SOIL MECHANICS

A one-dimensional introduction

This introductory course on soil mechanics presents the key concepts of stress, stiffness, seepage, consolidation, and strength within a one-dimensional framework. Consideration of the mechanical behaviour of soils requires us to consider density alongside stresses, thus permitting the unification of deformation and strength characteristics. Soils are described in a way which can be integrated with concurrent teaching of the properties of other engineering materials. The book includes a model of the shearing of soil and some examples of soil-structure interaction which are capable of theoretical analysis using one-dimensional governing equations. The text contains many worked examples, and exercises are given for private study at the end of all chapters. Some suggestions for laboratory demonstrations that could accompany such an introductory course are sprinkled through the book.

David Muir Wood has taught soil mechanics and geotechnical engineering at the universities of Cambridge, Glasgow and Bristol since 1975 and has contributed to courses on soil mechanics in many countries around the world. He is the author of numerous research papers and book chapters. His previous books include *Soil behaviour and critical state soil mechanics* (1990) and *Geotechnical modelling* (2004). He was co-chairman of the United Kingdom GeotechniCAL computer-aided learning project.
Soil mechanics

A ONE-DIMENSIONAL INTRODUCTION

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Preface

This book has emerged from a number of stimuli.

There is a view that soils are special: that their characteristics are so extraordinary that they can only be understood by a small band of specialists. Obviously, soils do have some special properties: the central importance of density and change of density merits particular attention. However, in the context of teaching principles of soil mechanics to undergraduates in the early years of their civil engineering degree programmes, I believe that there is advantage to be gained in trying to integrate this teaching with other teaching of properties of engineering materials to which the students are being exposed at the same time.

It is a fundamental tenet of critical state soil mechanics – with which I grew up in my undergraduate days – that consideration of the mechanical behaviour of soils requires us to consider density alongside effective stresses, thus permitting the unification of deformation and strength characteristics. This can be seen as a broad interpretation of the phrase critical state soil mechanics. I believe that such a unification can aid the teaching and understanding of soil mechanics.

There is an elegant book by A. J. Roberts which demonstrates in a unified way how a common mathematical framework can be applied to problems of solid mechanics, fluid mechanics, traffic flow and so on. While I cannot hope to emulate this elegance, the title prompted me to explore a similar one-dimensional theme for the presentation of many of the key concepts of soil mechanics: density, stress, stiffness, strength and fluid flow.

This one-dimensional approach to soil mechanics has formed the basis for an introductory course of ten one-hour lectures with ten one-hour problem classes and one three-hour laboratory afternoon for first-year civil engineering undergraduates at Bristol University. The material of that course is contained in this book. I have added a chapter on the analysis of one-dimensional consolidation, which fits neatly with the theme of the book. I have also included a model of the shearing of soil and some examples of soil-structure interaction which are capable of theoretical analysis using essentially one-dimensional governing equations.

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Simplification of more or less realistic problems leads to differential equations which can be readily solved: this is the essence of modelling with which engineers need to engage (and to realise that they are engaging) all the time. A few of these topics require some modest mathematical ability – a bit of integration, solution of ordinary and partial differential equations – but nothing beyond the eventual expectations of an undergraduate engineering degree programme. Sections that might, as a consequence, be omitted on a first reading, or until the classes in mathematics have caught up, are indicated by the symbol ♣.

Exercises are given for private study at the end of all chapters and some suggestions for laboratory demonstrations that could accompany such an introductory course are sprinkled through the book.

I am grateful to colleagues at Bristol and elsewhere – especially Danuta Lesniewska, Erdin Ibraim and Dick Clements – who have provided advice and comments on drafts of this book to which I have tried to respond. Erdin in particular has helped enormously by using material and examples from a draft of this book in his own teaching and has made many useful suggestions for clarification. However, the blame for any remaining errors must remain with me.

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