

1

The Standard Model and beyond

In this chapter we overview the structure of the Standard Model (SM) of particle physics, its shortcomings, and different ideas for physics beyond the Standard Model (BSM) devised to address them. We discuss the three fine-tuning puzzles of the SM (the cosmological constant, strong CP and hierarchy problems) and several ideas put forward to solve them. Among the extensions of the SM, this chapter overviews Grand Unified Theories (GUTs), which explore the embedding of the SM gauge symmetry into a larger unified gauge group like $SU(5)$ or $SO(10)$, and models with extra spatial dimensions beyond the observed three. These ideas, along with supersymmetry, studied in Chapter 2, will reappear in the study of particle physics models from string theory in later chapters.

1.1 The Standard Model of particle physics

The Standard Model of particle physics constitutes an impressive success of twentieth century physics. It describes elementary particles and their electromagnetic, weak and strong interactions in a remarkable wide range of energies, and with unprecedented precision. The SM is a quantum field theory based on a gauge group

$$G_{\text{SM}} = SU(3) \times SU(2)_L \times U(1)_Y, \tag{1.1}$$

with $SU(3)$ describing strong interactions via Quantum Chromodynamics (QCD), and $SU(2)_L \times U(1)_Y$ describing electroweak (EW) interactions. The matter fields form three generations (or families) of quarks and leptons, described as Weyl 2-component spinors, with the EW structure

$$Q_L = \begin{pmatrix} U_L^i \\ D_L^i \end{pmatrix}, \quad U_R^i, \quad D_R^i, \quad L = \begin{pmatrix} \nu_L^i \\ E_L^i \end{pmatrix}, \quad E_R^i; \quad i = 1, 2, 3. \tag{1.2}$$

We take all fields to be left-chirality Weyl spinors, independently of their L or R subindex, which merely denotes the $SU(2)_L$ transformation properties as doublets or singlets. In addition, quarks in Q_L transform as color triplets, while U_R, D_R transform as conjugate triplets. Gauge quantum numbers of the SM fermions are shown in Table 1.1.

The chirality of this fermionic spectrum is possibly one of the deepest properties of the SM. Describing particles in terms of Dirac spinors, it means that left- and right-chirality

Table 1.1 *Gauge quantum numbers of SM quarks, leptons and the Higgs scalars*

Field	$SU(3)$	$SU(2)_L$	$U(1)_Y$
$Q_L^i = (U^i, D^i)_L$	3	2	1/6
U_R^i	$\bar{\mathbf{3}}$	1	−2/3
D_R^i	$\bar{\mathbf{3}}$	1	+1/3
$L^i = (ν^i, E^i)_L$	1	2	−1/2
E_R^i	1	1	+1
$H = (H^-, H^0)$	1	2	−1/2

components actually have *different* EW quantum numbers. In the above more suitable description of all fermions as 2-component left-handed Weyl spinors, chirality is the statement that they fall in a complex representation of the SM gauge group. In any picture, the implication is that explicit Dirac mass terms $m \overline{f}_R f_L + \text{h.c.}$ are forbidden by gauge invariance, and fermions remain massless (until EW symmetry breaking, discussed below). The impossibility of mass terms makes chiral sets of fermions very robust, and easy to identify when deriving the light spectrum of particles from fundamental theories like string theory. Hence, the chirality of the fermion spectrum is a key guiding principle in building string models of particle physics.

In the SM the electroweak symmetry $SU(2)_L \times U(1)_Y$ is spontaneously broken down to the electromagnetic $U(1)_{\text{EM}}$ symmetry by a complex scalar Higgs field transforming as an $SU(2)_L$ doublet $H = (H^-, H^0)$ and with hypercharge $-1/2$. Its dynamics is parametrized in terms of a potential, devised to trigger a non-vanishing Higgs vacuum expectation value (vev) v

$$V = -\mu^2 |H|^2 + \lambda |H|^4 \quad \Rightarrow \quad v^2 \equiv \langle |H| \rangle^2 = \mu^2 / 2\lambda. \tag{1.3}$$

The vev defines the electrically neutral direction and is set to $\langle H^0 \rangle \simeq 170 \text{ GeV}$ in order to generate the W^\pm and Z vector boson masses. Simultaneously it produces masses for quarks and leptons through the Yukawa couplings

$$\mathcal{L}_{\text{Yuk}} = Y_U^{ij} \overline{Q}_L^i U_R^j H^* + Y_D^{ij} \overline{Q}_L^i D_R^j H + Y_L^{ij} \overline{L}^i E_R^j H + \text{h.c.} \tag{1.4}$$

These interactions are actually the most general consistent with gauge invariance and renormalizability, and accidentally are invariant under the global symmetries related to the baryon number B and the three family lepton numbers L_i . Regarding the SM as an effective theory, non-renormalizable operators violating these symmetries may, however, be present.

The hypercharges of the SM fermions in Table 1.1 are related to their usual electric charges by $Q_{\text{EM}} = Y + T_3$, where $T_3 = \text{diag}(\frac{1}{2}, -\frac{1}{2})$ is an $SU(2)_L$ generator. They thus reproduce electric charge quantization, e.g. the equality in magnitude of the proton and

Table 1.2 *Masses of quarks and charged leptons at the EW scale in GeV*

U-quarks	u 2×10^{-3}	c 5×10^{-1}	t 173
D-quarks	d 4×10^{-3}	s 1×10^{-1}	b 3
Leptons	e 0.51×10^{-3}	μ 1.05×10^{-1}	τ 1.7

electron charges. Although these hypercharge assignments look rather ad hoc, their values are dictated by quantum consistency of the theory. It is indeed easy to check that these are (modulo an irrelevant overall normalization) the only (family independent) assignments canceling all potential triangle gauge anomalies.

The above simple SM structure describes essentially all particle physics experimental data at present (with simple extensions to account for neutrino masses, see Section 1.2.6). Despite this character of a *Theory* of matter and interactions, it poses several intriguing questions, which probably hold the key to new physics at a more fundamental level, and thus motivate maintaining its status of *Model*. Most prominently, gravitational interactions are not included, in particular due to the difficulties in reconciling them with Quantum Mechanics. Gravity implies that the SM should be regarded as an effective theory with a cutoff at most the Planck scale

$$M_p = \frac{1}{\sqrt{8\pi} G_N^{1/2}} = 1.2 \times 10^{19} \text{ GeV}, \tag{1.5}$$

where G_N is Newton’s constant. Other examples of hints for BSM physics arise from the fine tuning issues in Section 1.3.

Finally, another suggestive hint for an underlying structure is that the SM has many free parameters, which are *external* to the model, rather than predicted by it. There are three gauge coupling constants g_1, g_2, g_3 , the QCD θ -parameter, the nine masses of quarks and leptons (plus those of neutrinos), as well as the quark CP violating phase (plus possible additional phases in the lepton sector). Finally, there are additional couplings in the Higgs sector. Most of the unknown parameters are related to the Higgs–Yukawa sector of the theory, which has not been fully tested experimentally, and is thus still poorly understood. One would hope for a more fundamental microscopic explanation of (at least some of) these parameters. Indeed, one of the most outstanding puzzles of the SM is the structure of fermion masses and mixing angles. The masses of quarks and leptons show a hierarchical structure, see Table 1.2, suggesting the possibility of some underlying pattern. The mass matrices m_{ij} obtained from the U, D, L Yukawa couplings are in general not diagonal, but may be diagonalized by bi-unitary transformations $V_L^{U,D,L}, V_R^{U,D,L}$, acting

on the left- and right-handed fermions respectively. After these rotations, the W^\pm gauge bosons couple to U- and D-quarks through the Cabibbo–Kobayashi–Maskawa (CKM) matrix, given by $U_{\text{CKM}} = V_L^U (V_L^D)^\dagger$. Experimental measurements yield the following approximate structure for this matrix

$$|V_{\text{CKM}}| = \begin{pmatrix} 0.974 & 0.22 & 0.003 \\ 0.22 & 0.973 & 0.04 \\ 0.008 & 0.04 & 0.999 \end{pmatrix}. \tag{1.6}$$

Again this displays an interesting structure. It is close to the identity matrix, with small off-diagonal mixing entries, except for the Cabibbo (1-2) entry, which is somewhat larger. Experiments also show evidence for CP-violation in the quark sector, whose size may be measured in terms of the Jarlskog invariant $J = (3.0 \pm 0.3) \times 10^{-5}$.

The structure of neutrino masses and mixings turns out to be quite different from that of quarks. Neutrinos are much lighter than quarks and charged leptons. Solar and atmospheric oscillation experiments allow for the measurement of some squared-mass differences

$$\Delta m_{12}^2 = 8 \times 10^{-5} \text{ eV}^2; \quad \Delta m_{23}^2 = 2.4 \times 10^{-3} \text{ eV}^2, \tag{1.7}$$

which also suggest a possible hierarchical structure. On the other hand the neutrino Pontecorvo–Maki–Nakagawa–Sakata (PMNS) mixing matrix is quite different from the CKM one, since oscillation experiments reveal a structure of typically large mixing angles

$$|V_{\text{PMNS}}| = \begin{pmatrix} 0.77 - 0.86 & 0.50 - 0.63 & 0.0 - 0.22 \\ 0.22 - 0.56 & 0.44 - 0.73 & 0.57 - 0.80 \\ 0.21 - 0.55 & 0.40 - 0.71 & 0.59 - 0.82 \end{pmatrix}. \tag{1.8}$$

The search for a fundamental explanation of the fermion spectrum, gauge coupling constants, and other *external* SM parameters, has constituted a driving force to propose theories beyond the SM. A prototypical example is provided by Grand Unification Theories, reviewed next.

1.2 Grand Unified Theories

In the SM there are three gauge factors, and five matter multiplets Q_L, U_R, D_R, L, E_R per family. Grand Unified Theories (GUTs) propose that there is an underlying gauge group G_{GUT} , in general assumed to be simple, which at very high energies experiences spontaneous symmetry breaking down to the SM group. Thus the SM gauge factors are unified into a single gauge force, and the different SM matter fields are also (possibly partially) unified into multiplets of the larger symmetry G_{GUT} . This gauge group G_{GUT} must be at least of rank four and contain the SM group, $SU(3) \times SU(2)_L \times U(1)_Y$, and must admit complex representations to accommodate a chiral fermion spectrum. GUT theories can thus be classified according to the choice of G_{GUT} satisfying these conditions, see later sections.

1.2.1 Gauge coupling unification

A particularly compelling motivation for unification of the SM gauge groups into a *simple* group is the unification of gauge coupling constants into a single one. Indeed, there is convincing evidence for such unification, coming from the evolution of the three SM gauge couplings to high energies with the renormalization group equations (RGE). The one-loop evolution equations for the couplings $\alpha_a = g_a^2/(4\pi)$ are

$$\frac{1}{\alpha_a(Q^2)} = \frac{1}{\alpha_a(M^2)} + \frac{b_a}{4\pi} \log \frac{M^2}{Q^2}; \quad a = 1, 2, 3, \tag{1.9}$$

where Q, M are two energy scales, and b_a are the one-loop β -function coefficients, which for a $SU(N)$ gauge theory are

$$b = -\frac{11}{3}N + \frac{2}{3}T(R)n_f + \frac{1}{3}T(R)n_s, \tag{1.10}$$

with n_f, n_s the number of Weyl fermions and complex scalar fields in the representation R , and $T(R)$ is the corresponding quadratic Casimir, in the normalization $T(R) = 1/2$ for fundamentals. Expression (1.10) also holds for $U(1)$, by setting $N = 0$ and replacing $T(R) \rightarrow Y^2$. For the SM one has

$$\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 0 \\ -22/3 \\ -11 \end{pmatrix} + N_{\text{gen}} \begin{pmatrix} 4/3 \\ 4/3 \\ 4/3 \end{pmatrix} + N_{\text{Higgs}} \begin{pmatrix} 1/10 \\ 1/6 \\ 0 \end{pmatrix}. \tag{1.11}$$

where $N_{\text{gen}}, N_{\text{Higgs}}$ denote the numbers of quark-lepton generations and Higgs doublets, and we have introduced an additional hypercharge normalization factor $Y^2 \rightarrow (3/5)Y^2$ to be justified later, just above (1.13).

Extrapolating the measured values of the three α_a from the EW scale up in energies, and assuming no further relevant degrees of freedom at intermediate scales (the so-called *desert hypothesis*), the three couplings tend to join around 10^{15} GeV into a single coupling α_{GUT} . An important general feature is that matter in full G_{GUT} multiplets does not modify the running of the *relative* gauge couplings at one loop, and hence does not spoil unification. The joining of the coupling constants using the minimal SM content is only in qualitative agreement with experiment, but becomes quite sharp in supersymmetric versions, as described in Section 2.6.2.

1.2.2 $SU(5)$ GUTs

There is a unique compact simple Lie group of rank four admitting complex representations: $SU(5)$. It moreover contains the SM group as a maximal subgroup. $SU(5)$ GUTs illustrate many features of grand unification, like gauge coupling unification and proton decay, valid for other choices of simple GUT group.

The $SU(5)$ theory has $5^2 - 1 = 24$ gauge bosons, which include the 12 SM ones and 12 extra gauge bosons transforming as $SU(2)_L$ doublets and $SU(3)$ triplets, denoted by (X_r^+, Y_r^+) , (X_r^-, Y_r^-) , $r = 1, 2, 3$. The $SU(5)$ symmetry can be broken to the SM group by introducing the “GUT-Higgs” scalars $\Phi_{\mathbf{24}}$ in the adjoint $\mathbf{24}$. It acquires a large vev $\langle \Phi_{\mathbf{24}} \rangle = \text{diag}(2v, 2v, 2v, -3v, -3v)$ commuting with the SM group, which is thus unbroken, whereas the X, Y gauge bosons get masses $M_{X,Y}^2 \simeq \alpha_{\text{GUT}} v^2$.

Each SM quark-lepton generation fits nicely into a reducible $SU(5)$ representation $\bar{\mathbf{5}} + \mathbf{10}$. For instance, fields in, e.g., the first generation can be written as

$$\bar{\mathbf{5}} = \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \\ e^- \\ \nu_e \end{pmatrix}; \quad \mathbf{10} = \begin{pmatrix} 0 & u_3^c & u_2^c & u_1 & d_1 \\ & 0 & u_1^c & u_2 & d_2 \\ & & 0 & u_3 & d_3 \\ & & & 0 & e^+ \\ & & & & 0 \end{pmatrix}, \quad (1.12)$$

where the $\mathbf{10}$ is displayed as a 5×5 antisymmetric matrix, whose lower triangle entries are ignored for clarity. As usual, we have taken all fermion fields left-handed, and our notation for the first family relates to (1.2) and Table 1.1 by $d^c = D_R^1$, $(\nu_e, e^-) = L^1$, $u^c = U_R^1$, $(u, d) = Q_L^1$, $e^+ = E_R^1$. The assignment (1.12) reproduces the correct SM gauge quantum numbers upon breaking $SU(5)$. Note also that the combination $\bar{\mathbf{5}} + \mathbf{10}$ is free of $SU(5)$ anomalies.

$SU(5)$ unification has some interesting implications, holding also in other GUTs:

- *Charge quantization:* The electromagnetic $U(1)_{\text{EM}}$ generator belongs to $SU(5)$, and hence it must be traceless, $\text{Tr } Q_{\text{EM}} = 0$. This implies that, e.g., fermions in the $\bar{\mathbf{5}}$ in (1.12) must have electric charges adding to zero, hence $Q_{d^c} = -\frac{1}{3} Q_{e^-}$. This and a similar expression for u-quarks imply equality of the proton and electron charge, and thus that all charges are quantized, i.e. multiples of a basic unit.
- *Relationships among gauge couplings:* As mentioned, unification into a simple GUT group implies a unique gauge coupling constant, leading to two relations among SM gauge couplings. At the unification scale the non-abelian couplings satisfy $g_3 = g_2$. Standard hypercharge Y is realized in $SU(5)$ with an additional factor of $\sqrt{3/5}$, to comply with the normalization $\text{tr } T_{SU(5)}^2 = 1/2$ in the fundamental representation. This corresponds to the GUT relation $g_1^2 = (3/5)g_2^2$. For the weak mixing angle we have

$$\sin^2 \theta_W = \frac{\text{Tr } (T_3^2)}{\text{Tr } (Q_{\text{EM}}^2)} = \frac{1/2}{4/3} = \frac{3}{8}, \quad (1.13)$$

where in the second equality we have taken traces in the fundamental representation and $Q_{\text{EM}} = T_3 + Y$. These relationships apply at the unification scale M_{GUT} , and provide boundary conditions for gauge coupling running at lower energies according to

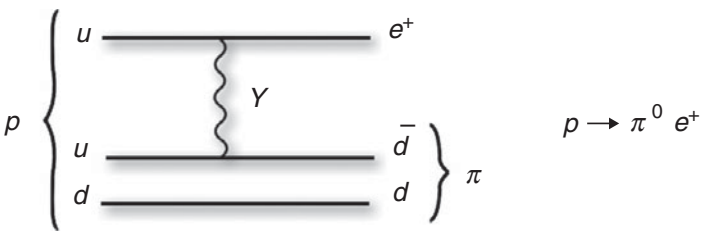


Figure 1.1 Dimension-6 contribution to the proton decay mode $p \rightarrow e^+ \pi^0$.

the RGE (1.9). Combining the equations for the three couplings, the relations among couplings at the EW scale M_W are

$$\begin{aligned} \frac{1}{\alpha_3(M_W)} &= \frac{3}{8} \left[\frac{1}{\alpha_{\text{EM}}(M_W)} - \frac{1}{2\pi} \left(b_1 + b_2 - \frac{8}{3} b_3 \right) \log \frac{M_{\text{GUT}}}{M_W} \right], \\ \sin^2 \theta_W(M_W) &= \frac{3}{8} + \frac{5}{16\pi} \alpha_{\text{EM}}(M_W) \left(b_2 - \frac{3}{5} b_1 \right) \log \frac{M_{\text{GUT}}}{M_W}. \end{aligned} \tag{1.14}$$

Using, e.g., the first equation, the unification scale can be computed to be $M_{\text{GUT}} \simeq 10^{15}$ GeV. The second then leads to the prediction $\sin^2 \theta_W = 0.214 \pm 0.004$, only in qualitative agreement with the experimental result 0.2312 ± 0.0002 . In the supersymmetric case in Section 2.6.2 the result gets much better, with an almost perfect quantitative agreement with the experimental value.

- *Baryon number violation and proton decay:* Since quarks and leptons live inside the same GUT multiplets, they can transform into each other by emission/absorption of the massive gauge bosons X, Y . This leads to baryon number violating transitions like the proton decay mode $p \rightarrow \pi^0 e^+$ in Figure 1.1. These transitions are suppressed by the large X, Y mass. On dimensional grounds, the amplitude scales as $1/M_X^2$, leading to a quite long proton lifetime $\tau_p \simeq M_X^4/m_p^5$, where m_p is the proton mass. A detailed computation yields $\tau_{p \rightarrow \pi^0 e^+} \simeq 4 \times 10^{29 \pm .7}$ years. This is actually well below the super-Kamiokande experiment bound $\tau_{p \rightarrow \pi^0 e^+} > 6.6 \times 10^{33}$ years, so the simplest $SU(5)$ GUT is excluded. However, in the supersymmetric version of this theory, proton decay through this kind of dimension-6 operator is more suppressed, as described in Section 2.6.2, so this variant of the theory is not ruled out.
- *Relationships among fermion masses:* In the SM the fermion masses arise from three independent kinds of Yukawa couplings (1.4). In $SU(5)$ the SM Higgs doublet sits in a GUT multiplet $H_{\bar{5}}$ in the representation $\bar{5}$ (along with a color triplet, to be made heavy as discussed in Section 2.6.2). There are only two kinds of Yukawa couplings, of the form

$$L_{\text{Yuk}}^{SU(5)} = Y_U^{ij} \bar{\psi}_{10}^i \psi_{10}^j H_{\bar{5}}^* + Y_{D,L}^{ij} \bar{\psi}_{\bar{5}}^i \psi_{10}^j H_{\bar{5}} + \text{h.c.} \tag{1.15}$$

Hence there are relations between the Yukawas of charged leptons and D-quarks, $Y_D^{ij} = Y_L^{ij}$ at the GUT scale. At lower energies, the Yukawa couplings run according to the

RGEs, and in particular the QCD loop corrections enhance the quark Yukawas compared to the charged lepton ones. The leading QCD correction, e.g., for the third generation yields

$$\frac{m_b(M_W)}{m_\tau(M_W)} = \left(\frac{\alpha_3(M_W)}{\alpha_3(M_X)} \right)^{-\frac{\gamma}{2b_3}} \simeq 2, \tag{1.16}$$

where $\gamma = 8$ is the quark QCD anomalous dimension coefficient. This result is in reasonable agreement with data for the third generation, but fails for the first two. The situation can be improved in more complicated GUT models involving additional Higgs multiplets, whose description is beyond this brief overview.

1.2.3 *SO(10) GUTs*

For rank five there are only two compact simple Lie groups admitting complex representations, $SU(6)$ and $SO(10)$. The use of $SU(6)$ for GUT model building is a simple extension of $SU(5)$ and does not lead to new remarkable features. On the other hand, $SO(10)$ GUTs extend $SU(5)$ in a more interesting way, leading to several genuinely new properties.

A remarkable property is that one SM family of quarks and leptons fits neatly into a single $SO(10)$ representation, the spinor representation **16**, which is the lowest-dimensional complex representation of $SO(10)$. For, e.g., the first family

$$\psi_{\mathbf{16}} = (\nu_e, \ u_1, \ u_2, \ u_3; \ e^-, \ d_1, \ d_2, \ d_3; \ d_3^c, \ d_2^c, \ d_1^c, \ e^+; \ u_3^c, \ u_2^c, \ u_1^c, \ \nu_R).$$

The **16** actually contains an additional fermion, singlet under all SM gauge interactions, as is manifest in its decomposition under the $SU(5)$ subgroup, $\mathbf{16} = \mathbf{10} + \bar{\mathbf{5}} + \mathbf{1}$. The extension of the SM by these singlets is aesthetically pleasing, since they can be regarded as right-handed neutrinos, completing the right-handed spectrum of leptons in analogy with that of quarks. Hence $SO(10)$ GUTs predict the existence of a right-handed neutrino ν_R for each generation. Right-handed neutrinos may play a key role in the structure of neutrino masses, as described in Section 1.2.6.

The group $SO(10)$ has a maximal $SU(5) \times U(1)$ subgroup, with the adjoint decomposing as $\mathbf{45} = \mathbf{24} + \mathbf{10} + \bar{\mathbf{10}} + \mathbf{1}$. Unlike the $SU(5)$ case, breaking of the gauge symmetry down to the SM group G_{SM} may proceed via an intermediate stage of partial unification. Different patterns are

$$\begin{array}{llll} SO(10) & \longrightarrow & & G_{\text{SM}} \\ SO(10) & \longrightarrow SU(3) \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} & \longrightarrow & G_{\text{SM}} \\ SO(10) & \longrightarrow & SU(5) \times U(1) & \longrightarrow G_{\text{SM}} \\ SO(10) & \longrightarrow & SU(4) \times SU(2)_L \times SU(2)_R & \longrightarrow G_{\text{SM}}. \end{array}$$

The breaking of $SO(10)$ to the SM group requires not only adjoint scalars $\Phi_{\mathbf{45}}$, but also extra scalars $\phi_{\mathbf{16}}$ transforming in the **16** in order to lower the rank. Models with direct $SO(10)$ breaking down to the SM group lead to predictions quite similar to those of $SU(5)$.

The intermediate step $SU(3) \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, with $B-L$ denoting baryon minus lepton number, is often called the left–right symmetric model, and $SU(4) \times SU(2)_L \times SU(2)_R$ is known as the Pati–Salam model. These partial unifications are interesting even without resorting to an ultimate $SO(10)$ unification, although they fit nicely in the latter.

The minimal embedding of the EW Higgs doublet is in a multiplet of scalars H_{10} in the representation **10**. It decomposes as $\mathbf{5} + \bar{\mathbf{5}}$ under $SU(5)$, and actually contains two ordinary Higgs doublets (plus additional triplets). They are often denoted H_u, H_d , since their vevs induce U - and D -quark masses. In the simplest $SO(10)$ GUT model, there is only one kind of Yukawa coupling

$$\mathcal{L}_{\text{Yuk}}^{SO(10)} = Y^{ij} \bar{\psi}_{16}^i \psi_{16}^j H_{10} + \text{h.c.} \tag{1.17}$$

This leads to a unification of Yukawas at the GUT scale,

$$Y_U^{ij} = Y_D^{ij} = Y_L^{ij} = Y_\nu^{ij}, \tag{1.18}$$

where Y_ν denotes the Yukawa coupling inducing neutrino Dirac masses, see Section 1.2.6. In particular, the relation for the third generation $Y_\tau = Y_b = Y_t$ may be consistent with experimental data for very large $\tan \beta = \langle H_u \rangle / \langle H_d \rangle \simeq 50 - 60$. However, this simplest scheme with a single Higgs H_{10} does not quite work, since it implies $Y_U^{ij} = Y_D^{ij}$ and hence aligned quark rotation matrices $V_L^U = V_L^D$, and there is no CKM mixing. This improves in models where the physical EW Higgs field involves further $SO(10)$ multiplets, like scalars in the **126**, or involving non-renormalizable couplings.

1.2.4 E_6 GUTs

At rank 6, the only compact simple Lie groups with complex representations are $SU(7)$ and E_6 . Again $SU(7)$ adds no essentially new feature, whereas the exceptional group E_6 contains $SO(10) \times U(1)$ as a maximal subgroup, and does introduce some novelties.

The lowest-dimensional non-trivial representation in E_6 has dimension 27 and decomposes under the $SO(10)$ and $SU(5)$ subgroups as

$$\mathbf{27} = \mathbf{16} + \mathbf{10} + \mathbf{1} = (\mathbf{10} + \bar{\mathbf{5}} + \mathbf{1}) + (\mathbf{5} + \bar{\mathbf{5}}) + \mathbf{1}. \tag{1.19}$$

Thus the **27** contains one $SO(10)$ generation, but also some extra non-chiral D -like quarks and L -like leptons. This relative abundance of extra fields is a shortcoming of E_6 GUTs.

There are 78 gauge bosons decomposing under $SO(10)$ as $\mathbf{78} = \mathbf{45} + \mathbf{16} + \bar{\mathbf{16}} + \mathbf{1}$. The E_6 gauge symmetry may be broken down to the SM by Higgs fields in the adjoint and the **27**. The symmetry breaking may proceed also in various steps, for instance $E_6 \rightarrow SO(10) \times U(1)$, but also the breaking

$$E_6 \longrightarrow SU(3)_c \times SU(3)_L \times SU(3)_R \rightarrow G_{\text{SM}}. \tag{1.20}$$

This intermediate step, in which the QCD symmetry is on equal footing with left–right symmetries, is sometimes called *trinification*. An E_6 generation decomposes under (1.20) as $\mathbf{27} = (\mathbf{3}, \bar{\mathbf{3}}, \mathbf{1}) + (\bar{\mathbf{3}}, \mathbf{1}, \mathbf{3}) + (\mathbf{1}, \mathbf{3}, \bar{\mathbf{3}})$.

Although E_6 unification has in principle no compelling advantage over $SU(5)$ or $SO(10)$, it arises in a simple class of heterotic string compactifications, constructed in Section 7.3.2.

1.2.5 Flipped $SU(5)$ unification

There are proposals in which the unification gauge group is not simple. The most prominent examples are “flipped” $SU(5)$ models, based on a $SU(5) \times U(1)$ gauge group. In the breaking $SU(5) \times U(1)_X \rightarrow SU(3) \times SU(2) \times U(1)_Y$, the hypercharge generator is identified with the linear combination

$$Y = \frac{1}{5} \left(Q_X - \frac{Q_{Y'}}{6} \right), \tag{1.21}$$

where Y' is the diagonal $SU(5)$ generator, given by $Q_{Y'} = \text{diag}(-2, -2, -2, 3, 3)$ in the $\mathbf{5}$ representation. The flipped gauge group $SU(5) \times U(1)_X$ may be embedded into $SO(10)$.

One quark-lepton generation fits into the reducible structure

$$\mathbf{10}_1 = (Q_L, D_R, \nu_R), \quad \bar{\mathbf{5}}_{-3} = (U_R, L), \quad \mathbf{1}_5 = E_R, \tag{1.22}$$

where subindices denote $U(1)_X$ charges. The name “flipped” is due to the particle content of the representations, related to the standard $SU(5)$ by flipping $U_R \leftrightarrow D_R, E_R \leftrightarrow \nu_R$.

The symmetry breaking is obtained through scalars transforming as

$$\mathbf{5}_{-2} = (T, H_d), \quad \bar{\mathbf{5}}_2 = (\bar{T}, H_u); \quad \Phi_{\mathbf{10}_1}, \quad \Phi_{\bar{\mathbf{10}}_{-1}}, \tag{1.23}$$

where T, \bar{T} are color triplets with hypercharge $\pm 1/3$. Vevs for the $\mathbf{10}, \bar{\mathbf{10}}$ along the hypercharge neutral components trigger the breaking down to the SM group. The $\mathbf{5}, \bar{\mathbf{5}}$ contain the EW Higgs doublets, plus color triplets T, \bar{T} .

The gauge symmetries allow for three independent kinds of Yukawa couplings

$$L_{\text{Yuk}}^{F, SU(5)} = Y_D^{ij} \bar{\psi}_{\mathbf{10}_1}^i \psi_{\mathbf{10}_1}^j H_{\mathbf{5}_{-2}} + Y_U^{ij} \bar{\psi}_{\bar{\mathbf{5}}_{-3}}^i \psi_{\mathbf{10}_1}^j H_{\bar{\mathbf{5}}_2} + Y_L^{ij} \bar{\psi}_{\bar{\mathbf{5}}_{-3}}^i E_R^j H_{\mathbf{5}_{-2}} + \text{h.c.}$$

Hence flipped $SU(5)$ models imply no unification predictions for quark and lepton masses. Also, since the GUT group is not simple, there is in principle no unification of the hypercharge coupling with the remaining two. Despite these less appealing features, an advantage of flipped $SU(5)$ models is that they do not require large representations for GUT symmetry breaking. Since some large classes of string models cannot lead to adjoint scalars, flipped $SU(5)$ models provide an interesting possibility for unification in these string constructions.

1.2.6 Neutrino masses, seesaw mechanism, and GUTs

Non-zero neutrino masses require the introduction of additional ingredients beyond the minimal SM, namely right-handed neutrinos and/or a high scale of lepton number violation. At energies below the EW breaking scale, neutrinos are electrically neutral singlets.