STOCHASTIC SCHEDULING

Stochastic scheduling occurs in the area of production scheduling. There is a dearth of work that analyzes the variability of schedules. In a stochastic environment, in which the processing time of a job is not known with certainty, a schedule typically is analyzed based on the expected value of a performance measure. This book addresses this problem and presents algorithms to determine the variability of a schedule under various machine configurations and objective functions. It is intended for graduate and advanced undergraduate students in manufacturing, operations management, applied mathematics, and computer science, and it is also a good reference book for practitioners. Computer software containing the algorithms is also provided on an accompanying Web site (www.cambridge.org/sarin) for ease of student and user implementation.

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Contents

Foreword

Over the years, I have taught introductory as well as advanced courses in job scheduling. The problem is fascinating in its variety, which is truly astounding, and in the methodologies that have been offered by many researchers for its resolution. But I have always felt let down when it came to job scheduling under uncertainty: The absence of a comprehensive treatment of these real-life scenarios was a handicap that I felt acutely. After reading this book, I do not feel that way any more.

The fixation on "averages" is often meaningless, if not downright misleading. True, while the statement "on average, such and such phenomenon behaves in a certain way" or "has a particular value" is often a useful bit of information (one is usually interested in the average water temperature at the seaside before taking a jump into the ocean), it often carries insufficient information for intelligent decision making (knowing that one's heartbeat is good "on average" does not help to diagnose the ailment if sometimes it stops beating altogether!). Here is where the "range of variation," as measured by the variance (or any other measure of dispersion), becomes invaluable.

Scheduling under stochastic conditions has received scant attention from authors in the field. Usually it appears, if at all, in the form of one or two (thin) chapters that introduce the problem and give a treatment based mostly on the assumption that the processing times of the jobs are exponentially distributed. The feeling always has persisted: Whoever heard of processing times that are exponentially distributed? And how to behave if they are not?

Heaven knows (also researchers in the field know) that scheduling under conditions of uncertainty with the objective of optimizing the expected value of the adopted criterion, be it job-focused or time-focused, is difficult enough. But adding another dimension on top of it in the form of the variance of the declared criterion seems like adding insult to injury and stretching credibility in the analyst's ability to come up with an "answer" – any answer! This is what this book is all about. Sarin makes no attempt to hide such difficulty – on the contrary, for each scenario he presents the mathematical model (if one exists)

in all its glorious complexity and then proceeds to discuss ways and means of approaching a solution through branch and bound or heuristics.

To my utter delight, throughout this monograph I discovered a few concepts that have general applicability well beyond the subject matter of the book. For instance, in Chapter 8, the reader is introduced to the concept of finite mixture of distributions, in which a random variable is represented as a convex combination of a finite set of random variables. Usually, the component random variables are assumed to be normally distributed, but they can be any other distribution. Sarin is meticulous in explaining the approach and citing the appropriate references. Anyone interested in "fitting a continuous distribution to a given set of data" or in "approximating a distribution by a set of normal distributions" would welcome the discussion. I particularly enjoyed being treated to the rare view of approximating the *uniform* distribution with a convex mixture of *normal* distributions as the number of iterations of the approximation increases from 1 to 113,179. The further application of these concepts to the approximation of the distribution of the completion time of a project (in which the "jobs" are further constrained by precedence relations) provides an excellent introduction to the myriad problems faced by analysts in treating project-related problems.

A pleasant surprise to the owner of this book is the software XVA-Sched (for the e*x*pectation-*v*ariance *a*nalysis of a *sched*ule) that accompanies it, as well as the instructions on how to use it. This software helps the user to implement the methodologies developed in previous chapters to determine the expectation-variance-efficient schedules.

Subhash C. Sarin has written a gem of a research monograph that shall find its way to the shelf of each researcher and worker in the field of scheduling. It is long overdue. And I feel privileged in having been given a peek at its contents before its publication.

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Preface

Scheduling is a decision-making process that is commonly encountered in practice – in both production and nonproduction-related environments. The effectiveness of a schedule depends on how well it performs in the environment for which it is designed. For an environment that involves a stochastic element, which is more often the case in practice, a typical approach presented in the literature has been to determine a schedule that optimizes the expected value of a performance measure. However, optimizing merely the expectation of a performance measure for a schedule is not enough for use in such an environment because variability of the measure plays an important role in influencing the performance of the schedule. It is, therefore, essential that a schedule be selected for implementation that has suitable values for both the expectation and variance of the performance measure in consideration. In this book, we present methodologies for determining such a schedule, and in this respect, this book is different from the other books on scheduling.

The book is organized as follows. In Chapter 1, we focus on the impact of uncertainty (variability) in scheduling, and the need for efficient modeling of stochastic scheduling problems and for devising effective scheduling strategies to counter the impact of variability. We specifically highlight the prevalence of variability in job processing times and elicit the issue of neglecting variance in schedule optimization and the significance of considering variance. Furthermore, we enunciate the need for a comprehensive analytical evaluation of the expectation and variance of different performance measures. Chapters 2 and 3 review methods presented in the literature to address the issue of variability of a performance measure for a schedule. In particular, Chapter 2 deals with robust scheduling approaches to hedge against processing time variability. Various model formulations and solution methodologies are presented in detail. Chapter 3 deals with another approach that determines a set of "nondominating schedules" or "expectation-variance efficient schedules" and selects a preferred schedule from such a set. We present our work in Chapters 4, 5, 6, 7, and 8 wherein a comprehensive analysis of

schedules for various scheduling environments and different performance measures is presented. Chapters 4, 5, 6, and 7, respectively, deal with scheduling in a single-machine, flow-shops, job-shops, and parallel-machines environments. We develop closed-form expressions (wherever possible, and devise methodologies otherwise) to evaluate the expectation and variance of various performance measures for a schedule. The methodologies are illustrated using example problems. The closed-form expressions for many performance measures rely on the assumption that job processing times follow normal distributions. This assumption is relaxed in Chapter 8, and we consider the case of general processing time distributions for jobs. We use a finite mixture model to represent a given processing time distribution of a job by a convex combination of normal distributions. Subsequently, we use the methodologies presented in previous chapters to perform the expectation-variance analysis of a schedule for each of the environments presented in those chapters whose analysis relies on the assumption of normal distribution. Also, we demonstrate, in this chapter, the use of a finite mixture model to perform the expectation-variance analysis for the completion time of an activity network.

An accompanying Web site (www.cambridge.org/sarin) contains a software package to help implement the methodologies developed in Chapters 4, 5, 6, 7, and 8. This is user-friendly software termed XVA-Sched (for the *ex*pectation-*va*riance *a*nalysis of a *sched*ule). The instructions to use the software are included in an appendix.

The material presented in the book can be used as a supplement to a course in sequencing and scheduling, and to courses in related areas at both graduate and advanced undergraduate levels. As background, it requires mathematical maturity and introductory knowledge of probability theory and optimization concepts and methodologies. The book provides useful ideas and algorithms for practitioners, and it can serve as a useful research reference.

My first and foremost thanks go to my graduate students Balaji Nagarajan and Lingrui Liao. For their many direct contributions, I consider them co-authors of this book. I would also like to extend my sincere thanks to the anonymous reviewers for their careful reading of the manuscript and insightful comments.

A project of this magnitude cannot be accomplished without the unconditional support, encouragement, and love of the family. For this, I would like to thank my wife, Veena, and our sons, Sumeet and Shivan.

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