

## Nanoscale MOS Transistors

Written from an engineering standpoint, this book provides the theoretical background and physical insight needed to understand new and future developments in the modeling and design of n- and p-MOS nanoscale transistors. A wealth of applications, illustrations, and examples connect the methods described to all the latest issues in nanoscale MOSFET design. Key areas covered include:

- Transport in arbitrary crystal orientations and strain conditions, and new channel and gate stack materials;
- All the relevant transport regimes, ranging from low field mobility to quasi-ballistic transport, described using a single modeling framework;
- Predictive capabilities of device models, discussed with systematic comparisons to experimental results.

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Cover illustration: the images represent the k-space carrier distributions at the end of the channel of nanoscale n- and p-MOSFETs biased in the saturation region of operation.

Cambridge University Press  
978-0-521-51684-6 - Nanoscale MOS Transistors: Semi-Classical Transport and Applications  
David Esseni, Pierpaolo Palestri and Luca Selmi  
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“In this comprehensive text, physicists and electrical engineers will find a thorough treatment of semiclassical carrier transport in the context of nanoscale MOSFETs. With only a very basic background in mathematics, physics, and electronic devices, the authors lead readers to a state-of-the-art understanding of the advanced transport physics and simulation methods used to describe modern transistors.”

*Mark Lundstrom, Purdue University*

“This is the most pedagogical and comprehensive book in the field of CMOS device physics I have ever seen.”

*Thomas Skotnicki, STMicroelectronics*

“This is a modern and rigorous treatment of transport in advanced CMOS devices. The detailed and complete description of the models and the simulation techniques makes the book fully self sufficient.”

*Asen Asenov, University of Glasgow*

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Semi-Classical Transport and Applications

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CAMBRIDGE UNIVERSITY PRESS  
Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore,  
São Paulo, Delhi, Dubai, Tokyo, Mexico City  
  
Cambridge University Press  
The Edinburgh Building, Cambridge CB2 8RU, UK

Published in the United States of America by Cambridge University Press, New York

[www.cambridge.org](http://www.cambridge.org)  
Information on this title: [www.cambridge.org/9780521516846](http://www.cambridge.org/9780521516846)

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First published 2011

Printed in the United Kingdom at the University Press, Cambridge

*A catalog record for this publication is available from the British Library*

ISBN 978-0-521-51684-6 Hardback

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# Preface

The traditional geometrical scaling of the CMOS technologies has recently evolved in a generalized scaling scenario where material innovations for different intrinsic regions of MOS transistors as well as new device architectures are considered as the main routes toward further performance improvements. In this regard, high- $\kappa$  dielectrics are used to reduce the gate leakage with respect to the  $\text{SiO}_2$  for a given drive capacitance, while the on-current of the MOS transistors is improved by using strained silicon and possibly with the introduction of alternative channel materials. Moreover, the ultra-thin body Silicon-On-Insulator (SOI) device architecture shows an excellent scalability even with a very lightly doped silicon film, while non-planar FinFETs are also of particular interest, because they are a viable way to obtain double-gate SOI MOSFETs and to realize in the same fabrication process  $n$ -MOS and  $p$ -MOS devices with different crystal orientations.

Given the large number of technology options, physically based device simulations will play an important role in indicating the most promising strategies for forthcoming CMOS technologies. In particular, most of the device architecture and material options discussed above are expected to affect the performance of the transistors through the band structure and the scattering rates of the carriers in the device channel. Hence microscopic modeling is necessary in order to gain a physical insight and develop a quantitative description of the carrier transport in advanced CMOS technologies.

In this context, our book illustrates semi-classical transport modeling for both  $n$ -MOS and  $p$ -MOS transistors, extending from the theoretical foundations to the challenges and opportunities related to the most recent developments in nanometric CMOS technologies. Moreover, we describe relevant implementations of the semi-classical models which rely on the momentum relaxation time approximation and on the Monte Carlo approach for solution of the transport equations. The book aims at giving a description of the models that, without sacrificing the rigor of the treatment, can be accessible to both physicists and electronic engineers working in the electron device community. In this spirit, the selection of topics is driven by the innovations recently introduced in the semiconductor industry and by the trends in CMOS technology forecast by the International Roadmap for Semiconductors. Furthermore, since the CMOS technologies make inherently equal use of  $n$ -type and  $p$ -type MOSFETs, and because the physically based transport modeling is far more complicated for  $p$ -MOS than for  $n$ -MOS transistors, we describe the models for the two devices separately and in the same detail, thus avoiding

leaving the reader with the misleading impression that modeling of *p*-MOS devices is a trivial extension of the *n*-MOS case.

With respect to implementations, we have highlighted the multi-subband Monte Carlo approach because of some distinct features compared to other methods. These are its generality (with a suitable choice of boundary conditions all transport regimes can be explored, including the uniform and the non-uniform, the low field and the high field regimes), accuracy (the Boltzmann transport equation is solved without a-priori assumptions about the carrier distribution functions), modularity (new scattering mechanisms can be added without changing the core of the Monte Carlo solver) and completeness (all the scattering mechanisms claimed to be relevant for nanoscale MOSFETs can be accounted for).

As for the modeling methodologies alternative to the semi-classical approach illustrated in this book, quantum transport and its application to nanoscale MOSFETs has recently made important progress, especially thanks to the non-equilibrium Green's function formalism. However, we believe that semi-classical transport will remain for a long time the reference framework to understand the transport and support the design and innovation of MOS transistors, because it is an adequate approach for both uniform transport in long devices and strongly non-local, quasi-ballistic transport in nanoscale MOSFETs. These characteristics fit well with the path to innovation followed in the CMOS technologies, which typically starts from observation of possible improvements in low field mobility and then tries to translate them into enhancements of the on-current for nanoscale transistors.

At the time of writing, several alternative devices are being investigated as complements to the traditional MOSFETs, such as nanowires, carbon nano-tubes, graphene nano-ribbon transistors, and tunnel-FETs, to name a few. Nevertheless, we believe that devoting a book to nanoscale MOS transistors is a well defendable choice, because on the one hand in the foreseeable future none of the above devices is expected to replace MOSFETs for mainstream applications, and, furthermore, we know from experience that the semi-classical transport methodologies described in this book can be extended quite naturally also to devices with a different carrier gas dimensionality or with different channel materials.

Due to the volume of literature related to semi-classical transport in MOSFETs, the references included in the book could not be exhaustive. Rather, for each topic we have tried to include a selection of the most relevant journal papers, books and also papers presented at the leading conferences, which are frequently the most dynamic vehicles for introduction of the latest developments into the electron device community.

We wrote this book to serve as a reference for graduate student courses devoted to the theoretical foundations of, and recent developments in, carrier transport in nanoscale CMOS technologies, and also as a reference book for researchers and practitioners working in development and optimization of advanced MOS devices.

The prerequisite knowledge of physics for this book is limited to the basic concepts of classical electrostatics and electrodynamics, to the basic notions and methods of quantum mechanics and, in particular, to a familiarity with the Schrödinger equation and with the meaning of the corresponding eigenvalues and wave-functions. A previous

basic knowledge of the band structure in crystals would be useful for the reader, however, the second chapter aims at making the book self-contained also in this respect. The mathematical prerequisite knowledge is instead related to matrix algebra and to differential equations and differential eigenvalue problems. The book also assumes that the reader has a basic acquaintance with the working principle of semiconductor devices and, in particular, of MOS transistors.

The book was written to be as much as possible self-contained, so that most of the derivations are included in detail, also by resorting to appendixes in the cases where we thought that they resulted in too long a digression from the main flow of the discussion. The availability of the derivations allows the reader to trace back the origin and understand the validity limits of some results which may be very widely quoted and used in the literature but not as often fully justified and explained.

Essentially all the models described in the book have been implemented by the authors in benchmark codes or in complete simulators, so that it has been possible to include many simulation results in order either to clarify some theoretical aspects or to exemplify the insight provided by the models in practically relevant case studies.

David Esseni  
Pierpaolo Palestri  
Luca Selmi

# Acknowledgements

Many people contributed to this book and to the work which is behind it. Among them, we would like to express our sincere gratitude to M.De Michielis, F.Conzatti, N.Serra, P.Toniutti, L.Lucci, Q.Raphay, and M.Iellina for their contributions to the development of the simulation tools used to obtain many of the results included in the book, for their help in producing some of the figures and also for their careful reading of the manuscript. M.Bresciani, A.Cristofoli, A.Paussa, M.Panozzo, and E.Beaudoin helped us with the bibliographic entries in order to make the style of the references uniform throughout the book and also with editing some of the figures.

We are also in debt to our colleagues F.Driussi, A.Gambi, and P.Gardonio for the critical reading of some sections of the book, that was really invaluable for correcting mistakes and improving the text clarity.

This work has benefited substantially from interactions with colleagues with whom we have had a fruitful and stimulating collaboration over the years; among them, we would like to thank E.Sangiorgi, A.Abramo, C.Fiegna, and R.Clerc.

Our special thanks go also to J.Lancashire and S.Matthews at Cambridge University Press for following the progress of our work in all its phases, and to S.Tahir for support with all the LaTeX related troubles that inevitably occurred during the writing.

The understanding of our families for our devoting to this project much of our supposedly free time during the last two years has been at least as necessary as all the previously mentioned contributions in making possible the completion of the writing. To our families we gratefully dedicate this book.

David Esseni  
Pierpaolo Palestri  
Luca Selmi

# Terminology

## Abbreviations and acronyms

BTE	Boltzmann transport equation
DG	Double gate
DIBL	Drain induced barrier lowering
DoS	Density of states
EMA	Effective mass approximation
EOT	Equivalent oxide thickness
EPM	Empirical pseudo-potential method
ITRS	International technology roadmap for semiconductors
MC	Monte Carlo
MOS	Metal-oxide-semiconductor
MOSFET	MOS field effect transistor
CMOS	Complementary metal-oxide-semiconductor
MSMC	Multi-subband Monte Carlo
MRT	Momentum relaxation time
SG	Single gate
SOI	Silicon on insulator
SS	Subthreshold swing
TCAD	Technology computer-aided design
VLSI	Very large scale integration
VS	Virtual source

## Notation

$x$	Scalar
$x^\dagger$	Complex conjugate of the scalar $x$
$x + (c.c)$	A scalar plus the complex conjugate, namely $(x + x^\dagger)$
$\mathbf{x}$	Vector, matrix or multi-dimensional tensor
$x_{ij}$	Element of the matrix $\mathbf{x}$
$\mathbf{x}^T$	Transpose of the vector or matrix $\mathbf{x}$
$\mathbf{x}^\dagger$	Transpose conjugate of the vector or matrix $\mathbf{x}$
$\mathbf{x} \cdot \mathbf{y}$	Scalar product between vectors $\mathbf{x}$ and $\mathbf{y}$

$\hat{\mathbf{e}}_x, \hat{\mathbf{x}}, \hat{\mathbf{e}}_y, \hat{\mathbf{y}}, \hat{\mathbf{e}}_z, \hat{\mathbf{z}}$	Unit vectors along the direction $x$ , $y$ and $z$
$\hat{H}$	Operator: typically consisting of a differential and an algebraic part
$H_v(x)$	Heaviside function: 0 for negative $x$ values and 1 otherwise
$F\{f(x)\} = F(q)$	Fourier transform of the function $f(x)$
$(f*g)(x)$	Convolution of the functions $f(x)$ and $g(x)$
$\nabla$ or $\nabla_{\mathbf{R}}$	Gradient with respect to real space three-dimensional coordinates $\mathbf{R}$
$\nabla$ or $\nabla_{\mathbf{r}}$	Gradient with respect to real space two-dimensional coordinates $\mathbf{r}$
$\nabla_{\mathbf{K}}$ or $\nabla_{\mathbf{k}}$	Gradient with respect to wave-vectors $\mathbf{K}$ or $\mathbf{k}$
$[hkl]$	Miller indices that specify a crystal direction
$\langle hkl \rangle$	Miller indices that specify equivalent crystal directions
$(hkl)$	Miller indices that specify the crystal plane normal to $[hkl]$
$\{hkl\}$	Miller indices that specify the equivalent crystal planes normal to $\langle hkl \rangle$

Symbols:

$a_0$	Direct lattice constant of a crystal	m
$E_F$	Fermi level	J
$g(E)$	Density of the states for a $d$ dimensional carrier gas	$\text{m}^{-d} \text{J}^{-1}$
$n_{sp}$	Spin degeneracy factor: can be either 1 or 2	unitless
$\mathbf{F}$	Electric field	$\text{V m}^{-1}$
$F_x, F_y, F_z$	Electric field components in the $x$ , $y$ and $z$ direction	$\text{V m}^{-1}$
$F_{eff}$	Effective electrical field in an inversion layer	$\text{V m}^{-1}$
$\mathbb{F}$	Driving force for carrier motion	Newton
$\mathbf{V}_g, \mathbf{v}_g$	Group velocity for a 3D or a 2D carrier gas	$\text{m s}^{-1}$
$m_x, m_y, m_z$	Effective electron masses in the $x$ , $y$ and $z$ direction	kg
$\Omega$	Normalization volume	$\text{m}^3$
$A$	Normalization area	$\text{m}^2$
$\phi$	Electrostatic potential	V
$U$	Potential energy	J
$T$	Temperature	K
$V$	Voltage at device terminals	V
$V_{GS}$	Intrinsic terminal voltage difference from gate to source	V
$V_{DS}$	Intrinsic terminal voltage difference from drain to source	V
$L_G$	Gate length	m
$I_{ON}$	Drain current per unit width at $ V_{GS}  =  V_{DS}  = V_{DD}$	A/m
$I_{OFF}$	Drain current per unit width at $V_{GS} = 0,  V_{DS}  = V_{DD}$	A/m
$t_{ox}$	Physical oxide thickness	m
$N_{inv}$	Electron inversion layer density	$\text{m}^{-2}$
$P_{inv}$	Hole inversion layer density	$\text{m}^{-2}$
$N^+$	Inversion density of carriers moving from source to drain	$\text{m}^{-2}$
$v^+$	Average velocity of carriers moving from source to drain	m/s
$N^-$	Inversion density of carriers moving from drain to source	$\text{m}^{-2}$



Terminology

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$v^-$	Average velocity of carriers moving from drain to source	m/s
$v_{sat}$	Saturation velocity	m/s
$r$	Back-scattering coefficient	unitless

Physical constants

$h$	Planck’s constant	$6.626075 \times 10^{-34}$ Js
$\hbar$	Reduced Planck’s constant	$h/(2\pi)$
$K_B$	Boltzmann’s constant	$1.380662 \times 10^{-23}$ JK <sup>-1</sup>
$e$	Positive electron charge	$1.602189 \times 10^{-19}$ C
$m_0$	Electron rest mass	$9.109390 \times 10^{-31}$ kg
$\epsilon_0$	Dielectric constant of vacuum	$8.854188 \times 10^{-12}$ CV <sup>-1</sup> m <sup>-1</sup>