tokyo, Mexico City

Cambridge University Press  
32 Avenue of the Americas, New York, NY 10013-2473, USA  
[www.cambridge.org](http://www.cambridge.org)  
Information on this title: [www.cambridge.org/9780521518734](http://www.cambridge.org/9780521518734)

© Alok Sinha 2010

This publication is in copyright. Subject to statutory exception  
and to the provisions of relevant collective licensing agreements,  
no reproduction of any part may take place without the written  
permission of Cambridge University Press.

First published 2010

Printed in the United States of America

*A catalog record for this publication is available from the British Library.*

*Library of Congress Cataloging in Publication Data*

Sinha, Alok  
Vibration of mechanical systems / Alok Sinha.  
p. cm.  
Includes bibliographical references and index.  
ISBN 978-0-521-51873-4 (hardback)  
1. Machinery – Vibration. I. Title.  
TJ177.S56 2010  
621.8'11–dc22 2010021143

ISBN 978-0-521-51873-4 Hardback

Cambridge University Press has no responsibility for the persistence or accuracy of URLs  
for external or third-party Internet Web sites referred to in this publication and does not  
guarantee that any content on such Web sites is, or will remain, accurate or appropriate.

Cambridge University Press  
978-0-521-51873-4 - Vibration of Mechanical Systems  
Alok Sinha  
Frontmatter  
[More information](#)

---

*To  
My Wife Hansa  
and  
My Daughters Divya and Swarna*

## CONTENTS

Preface	page xiii
<b>1 Equivalent Single-Degree-of-Freedom System and Free Vibration .....</b>	<b>1</b>
1.1 Degrees of Freedom	3
1.2 Elements of a Vibratory System	5
1.2.1 <i>Mass and/or Mass-Moment of Inertia</i>	5
Pure Translational Motion	5
Pure Rotational Motion	6
Planar Motion (Combined Rotation and Translation) of a Rigid Body	6
Special Case: Pure Rotation about a Fixed Point	8
1.2.2 <i>Spring</i>	8
Pure Translational Motion	8
Pure Rotational Motion	9
1.2.3 <i>Damper</i>	10
Pure Translational Motion	10
Pure Rotational Motion	11
1.3 Equivalent Mass, Equivalent Stiffness, and Equivalent Damping Constant for an SDOF System	12
1.3.1 <i>A Rotor–Shaft System</i>	13
1.3.2 <i>Equivalent Mass of a Spring</i>	14
1.3.3 <i>Springs in Series and Parallel</i>	16
Springs in Series	16
Springs in Parallel	17
1.3.4 <i>An SDOF System with Two Springs and Combined Rotational and Translational Motion</i>	19
1.3.5 <i>Viscous Dampers in Series and Parallel</i>	22

Dampers in Series	22
Dampers in Parallel	23
1.4 Free Vibration of an Undamped SDOF System	25
1.4.1 <i>Differential Equation of Motion</i>	25
Energy Approach	27
1.4.2 <i>Solution of the Differential Equation of Motion</i>	
<i>Governing Free Vibration of an Undamped</i>	
<i>Spring–Mass System</i>	34
1.5 Free Vibration of a Viscously Damped SDOF System	40
1.5.1 <i>Differential Equation of Motion</i>	40
1.5.2 <i>Solution of the Differential Equation of Motion</i>	
<i>Governing Free Vibration of a Damped</i>	
<i>Spring–Mass System</i>	41
Case I: Underdamped ( $0 < \xi < 1$ or $0 < c_{eq} < c_c$ )	42
Case II: Critically Damped ( $\xi = 1$ or $c_{eq} = c_c$ )	45
Case III: Overdamped ( $\xi > 1$ or $c_{eq} > c_c$ )	46
1.5.3 <i>Logarithmic Decrement: Identification of Damping</i>	
<i>Ratio from Free Response of an Underdamped</i>	
<i>System (<math>0 &lt; \xi &lt; 1</math>)</i>	51
Solution	55
1.6 Stability of an SDOF Spring–Mass–Damper System	58
Exercise Problems	63
<b>2 Vibration of a Single-Degree-of-Freedom System Under</b>	
<b>Constant and Purely Harmonic Excitation .....</b>	<b>72</b>
2.1 Responses of Undamped and Damped SDOF Systems	
to a Constant Force	72
Case I: Undamped ( $\xi = 0$ ) and Underdamped	
( $0 < \xi < 1$ )	74
Case II: Critically Damped ( $\xi = 1$ or $c_{eq} = c_c$ )	75
Case III: Overdamped ( $\xi > 1$ or $c_{eq} > c_c$ )	76
2.2 Response of an Undamped SDOF System	
to a Harmonic Excitation	82
Case I: $\omega \neq \omega_n$	83
Case II: $\omega = \omega_n$ (Resonance)	84
Case I: $\omega \neq \omega_n$	87
Case II: $\omega = \omega_n$	87
2.3 Response of a Damped SDOF System to a Harmonic	
Excitation	88
Particular Solution	89
Case I: Underdamped ( $0 < \xi < 1$ or $0 < c_{eq} < c_c$ )	92

Case II: Critically Damped ( $\xi = 1$ or $c_{eq} = c_c$ )	92
Case III: Overdamped ( $\xi > 1$ or $c_{eq} > c_c$ )	94
2.3.1 <i>Steady State Response</i>	95
2.3.2 <i>Force Transmissibility</i>	101
2.3.3 <i>Quality Factor and Bandwidth</i>	106
Quality Factor	106
Bandwidth	107
2.4 Rotating Unbalance	109
2.5 Base Excitation	116
2.6 Vibration Measuring Instruments	121
2.6.1 <i>Vibrometer</i>	123
2.6.2 <i>Accelerometer</i>	126
2.7 Equivalent Viscous Damping for Nonviscous Energy	
Dissipation	128
Exercise Problems	132
<b>3 Responses of an SDOF Spring–Mass–Damper System to Periodic and Arbitrary Forces</b>	<b>138</b>
3.1 Response of an SDOF System to a Periodic Force	138
3.1.1 <i>Periodic Function and its Fourier Series Expansion</i>	139
3.1.2 <i>Even and Odd Periodic Functions</i>	142
Fourier Coefficients for Even Periodic Functions	143
Fourier Coefficients for Odd Periodic Functions	145
3.1.3 <i>Fourier Series Expansion of a Function with a Finite Duration</i>	147
3.1.4 <i>Particular Integral (Steady-State Response with Damping) Under Periodic Excitation</i>	151
3.2 Response to an Excitation with Arbitrary Nature	154
3.2.1 <i>Unit Impulse Function <math>\delta(t - a)</math></i>	155
3.2.2 <i>Unit Impulse Response of an SDOF System with Zero Initial Conditions</i>	156
Case I: Undamped and Underdamped System ( $0 \leq \xi < 1$ )	158
Case II: Critically Damped ( $\xi = 1$ or $c_{eq} = c_c$ )	158
Case III: Overdamped ( $\xi > 1$ or $c_{eq} > c_c$ )	159
3.2.3 <i>Convolution Integral: Response to an Arbitrary Excitation with Zero Initial Conditions</i>	160
3.2.4 <i>Convolution Integral: Response to an Arbitrary Excitation with Nonzero Initial Conditions</i>	165
Case I: Undamped and Underdamped ( $0 \leq \xi < 1$ or $0 \leq c_{eq} < c_c$ )	166

Case II: Critically Damped ( $\xi = 1$ or $c_{eq} = c_c$ )	166
Case III: Overdamped ( $\xi > 1$ or $c_{eq} > c_c$ )	166
3.3 Laplace Transformation	168
3.3.1 <i>Properties of Laplace Transformation</i>	169
3.3.2 <i>Response of an SDOF System via Laplace Transformation</i>	170
3.3.3 <i>Transfer Function and Frequency Response Function</i>	173
Significance of Transfer Function	175
Poles and Zeros of Transfer Function	175
Frequency Response Function	176
Exercise Problems	179
<b>4 Vibration of Two-Degree-of-Freedom-Systems . . . . .</b>	<b>186</b>
4.1 Mass, Stiffness, and Damping Matrices	187
4.2 Natural Frequencies and Mode Shapes	192
4.2.1 <i>Eigenvalue/Eigenvector Interpretation</i>	197
4.3 Free Response of an Undamped 2DOF System Solution	198
4.4 Forced Response of an Undamped 2DOF System Under Sinusoidal Excitation	201
4.5 Free Vibration of a Damped 2DOF System	203
4.6 Steady-State Response of a Damped 2DOF System Under Sinusoidal Excitation	209
4.7 Vibration Absorber	212
4.7.1 <i>Undamped Vibration Absorber</i>	212
4.7.2 <i>Damped Vibration Absorber</i>	220
Case I: Tuned Case ( $f = 1$ or $\omega_{22} = \omega_{11}$ )	224
Case II: No restriction on $f$ (Absorber not tuned to main system)	224
4.8 Modal Decomposition of Response	227
Case I: Undamped System ( $C = 0$ )	228
Case II: Damped System ( $C \neq 0$ )	228
Exercise Problems	231
<b>5 Finite and Infinite (Continuous) Dimensional Systems . . . . .</b>	<b>237</b>
5.1 Multi-Degree-of-Freedom Systems	237
5.1.1 <i>Natural Frequencies and Modal Vectors (Mode Shapes)</i>	239
5.1.2 <i>Orthogonality of Eigenvectors for Symmetric Mass and Symmetric Stiffness Matrices</i>	242



5.1.3 <i>Modal Decomposition</i>	245
Case I: Undamped System ( $C = 0$ )	246
Case II: Proportional or Rayleigh Damping	249
5.2 Continuous Systems Governed by Wave Equations	250
5.2.1 <i>Transverse Vibration of a String</i>	250
Natural Frequencies and Mode Shapes	251
Computation of Response	255
5.2.2 <i>Longitudinal Vibration of a Bar</i>	258
5.2.3 <i>Torsional Vibration of a Circular Shaft</i>	261
5.3 Continuous Systems: Transverse Vibration of a Beam	265
5.3.1 <i>Governing Partial Differential Equation of Motion</i>	265
5.3.2 <i>Natural Frequencies and Mode Shapes</i>	267
Simply Supported Beam	269
Cantilever Beam	271
5.3.3 <i>Computation of Response</i>	273
5.4 Finite Element Analysis	279
5.4.1 <i>Longitudinal Vibration of a Bar</i>	279
Total Kinetic and Potential Energies of the Bar	283
5.4.2 <i>Transverse Vibration of a Beam</i>	286
Total Kinetic and Potential Energies of the Beam	291
Exercise Problems	295
<b>APPENDIX A: EQUIVALENT STIFFNESSES (SPRING CONSTANTS) OF BEAMS, TORSIONAL SHAFT, AND LONGITUDINAL BAR.....</b>	<b>299</b>
<b>APPENDIX B: SOME MATHEMATICAL FORMULAE .....</b>	<b>302</b>
<b>APPENDIX C: LAPLACE TRANSFORM TABLE.....</b>	<b>304</b>
References	305
Index	307

## PREFACE

This book is intended for a vibration course in an undergraduate Mechanical Engineering curriculum. It is based on my lecture notes of a course (ME370) that I have been teaching for many years at The Pennsylvania State University (PSU), University Park. This vibration course is a required core course in the PSU mechanical engineering curriculum and is taken by junior-level or third-year students. Textbooks that have been used at PSU are as follows: Hutton (1981) and Rao (1995, First Edition 1986). In addition, I have used the book by Thomson and Dahleh (1993, First Edition 1972) as an important reference book while teaching this course. It will be a valid question if one asks why I am writing another book when there are already a large number of excellent textbooks on vibration since Den Hartog wrote the classic book in 1956. One reason is that most of the books are intended for senior-level undergraduate and graduate students. As a result, our faculties have not found any book that can be called ideal for our junior-level course. Another motivation for writing this book is that I have developed certain unique ways of presenting vibration concepts in response to my understanding of the background of a typical undergraduate student in our department and the available time during a semester. Some of the examples are as follows: review of selected topics in mechanics; the description of the chapter on single-degree-of-freedom (SDOF) systems in terms of equivalent mass, equivalent stiffness, and equivalent damping; unified treatment of various forced

response problems such as base excitation and rotating balance; introduction of system thinking, highlighting the fact that SDOF analysis is a building block for multi-degree-of-freedom (MDOF) and continuous system analyses via modal analysis; and a simple introduction of finite element analysis to connect continuous system and MDOF analyses.

As mentioned before, there are a large number of excellent books on vibration. But, because of a desire to include everything, many of these books often become difficult for undergraduate students. In this book, all the basic concepts in mechanical vibration are clearly identified and presented in a simple manner with illustrative and practical examples. I have also attempted to make this book self-contained as much as possible; for example, materials needed from previous courses, such as differential equation and engineering mechanics, are presented. At the end of each chapter, exercise problems are included. The use of MATLAB software is also included.

## ORGANIZATION OF THE BOOK

In Chapter 1, the degrees of freedom and the basic elements of a vibratory mechanical system are presented. Then the concepts of equivalent mass, equivalent stiffness, and equivalent damping are introduced to construct an equivalent single-degree-of-freedom model. Next, the differential equation of motion of an undamped SDOF spring–mass system is derived along with its solution. Then the solution of the differential equation of motion of an SDOF spring–mass–damper system is obtained. Three cases of damping levels – underdamped, critically damped, and overdamped – are treated in detail. Last, the concept of stability of an SDOF spring–mass–damper system is presented.

In Chapter 2, the responses of undamped and damped SDOF spring–mass systems are presented. An important example of input shaping is shown. Next, the complete solutions of both undamped and

damped spring–mass systems under sinusoidal excitation are derived. Amplitudes and phases of steady-state responses are examined along with force transmissibility, quality factor, and bandwidth. Then the solutions to rotating unbalance and base excitation problems are provided. Next, the basic principles behind the designs of a vibrometer and an accelerometer are presented. Last, the concept of equivalent viscous damping is presented for nonviscous energy dissipation.

In Chapter 3, the techniques to compute the response of an SDOF system to a periodic excitation are presented via the Fourier series expansion. Then it is shown how the response to an arbitrary excitation is obtained via the convolution integral and the unit impulse response. Last, the Laplace transform technique is presented. The concepts of transfer function, poles, zeros, and frequency response function are also introduced.

In Chapter 4, mass matrix, stiffness matrix, damping matrix, and forcing vector are defined. Then the method to compute the natural frequencies and the mode shapes is provided. Next, free and forced vibration of both undamped and damped two-degree-of-freedom systems are analyzed. Last, the techniques to design undamped and damped vibration absorbers are presented.

In Chapter 5, the computation of the natural frequencies and the mode shapes of discrete multi-degree-of-freedom and continuous systems is illustrated. Then the orthogonality of the mode shapes is shown. The method of modal decomposition is presented for the computation of both free and forced responses. The following cases of continuous systems are considered: transverse vibration of a string, longitudinal vibration of a bar, torsional vibration of a circular shaft, and transverse vibration of a beam. Last, the finite element method is introduced via examples of the longitudinal vibration of a bar and the transverse vibration of a beam.