Graph Algorithms, 2nd Edition

Shimon Even's *Graph Algorithms*, published in 1979, was a seminal introductory book on algorithms read by everyone engaged in the field. This thoroughly revised second edition, with a foreword by Richard M. Karp and notes by Andrew V. Goldberg, continues the exceptional presentation from the first edition and explains algorithms in formal but simple language with a direct and intuitive presentation.

The material covered by the book begins with basic material, including graphs and shortest paths, trees, depth-first search, and breadth-first search. The main part of the book is devoted to network flows and applications of network flows. The book ends with two chapters on planar graphs and on testing graph planarity.

SHIMON EVEN (1935–2004) was a pioneering researcher on graph algorithms and cryptography. He was a highly influential educator who played a major role in establishing computer science education in Israel at the Weizmann Institute and the Technion. He served as a source of professional inspiration and as a role model for generations of students and researchers. He is the author of *Algorithmic Combinatorics* (1973) and *Graph Algorithms* (1979).

Graph Algorithms

2nd Edition

SHIMON EVEN

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Foreword

In Appreciation of Shimon Even

Shimon was a great computer scientist who inspired generations of Israeli stutents and young researchers, including many future leaders of theoretical computer science.

He was a master at creating combinatorial algorithms, constructions, and proofs. He always sought the simplest and most lucid solutions. Because he never allowed himself to use a known theorem unless he understood its proof, his discoveries were often based on original methods. His lectures were legendary for their clarity.

Shimon was devoted to his family, generous to his colleagues, and freely available to the students in his classes.

He expressed his views forcefully and with complete honesty. He expected honesty in return, and reserved his disapproval for those who tried to obfuscate or mislead.

Shimon had an unending supply of interesting anecdotes, and would laugh uproariously at good jokes, including his own.

In sum, he was a great and unforgettable man and a great scientist, and his name has a permanent place in the annals of theoretical computer science.

Richard M. Karp
Berkeley, April 2011

Preface to the Second Edition

My father, Shimon Even, died on May 1, 2004. In the year prior to his illness, he began revising this book. He used to tell me with great satisfaction whenever he completed the revision of a chapter. To his surprise, he often discovered that, after twenty-five years, he preferred to present the material differently (the first edition was published in 1979). Unfortunately, he only managed to revise Chapters 1, 2, 3, and 5. These revised chapters appear in this edition. However, since the material in Chapters 9 and 10 on NP-completeness is well covered in a few other books, we decided to omit these chapters from the second edition. Therefore, the second edition contains only the first eight chapters.

As I was reading the manuscript for the second edition, my father's deep voice resonated clearly in my mind. Not only his voice, but also his passion for teaching, for elegant explanations, and, most importantly, for distilling the essence. As an exceptional teacher, he used his voice and his physique to reinforce his arguments. His smile revealed how happy he was to have the opportunity to tell newcomers about this wonderful topic. One cannot overvalue the power of such enthusiasm. Luckily, this enthusiasm is conveyed in this book.

Many people tell me (with a smile) about being introduced to the topic of algorithms through this book. I believe the source of their smiles is its outstanding balance between clarity and preciseness. When one writes mathematical text, it is very easy to get carried away with the desire to be precise. The written letter is long lasting, and being aware that one's text leaves certain gaps requires boldness. For my father this task was trivial. The audience he had in mind consisted simply of himself. He wrote as he would have wanted the material to be presented to him. This meant that he elaborated where he needed to, and he did not hesitate to skim over details he felt comfortable with. As a child, I recall seeing him prepare for class by reading a chapter from his book. I asked him:

"Why are you reading your *own* book? Presumably, you know what is there." "True," he replied, "but I don't remember!"

This second edition would have never been completed without Oded Goldreich. Oded Goldreich began by convincing me to prepare the second edition. Then he put me in touch with Lauren Cowles from Cambridge University Press. Finally, he continuously encouraged me to complete this project. It took almost seven years! There is no good excuse for it. We all know how such a task can be pushed aside by more "urgent" and "demanding" tasks. Apparently, it took me some time to realize how important this task was, and that it could not be completed without a coordinated effort. Only after I recruited Lotem Kaplan to do the typesetting of the unrevised chapters and complete the missing figure and index terms did this project begin to progress seriously. I am truly grateful to Oded for his insistence, to Lotem for her assistance, and to Lauren for her kind patience.

Finally, I wish to thank Richard M. Karp, an old friend of my father's, for his foreword. I also wish to thank Andrew Goldberg, the expert in network flow algorithms, for the notes he contributed in Chapter 5. These notes outline the major developments in the algorithms for maximum flow that have taken place since the first edition of this book was published.

Guy Even
Tel-Aviv, March 2011

Preface to the First Edition

Graph theory has long been recognized as one of the more useful mathematical subjects for the computer science student to master. The approach that is natural to computer science is the algorithmic one; our interest is not so much in the existence proofs or enumeration techniques as it is in finding efficient algorithms for solving relevant problems or, alternatively, in showing evidence that no such algorithm exists. Although algorithmic graph theory was started by Euler, if not earlier, its development in the last ten years has been dramatic and revolutionary. Much of the material in Chapters 3, 5, 6, 8, 9, and 10 is less than ten years old.

This book is meant to be a textbook for an upper-level undergraduate, or a graduate course. It is the result of my experience in teaching such a course numerous times, since 1967, at Harvard, the Weizmann Institute of Science, Tel-Aviv University, the University of California at Berkeley, and the Technion. There is more than enough material for a one-semester course; I am sure that most teachers will have to omit parts of the book. If the course is for undergraduates, Chapters 1 to 5 provide enough material, and even then, the teacher may choose to omit a few sections, such as 2.6, 2.7, 3.3, and 3.4.[1] Chapter 7 consists of classical nonalgorithmic studies of planar graphs, which are necessary in order to understand the tests of planarity, described in Chapter 8; it may be assigned as a preparatory reading assignment. The mathematical background needed for understanding Chapters 1 to 8 includes some knowledge of set theory, combinatorics, and algebra, which the computer science student usually masters during his freshman year through courses on discrete mathematics and on linear algebra. However, the student also needs to know something about data structures and programming techniques, or he may not

The first edition was published in 1979 (G.E.).

[1] Sections 2.6 and 2.7 were removed from the second edition by Shimon Even.

appreciate the algorithmic side or may miss the complexity considerations. It is my experience that after two courses in programming, students have the necessary knowledge. However, in order to follow Chapters 9 and 10,[2] additional background is necessary, namely, in theory of computation. Specifically, the student should know about Turing machines and Church's thesis.

The book is self-contained. No previous knowledge is needed beyond the general background just described. No comments such as "the rest of the proof is left to the reader" or "this is beyond the scope of this book" are ever made. Some unproved results are mentioned, with a reference, but are not used later in the book.

At the end of each chapter, there are a few problems teachers can use for homework assignments. The teacher is advised to use them discriminately, since some of them may be too hard for his students.

I would like to thank some of my past colleagues for our joint work and for the influence they have had on my work, and therefore on this book: I. Cederbaum, M. R. Garey, J. E. Hopcroft, R. M. Karp, A. Lempel, A. Pnuely, A. Shamir, and R. E. Tarjan. Also, I would like to thank some of my former Ph.D. students for all that I have learned from them: O. Kariv, A. Itai, Y. Perl, M. Rodeh, and Y. Shiloach. Finally, I would like to thank E. Horowitz for his continuing encouragement.

S.E., Techinion, Haifa, Israel

[2] Chapters 9 and 10 are not included in the second edition.