Optimal Combining and Detection

Statistical Signal Processing for Communications

With signal combining and detection methods now representing a key application of signal processing in communication systems, this book provides a range of key techniques for receiver design when multiple received signals are available. Various optimal and suboptimal signal combining and detection techniques are explained in the context of multiple-input multiple-output (MIMO) systems, including successive interference cancellation (SIC) based detection and lattice reduction (LR) aided detection. The techniques are then analyzed using performance analysis tools. The fundamentals of statistical signal processing are also covered, with two chapters dedicated to important background material. With a carefully balanced blend of theoretical elements and applications, this book is ideal for both graduate students and practicing engineers in wireless communications.

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Preface

Statistical signal processing is a set of statistical techniques that have been developed to deal with random signals in a number of applications. Since it is rooted in detection and estimation theory, which are well established in statistics, the fundamentals are not changed although new applications have emerged. Thus, I did not have any strong motivation to write another book on statistical signal processing until I was convinced that there was a sufficient amount of new results to be put together with fundamentals of detection and estimation theory in a single book.

These new results have emerged in applying statistical signal processing techniques to wireless communications since 1990. We can consider a few examples here. The first example is smart antenna. Smart antenna is an application of array signal processing to cellular systems to exploit spatial selectivity for improving spectral efficiency. Using antenna arrays, the spatial selectivity can be used to mitigate incoming interfering signals at a receiver or control the transmission direction of signals from a transmitter to avoid any interference with the receivers which do not want to receive the signal. The second example is based on the development of code division multiple access (CDMA) systems for cellular systems. In CDMA systems, multiple users are allowed to transmit their signals simultaneously with different signature waveforms. The matched filter can be employed to detect a desired signal with its signature waveform. This detector is referred to as the single-user detector as it only detects one user's signal. Although this single-user detector is able to provide a reasonable performance, it is also possible to improve the performance to detect multiple signals simultaneously. This detector is called the multiuser detector. The third example is multiple-input multiple-output (MIMO) systems. In MIMO systems, multiple signals are transmitted and multiple signals are received. Thus, it is required to detect multiple signals simultaneously. These new applications promote advances of statistical signal processing. In particular, new and advanced techniques for signal combining and detection have emerged.

This book is intended to provide fundamentals of signal detection and estimation together with new results that have been developed for the new applications mentioned above.

I would like to thank many people for supporting this work, in particular: I. M. Kim (Queens University), C. Ling (Imperial College), and F. Adachi (Tohoku University). They helped me by providing constructive comments and proofreading. Needless to say the responsibility for the remaining errors, typos, unclear passages, and weaknesses is mine. I would also like to thank those people who inspire and encourage me all the time.
F. Adachi (Tohoku University) for encouragement as my mentor, J. Ritcey (University of Washington) for long-term friendship, and many others including my students for useful discussions.

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Finally, I would like to offer very special thanks to my wife, Kila, and children, Seji and Wooji, for their generous support, understanding, and love.
Symbols

General

\[ j = \sqrt{-1} \]
\[ \mathbb{F}_2: \text{binary field} \]
\[ \mathbb{Z}: \text{set of integer numbers} \]
\[ \mathbb{R}^n: \text{real-valued } n\text{-dimensional vector space} \]
\[ \mathbb{C}^n: \text{complex-valued } n\text{-dimensional vector space} \]
\[ \times: \text{Cartesian product (if it does not mean the product)} \]
\[ |\mathcal{A}|: \text{cardinality of set } \mathcal{A} \text{ or the number of the elements in } \mathcal{A} \]
\[ \emptyset: \text{empty set} \]
\[ \cup: \text{set union} \]
\[ \cap: \text{set intersection} \]
\[ \setminus: \text{set difference or set-minus} \]
\[ \mathcal{A}^c: \text{the complementary set of set } \mathcal{A} \]
\[ u(x): \text{step function} \]
\[ \delta(x): \text{Dirac delta function} \]

Statistics related symbols

\[ f_X(x): \text{pdf of random variable } X \]
\[ F_X(x): \text{cdf of random variable } X \]
\[ \Pr(\mathcal{A}): \text{probability of random event } \mathcal{A} \]
\[ \mathbb{E}[X]: \text{statistical expectation of } X \]
\[ \text{Var}(X): \text{variance of } X \]
\[ Q(x): \text{Q-function, } Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} \, dz \]
\[ \mathcal{N}(x, \mathbf{R}): \text{Gaussian probability density function with mean } x \text{ and covariance } \mathbf{R} \]
\[ \mathcal{CN}(x, \mathbf{R}): \text{circularly symmetric complex Gaussian probability density function with mean } x \text{ and covariance } \mathbf{R} \]

Vector/Matrix related symbols

\[ || \cdot ||_p: p\text{-norm} \]
\[ || \cdot ||_F: \text{Frobenius norm} \]
\[ (\cdot)^T: \text{transpose} \]
\[ (\cdot)^H: \text{Hermitian transpose} \]
\[ \text{det}(\cdot): \text{determinant of a square matrix} \]
\[ \text{tr}(\cdot): \text{trace of a square matrix} \]
\[ \text{Diag}(a_1, a_2, \ldots, a_N): N \times N \text{ diagonal matrix whose elements are } a_1, a_2, \ldots, a_N \]
List of symbols

\[\mathbf{a}\]_n: \text{n}th element of a vector \(\mathbf{a}\)

\[\mathbf{A}\]_{m,n}: \text{(m, n)th element of a matrix} \ \mathbf{A}

\[\mathbf{A}\]_{m_1:m_2,n_1:n_2}: \text{a submatrix of} \ \mathbf{A} \ \text{obtained by taking the elements in the} \ m_1 \text{th to} \ m_2 \text{th columns and the} \ n_1 \text{th to} \ n_2 \text{th rows}

\[\mathbf{A}\]_{:,n}: \text{n}th column vector of \ \mathbf{A}

\[\mathbf{A}\]_{n,:}: \text{n}th row vector of \ \mathbf{A}
## Abbreviations

- **AR** autoregressive
- **AR V** array response vector
- **ASK** amplitude shift keying
- **AWGN** additive white Gaussian noise
- **BER** bit error rate
- **BSC** binary symmetric channel
- **cdf** cumulative distribution function
- **CDMA** code division multiple access
- **CLT** central limit theorem
- **CRB** Cramer–Rao Bound
- **CSCG** circularly symmetric complex Gaussian
- **CVP** closed vector problem
- **DFE** decision feedback equalizer
- **DMC** discrete memoryless channel
- **DMI** direct matrix inversion
- **DPSK** differential phase shift keying
- **EGC** equal gain combining
- **FA** false alarm
- **GLR** generalized likelihood ratio
- **GLRT** generalized likelihood ratio test
- **GSDC** generalized selection diversity combining
- **iid** independent and identically distributed
- **ISI** intersymbol interference
- **LCMV** linearly constrained minimum variance
- **LLR** log-likelihood ratio
- **LMS** least mean square
- **LR** lattice reduction or likelihood ratio
- **LS** least square
- **MAC** multiple access channel
- **MAP** maximum a posteriori probability
- **MIMO** multiple-input multiple-output
- **MISO** multiple-input single-output
- **ML** maximum likelihood
- **MLE** maximum likelihood estimate
List of abbreviations

MMSE  minimum mean square error
MRC   maximal ratio combining
MSE   mean square error
MSNR  maximum signal to noise ratio
MUSIC multiple signal classification
MVDR  minimum variance distortionless response
MVUE  minimum variance unbiased
PAM   pulse amplitude modulation
pdf   probability density function
PEP   pairwise error probability
QAM   quadrature amplitude modulation
QPSK  quadrature phase shift keying
RLS   recursive least square
ROC   receiver operating characteristics
SD    selection diversity
SDR   software defined radio
SLLN  strong law of large numbers
SMI   sample matrix inversion
SIC   successive interference cancellation
SIMO  single-input multiple-output
SISO  single-input single-output
SINR  signal to interference-plus-noise ratio
SNR   signal to noise ratio
SVP   shortest vector problem
ULA   uniform linear array
WLLN  weak law of large numbers
WLS   weighted least square
WSS   wide-sense stationary
ZF    zero-forcing