Spouted and Spout-Fluid Beds
Fundamentals and Applications

Since the pioneering text by Mathur and Epstein over 35 years ago, much of the work on this subject has been extended or superseded, producing an enormous body of scattered literature. This edited volume unifies the subject, pulling material together and underpinning it with fundamental theory to produce the only complete, up-to-date reference on all major areas of spouted bed research and practice. With contributions from internationally renowned research groups, this book guides the reader through new developments, insights, and models. The hydrodynamic and reactor models of spouted and spout-fluid beds are examined, as well as such topics as particle segregation, heat and mass transfer, mixing, and scale-up. Later chapters focus on drying, particle-coating, and energy-related applications based on spouted and spout-fluid beds. This is a valuable resource for chemical and mechanical engineers in research and industry.

Norman Epstein is Honorary Professor in the Department of Chemical and Biological Engineering at the University of British Columbia. His research areas during the past 60 years have focused on heat, mass, and momentum transfer; on the fluid-particle dynamics of spouted beds and liquid-fluidized beds; and on various aspects of heat exchanger fouling. He is a former Editor of the Canadian Journal of Chemical Engineering and has published widely.

John R. Grace is Professor and Canada Research Chair in the Department of Chemical and Biological Engineering at the University of British Columbia. He has more than 40 years’ experience working on fluidized and spouted beds, fluid-particle systems, and multiphase flow. His work includes reactor design, hydrodynamics, heat and mass transfer, and applications. Professor Grace has published widely, and has lectured and consulted for industry in these fields.

Both Epstein and Grace are Fellows of the Canadian Academy of Engineering and of the Chemical Institute of Canada, and former Presidents of the Canadian Society for Chemical Engineering.
Spouted and Spout-Fluid Beds

Fundamentals and Applications

Edited by

NORMAN EPSTEIN
University of British Columbia, Vancouver

JOHN R. GRACE
University of British Columbia, Vancouver
Contents

Contributors xi
Preface xvii
Common nomenclature xix

1 Introduction
Norman Epstein and John R. Grace 1

1.1 The spouted bed 1
1.2 Brief history 3
1.3 Flow regime maps encompassing conventional spouting 5
1.4 Nonaxisymmetric geometries of spouted beds 7
1.5 Spouted beds in the gas–solid contacting spectrum 9
1.6 Layout of chapter topics 12
References 14

2 Initiation of spouting
Xiaotao Bi 17

2.1 Introduction 17
2.2 Evolution of internal spout 19
2.3 Peak pressure drop 22
2.4 Onset of external spouting and minimum spouting velocity 23
Chapter-specific nomenclature 26
References 27

3 Empirical and analytical hydrodynamics
Norman Epstein 29

3.1 Constraints on fluid inlet diameter 29
3.2 Minimum spouting velocity 30
3.3 Maximum spoutable bed depth 35
3.4 Annular fluid flow 39
3.5 Pressure drops, profiles, and gradients 47
3.6 Spout diameter 51
3.7 Flow split between spout and annulus 53
References 54
<table>
<thead>
<tr>
<th>Chapter</th>
<th>Title</th>
<th>Authors</th>
<th>Pages</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>Computational fluid dynamic modeling of spouted beds</td>
<td>Xiaojun Bao, Wei Du, and Jian Xu</td>
<td>57</td>
</tr>
<tr>
<td></td>
<td>Introduction</td>
<td></td>
<td>57</td>
</tr>
<tr>
<td></td>
<td>Eulerian-Eulerian approach</td>
<td></td>
<td>57</td>
</tr>
<tr>
<td></td>
<td>Eulerian-Lagrangian approach</td>
<td></td>
<td>68</td>
</tr>
<tr>
<td></td>
<td>Concluding remarks</td>
<td></td>
<td>76</td>
</tr>
<tr>
<td></td>
<td>Chapter-specific nomenclature</td>
<td></td>
<td>77</td>
</tr>
<tr>
<td></td>
<td>References</td>
<td></td>
<td>78</td>
</tr>
<tr>
<td>5</td>
<td>Conical spouted beds</td>
<td>Martin Olazar, Maria J. San José, and Javier Bilbao</td>
<td>82</td>
</tr>
<tr>
<td></td>
<td>Introduction</td>
<td></td>
<td>82</td>
</tr>
<tr>
<td></td>
<td>Conditions for stable operation and design geometric factors</td>
<td></td>
<td>82</td>
</tr>
<tr>
<td></td>
<td>Hydrodynamics</td>
<td></td>
<td>84</td>
</tr>
<tr>
<td></td>
<td>Gas flow modeling</td>
<td></td>
<td>87</td>
</tr>
<tr>
<td></td>
<td>Particle segregation</td>
<td></td>
<td>90</td>
</tr>
<tr>
<td></td>
<td>Local properties</td>
<td></td>
<td>92</td>
</tr>
<tr>
<td></td>
<td>Numerical simulation</td>
<td></td>
<td>96</td>
</tr>
<tr>
<td></td>
<td>Applications</td>
<td></td>
<td>97</td>
</tr>
<tr>
<td></td>
<td>Chapter-specific nomenclature</td>
<td></td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>References</td>
<td></td>
<td>101</td>
</tr>
<tr>
<td>6</td>
<td>Hydrodynamics of spout-fluid beds</td>
<td>Wenqi Zhong, Baosheng Jin, Mingyao Zhang, and Rui Xiao</td>
<td>105</td>
</tr>
<tr>
<td></td>
<td>Hydrodynamic characteristics</td>
<td></td>
<td>105</td>
</tr>
<tr>
<td></td>
<td>Typical applications</td>
<td></td>
<td>119</td>
</tr>
<tr>
<td></td>
<td>Closing remarks</td>
<td></td>
<td>122</td>
</tr>
<tr>
<td></td>
<td>Chapter-specific nomenclature</td>
<td></td>
<td>123</td>
</tr>
<tr>
<td></td>
<td>References</td>
<td></td>
<td>123</td>
</tr>
<tr>
<td>7</td>
<td>Spouted and spout-fluid beds with draft tubes</td>
<td>Željko B. Grbavčić, Howard Littman, Morris H. Morgan III, and John D. Paccione</td>
<td>128</td>
</tr>
<tr>
<td></td>
<td>Operation of draft tube spout-fluid beds</td>
<td></td>
<td>128</td>
</tr>
<tr>
<td></td>
<td>Novel applications and experimental studies</td>
<td></td>
<td>133</td>
</tr>
<tr>
<td></td>
<td>Chapter-specific nomenclature</td>
<td></td>
<td>138</td>
</tr>
<tr>
<td></td>
<td>References</td>
<td></td>
<td>138</td>
</tr>
<tr>
<td>8</td>
<td>Particle mixing and segregation</td>
<td>Giorgio Rovero and Norberto Piccinini</td>
<td>141</td>
</tr>
<tr>
<td></td>
<td>Gross solids mixing behavior</td>
<td></td>
<td>141</td>
</tr>
<tr>
<td></td>
<td>Mixing in a pulsed spouted bed</td>
<td></td>
<td>148</td>
</tr>
<tr>
<td>Chapter</td>
<td>Title</td>
<td>Page</td>
<td></td>
</tr>
<tr>
<td>---------</td>
<td>--------------------------------------------</td>
<td>------</td>
<td></td>
</tr>
<tr>
<td>12.3</td>
<td>Influence of paste on the fluid dynamic parameters</td>
<td>211</td>
<td></td>
</tr>
<tr>
<td>12.4</td>
<td>Modeling of drying</td>
<td>215</td>
<td></td>
</tr>
<tr>
<td>12.5</td>
<td>Final remarks</td>
<td>218</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Chapter-specific nomenclature</td>
<td>219</td>
<td></td>
</tr>
<tr>
<td></td>
<td>References</td>
<td>219</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>Granulation and particle coating</td>
<td>222</td>
<td></td>
</tr>
<tr>
<td>13.1</td>
<td>Introduction</td>
<td>222</td>
<td></td>
</tr>
<tr>
<td>13.2</td>
<td>Granulating and coating mechanisms</td>
<td>223</td>
<td></td>
</tr>
<tr>
<td>13.3</td>
<td>Advances in spouted bed granulation/coating</td>
<td>225</td>
<td></td>
</tr>
<tr>
<td>13.4</td>
<td>Surface properties influencing granulation/coating processes</td>
<td>230</td>
<td></td>
</tr>
<tr>
<td>13.5</td>
<td>Concluding remarks</td>
<td>233</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Chapter-specific nomenclature</td>
<td>233</td>
<td></td>
</tr>
<tr>
<td></td>
<td>References</td>
<td>234</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>The Wurster coater</td>
<td>238</td>
<td></td>
</tr>
<tr>
<td>14.1</td>
<td>Introduction: particle coating techniques</td>
<td>238</td>
<td></td>
</tr>
<tr>
<td>14.2</td>
<td>Features of the Wurster coater</td>
<td>240</td>
<td></td>
</tr>
<tr>
<td>14.3</td>
<td>Uniformity and quality of coating</td>
<td>241</td>
<td></td>
</tr>
<tr>
<td>14.4</td>
<td>Particle motion studies</td>
<td>243</td>
<td></td>
</tr>
<tr>
<td>14.5</td>
<td>Modeling of coating thickness variation</td>
<td>245</td>
<td></td>
</tr>
<tr>
<td>14.6</td>
<td>Developments in design</td>
<td>246</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Acknowledgments</td>
<td>246</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Chapter-specific nomenclature</td>
<td>247</td>
<td></td>
</tr>
<tr>
<td></td>
<td>References</td>
<td>248</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>Gasification, pyrolysis, and combustion</td>
<td>250</td>
<td></td>
</tr>
<tr>
<td>15.1</td>
<td>Gasification background</td>
<td>250</td>
<td></td>
</tr>
<tr>
<td>15.2</td>
<td>Spouted bed gasification of coal and coke</td>
<td>251</td>
<td></td>
</tr>
<tr>
<td>15.3</td>
<td>Spouted bed gasification of biomass and wastes</td>
<td>256</td>
<td></td>
</tr>
<tr>
<td>15.4</td>
<td>Throughput and scale of spouted bed gasification</td>
<td>256</td>
<td></td>
</tr>
<tr>
<td>15.5</td>
<td>Modeling of spouted bed gasification</td>
<td>257</td>
<td></td>
</tr>
<tr>
<td>15.6</td>
<td>Spouted bed pyrolysis of coal, shale, biomass, and solid wastes</td>
<td>258</td>
<td></td>
</tr>
<tr>
<td>15.7</td>
<td>Combustion of solid fuels</td>
<td>260</td>
<td></td>
</tr>
<tr>
<td>15.8</td>
<td>Spouted bed combustion of coal and other solids</td>
<td>261</td>
<td></td>
</tr>
<tr>
<td>15.9</td>
<td>Concluding remarks</td>
<td>264</td>
<td></td>
</tr>
<tr>
<td></td>
<td>References</td>
<td>265</td>
<td></td>
</tr>
<tr>
<td>Chapter</td>
<td>Title</td>
<td>Page</td>
<td></td>
</tr>
<tr>
<td>---------</td>
<td>-------</td>
<td>------</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>Spouted bed electrochemical reactors</td>
<td>269</td>
<td></td>
</tr>
<tr>
<td>J. W. Evans and Vladimír Jiřičný</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>16.1 Introduction</td>
<td>269</td>
<td></td>
<td></td>
</tr>
<tr>
<td>16.2 Spouted bed electrodes</td>
<td>270</td>
<td></td>
<td></td>
</tr>
<tr>
<td>16.3 Applications of SBEs</td>
<td>275</td>
<td></td>
<td></td>
</tr>
<tr>
<td>16.4 Concluding remarks</td>
<td>280</td>
<td></td>
<td></td>
</tr>
<tr>
<td>References</td>
<td>280</td>
<td></td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>Scaleup, slot-rectangular, and multiple spouting</td>
<td>283</td>
<td></td>
</tr>
<tr>
<td>John R. Grace and C. Jim Lim</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>17.1 Introduction</td>
<td>283</td>
<td></td>
<td></td>
</tr>
<tr>
<td>17.2 Maximum size of spouted beds</td>
<td>284</td>
<td></td>
<td></td>
</tr>
<tr>
<td>17.3 Influence of column diameter on hydrodynamics</td>
<td>284</td>
<td></td>
<td></td>
</tr>
<tr>
<td>17.4 Application of dimensional similitude to scaling of spouted beds</td>
<td>285</td>
<td></td>
<td></td>
</tr>
<tr>
<td>17.5 Slot-rectangular spouted beds</td>
<td>289</td>
<td></td>
<td></td>
</tr>
<tr>
<td>17.6 Multiple spout geometries</td>
<td>291</td>
<td></td>
<td></td>
</tr>
<tr>
<td>17.7 Scaleup of spout-fluid beds</td>
<td>293</td>
<td></td>
<td></td>
</tr>
<tr>
<td>17.8 Final comments and recommendations related to scaleup</td>
<td>293</td>
<td></td>
<td></td>
</tr>
<tr>
<td>References</td>
<td>294</td>
<td></td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>Mechanical spouting</td>
<td>297</td>
<td></td>
</tr>
<tr>
<td>Tibor Szentmarjay, Elizabeth Pallai, and Judith Tóth</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>18.1 Principle of mechanical spouting</td>
<td>297</td>
<td></td>
<td></td>
</tr>
<tr>
<td>18.2 Mechanically spouted bed drying</td>
<td>298</td>
<td></td>
<td></td>
</tr>
<tr>
<td>18.3 Dimensions</td>
<td>300</td>
<td></td>
<td></td>
</tr>
<tr>
<td>18.4 Inert particles</td>
<td>300</td>
<td></td>
<td></td>
</tr>
<tr>
<td>18.5 Cycle time distribution</td>
<td>300</td>
<td></td>
<td></td>
</tr>
<tr>
<td>18.6 Pressure drop</td>
<td>302</td>
<td></td>
<td></td>
</tr>
<tr>
<td>18.7 Drying mechanism</td>
<td>303</td>
<td></td>
<td></td>
</tr>
<tr>
<td>18.8 Scaleup</td>
<td>303</td>
<td></td>
<td></td>
</tr>
<tr>
<td>18.9 Developments</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Acknowledgment</td>
<td>304</td>
<td></td>
<td></td>
</tr>
<tr>
<td>References</td>
<td>304</td>
<td></td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>Catalytic reactors and their modeling</td>
<td>305</td>
<td></td>
</tr>
<tr>
<td>Giorgio Rovero and Norberto Piccinini</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>19.1 Fundamental modeling</td>
<td>305</td>
<td></td>
<td></td>
</tr>
<tr>
<td>19.2 Applied studies</td>
<td>310</td>
<td></td>
<td></td>
</tr>
<tr>
<td>19.3 Conical spouted bed reactors</td>
<td>313</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Contents

19.4 Modified spouted bed reactors 317
   Acknowledgment 318
   Chapter-specific nomenclature 318
   References 319

20 Liquid and liquid–gas spouting of solids 321
   Željko B. Grbavčić, Howard Littman, and Morris H. Morgan III

   20.1 Liquid spouting 321
   20.2 Liquid–gas spouting 328
      Chapter-specific nomenclature 332
      References 333

Index 337
Contributors

Prof. Xiaojun Bao
Key Laboratory of Catalysis
Faculty of Chemical Science and Engineering
China University of Petroleum
Beijing, China

Prof. Xiaotao Bi
Department of Chemical and Biological Engineering
University of British Columbia
Vancouver, Canada

Prof. Javier Bilbao
University of the Basque Country
Faculty of Science and Technology
Department of Chemical Engineering
Bilbao, Spain

Dr. Esly Ferreira da Costa Jr.
Federal University of Espirito Santo (UFES)
Rural Engineering Department
São Mateus, Brazil

Dr. Wei Du
Key Laboratory of Catalysis
Faculty of Chemical Science and Engineering
China University of Petroleum
Beijing, China

Dr. Eng. Sebastian Englart
Institute of Air-Conditioning and District Heating
Faculty of Environmental Engineering
Wroclaw University of Technology
Wroclaw, Poland
List of contributors

Dr. Norman Epstein
Department of Chemical and Biological Engineering
University of British Columbia
Vancouver, Canada

Prof. J. W. Evans
Department of Materials Science and Engineering
University of California
Berkeley, CA, USA

Dr. Maria do Carmo Ferreira
Chemical Engineering Department
Universidade Federal de São Carlos – UFSCar
São Carlos, Brazil

Dr. Fábio Bentes Freire
Chemical Engineering Department
Universidade Federal de São Carlos
São Carlos, Brazil

Prof. José Teixeira Freire
Chemical Engineering Department
Universidade Federal de São Carlos
São Carlos, Brazil

Prof. John R. Grace
Department of Chemical and Biological Engineering
University of British Columbia
Vancouver, Canada

Prof. Željko B. Grbavčić
Faculty of Technology and Metallurgy
University of Belgrade
Beograd, Serbia

Dr. Andrew Ingram
School of Chemical Engineering
University of Birmingham
Birmingham, UK

Prof. Toshifumi Ishikura
Department of Chemical Engineering
Fukuoka University
Fukuoka, Japan
List of contributors

Prof. Baosheng Jin
Key Laboratory of Clean Power Generation and Combustion Technology of
Ministry of Education
Thermo-Energy Engineering Research Institute
Southeast University
Nanjing, China

Dr. Vladimír Jiřičný
Institute of Chemical Process Fundamentals
Academy of Sciences of the Czech Republic
Prague, Czech Republic

Prof. Andrzej Kmiec
Institute of Chemical Engineering and Heating Equipment
Technical University of Wroclaw
Wroclaw, Poland

Prof. C. Jim Lim
Department of Chemical and Biological Engineering
University of British Columbia
Vancouver, Canada

Dr. Antonio C. L. Lisboa
Universidade Estadual de Campinas (UNICAMP)
School of Chemical Engineering
Campinas, Brazil

Prof. Emeritus Howard Littman
Department of Chemical and Biological Engineering
Rensselaer Polytechnic Institute
Troy, NY, USA

Prof. Morris H. Morgan III
Department of Chemical Engineering
Hampton University
Hampton, VA, USA

Prof. Arun Sadashiv Mujumdar
National University of Singapore (NUS)
Minerals, Metals and Materials Technology Centre (M3TC)
Faculty of Engineering
Singapore
List of contributors

Dr. Hiroshi Nagashima
Department of Chemical Engineering
Fukuoka University
Fukuoka, Japan

Prof. Martin Olazar
University of the Basque Country
Faculty of Science and Technology
Department of Chemical Engineering
Bilbao, Spain

Dr. John D. Paccione
Department of Environmental Health Sciences
School of Public Health
State University of New York at Albany
Albany, NY, USA

Dr. Elizabeth Pallai
Pannon University, FIT
Research Institute of Chemical and Process Engineering
Veszprém, Hungary

Dr. Sarah Palmer
School of Engineering
University of Warwick
Coventry, UK

Dr. Maria Laura Passos
Laval, QC, Canada

Prof. Norberto Piccinini
Dipartimento di Scienza dei Materiali e Ingegneria Chimica
Politecnico di Torino
Turin, Italy

Prof. Sandra Cristina dos Santos Rocha
Universidade Estadual de Campinas (UNICAMP)
School of Chemical Engineering
Campinas, Brazil

Dr. Giorgio Rovero
Dipartimento di Scienza dei Materiali e Ingegneria Chimica
Politecnico di Torino
Turin, Italy
Prof. Maria J. San José  
University of the Basque Country  
Faculty of Science and Technology  
Department of Chemical Engineering  
Bilbao, Spain

Prof. Jonathan Seville  
Dean of Engineering  
School of Engineering  
University of Warwick  
Coventry, UK

Dr. Tibor Szentmarjay  
Testing Laboratory of Environmental Protection  
Veszprém, Hungary

Dr. Osvaldir Pereira Taranto  
Universidade Estadual de Campinas (UNICAMP)  
School of Chemical Engineering  
Campinas, Brazil

Dr. Judith Tóth  
Chemical Research Center  
Hungarian Academy of Sciences  
Institute of Material and Environmental Chemistry  
Veszprém, Hungary

Prof. A. Paul Watkinson  
Department of Chemical and Biological Engineering  
University of British Columbia  
Vancouver, Canada

Prof. Rui Xiao  
Key Laboratory of Clean Power Generation and Combustion Technology of  
Ministry of Education  
Thermo-Energy Engineering Research Institute  
Southeast University  
Nanjing, China

Dr. Jian Xu  
Key Laboratory of Catalysis  
Faculty of Chemical Science and Engineering  
China University of Petroleum  
Beijing, China
List of contributors

**Prof. Mingyao Zhang**
Key Laboratory of Clean Power Generation and Combustion Technology of Ministry of Education
Thermo-Energy Engineering Research Institute
Southeast University
Nanjing, China

**Dr. Wenqi Zhong**
Key Laboratory of Clean Power Generation and Combustion Technology of Ministry of Education
Thermo-Energy Engineering Research Institute
Southeast University
Nanjing, China
Preface

Spouted beds have now been studied and applied for more than 50 years; during this period there has been a continual output of research papers in the engineering literature, considerable efforts to apply spouted beds in agriculture-related and industrial operations, and five international symposia dedicated solely to spouted beds. The book *Spouted Beds* by Kishan Mathur and the first-named editor of this volume summarized the field up to 1974. Since then there have been several reviews, but none that have surveyed the entire field comprehensively, including aspects that were barely touched in the earlier book or that were entirely absent. Examples of new areas include mechanically assisted spouting, slot-rectangular spouted beds, spouted and spout-fluid bed gasifiers, spouted bed electrolysis, and application of computational fluid dynamics (CFD) to spouted beds.

Our original intention was to prepare a sequel to the Mathur and Epstein book, but we soon realized that this chore would be too daunting, especially in view of competing time commitments. We therefore adopted the idea of a multiauthored book for which we would provide editing and prepare a subset of the chapters ourselves. Our intent was to choose an international array of authors able to provide a truly comprehensive view of the field, fundamentals as well as applications. Almost all those whom we asked to participate agreed to do so, and they have been remarkably cooperative in submitting material, following instructions, and responding to requests for changes, many of these being editorial in nature.

In addition to acknowledging the authors and the cooperation of Cambridge University Press, we especially acknowledge the secretarial assistance of Helsa Leong, without whom we probably would never have finished. We are also indebted to the Natural Sciences and Engineering Research Council of Canada for continuing financial support. The book is dedicated to Kishan B. Mathur, co-inventor and a pioneer in spouted beds and an inspiration to us both.

References


Common nomenclature

- $A$: cross-sectional area of column, m$^2$
- $A_a$: cross-sectional area of annulus, m$^2$
- $A_i$: fluid inlet cross-sectional area, m$^2$
- $A_s$: cross-sectional area of spout, m$^2$
- $Ar$: Archimedes number
- $Bi$: Biot number
- $C_A$: concentration of component A in fluid, mol/m$^3$
- $CD$: drag coefficient
- $cp$: heat capacity at constant pressure, J/(kg $\cdot$ K)
- $D$: diameter of cylindrical column, m
- $D$: diffusivity or dispersion coefficient, m$^2$s$^{-1}$
- $DH$: diameter of upper surface of bed, m
- $D_i$: diameter of fluid inlet, m
- $D_o$: diameter of cone base, m
- $D_s$: spout diameter, m
- $d_p$: particle diameter or mean diameter, m
- $d_s$: sphere-equivalent diameter of particle based on its surface area, m
- $d_v$: sphere-equivalent diameter of particle based on its volume, m
- $F(t)$: output tracer concentration/step input tracer concentration
- $f$: Fanning friction factor
- $G$: superficial mass flux of fluid, kg/m$^2$s
- $g$: acceleration of gravity, m/s$^2$
- $H$: bed depth, usually measured as loose-packed static bed depth after spouting, m
- $H_c$: cone height, m
- $H_f$: fountain height, measured from bed surface, m
- $H_m$: maximum spoutable bed depth, m
- $H_o$: static bed depth, m
- $h$: heat transfer coefficient, W/m$^2$K
- $i, j$: integers
- $k$: chemical reaction rate constant, s$^{-1}$ if first order
- $L$: length, m
- $M$: inventory of particles, kg
- $m_p$: particle mass, kg
- $Nu$: Nusselt number
### Common nomenclature

- **$P$**: pressure, Pa
- **$Pr$**: Prandtl number
- **$Q$**: volumetric flow rate, m$^3$/s
- **$q$**: heat transfer rate, W
- **$R$**: radius of cylindrical column, m
- **$R_g$**: universal gas constant, 8.315 J/mol·K
- **$Re$**: Reynolds number
- **$r$**: radial coordinate, m
- **$S$**: surface area, m$^2$
- **$Sc$**: Schmidt number
- **$Sh$**: Sherwood number
- **$T$**: temperature, K
- **$t$**: time, s
- **$U$**: superficial velocity of spouting fluid based on $D$, m/s
- **$U_a$**: upward superficial velocity in annulus, m/s
- **$U_{alt}$**: value of $U_a$ at $z = H$, m/s
- **$U_M$**: superficial velocity corresponding to $\Delta P_M$, m/s
- **$U_m$**: value of $U_m$ at $H = H_m$, m/s
- **$U_{msf}$**: superficial velocity at minimum fluidization, m/s
- **$U_{ms}$**: superficial velocity at minimum spouting, m/s
- **$U_t$**: free settling terminal velocity, m/s
- **$u$**: local fluid velocity, m/s
- **$u_i$**: average fluid velocity at fluid inlet, m/s
- **$u_{msi}$**: minimum spouting velocity based on $D_i$, m/s
- **$V_p$**: particle volume, m$^3$
- **$v$**: local particle velocity, m/s
- **$x, y$**: horizontal Cartesian coordinates, m
- **$z$**: vertical coordinate measured from fluid inlet, m

### Greek letters

- **$\alpha$**: thermal diffusivity, m$^2$/s
- **$\beta$**: mass transfer coefficient, m/s
- **$\Delta P$**: pressure drop, Pa
- **$\Delta P_M$**: maximum pressure drop across bed, Pa
- **$\Delta P_S$**: spouting pressure drop across bed, Pa
- **$\varepsilon$**: fractional void volume
- **$\theta$**: total included angle of cone
- **$\lambda$**: thermal conductivity of fluid, W/(m·K)
- **$\lambda_p$**: thermal conductivity of particles, W/(m·K)
- **$\mu$**: absolute viscosity, Pa·s
- **$\vartheta$**: total included angle of cone
- **$\rho$**: density of fluid, kg/m$^3$
\( \rho_p \) density of particles, kg/m\(^3\)

\( \tau \) mean residence time, s

**Subscripts**

\( A, B \) component A, B

\( a \) annulus, auxiliary

\( f \) fountain

\( ms \) minimum spouting

\( p \) particle

\( s \) spout

**Abbreviations**

CSB conventional spouted bed

CCSB conical-cylindrical spouted bed = CSB

CcSB conical (Coni-cal) spouted bed