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978-0-521-51605-1 - Disposing Dictators, Demystifying Voting Paradoxes: Social Choice Analysis

Donald G. Saari

Excerpt

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## ONE

## Subtle Complexity of Social Choice

1.1 Does *Everything* Go Wrong?

Remember your last important election? Maybe it was to select the chair of a department, business unit, or social group. Maybe it was to determine who to hire, what alternative to select, which material to use in a construction project, or where to locate a new plant. Maybe it was in a presidential primary. Were you happy? It is not uncommon to be disappointed with the outcome and complain that the wrong choice won. A natural response is to dismiss such complaints as sour grapes: “Get over it already! You lost!” But surprisingly often the “wrong person” did win. The mathematical study of whether decision and election rules elect “whom the voters really want” is called *social choice*.

For an outsider, the area of social choice can leave the impression of a mysterious subject discouragingly consumed with disturbing voting paradoxes. These examples of voting inconsistencies, which permeate the literature, produce realistic worries about whether we might elect someone whom the voters really do not want. It is worth worrying about this fear because, in fact, surprisingly often that is precisely what happens.

Even more bothersome is the fact that this disease where societal outcomes can flaunt voters’ wishes appears to have reached the epidemic level by afflicting *all* conceivable voting rules. Worse news comes from the seminal Arrow’s and Sen’s theorems, which are introduced in Chapter 2. These theorems state that it is impossible to do what seems to be quite natural to do. There is also a severe language barrier where many of these published articles, which seemingly promise darker and deeper levels

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of dismal assertions and conclusions, are described in dense technical terms that even a mathematician can find difficult to parse. I know; I am a mathematician.

But the news isn't all bad. In recent years, positive conclusions have been discovered, while negative assertions have been put to rest or placed in perspective. In this book, I will put a cheerier tone on central issues in the field by replacing gloom with some "good news."

I do so by providing new perspectives for essential difficulties in this area. This includes addressing those troubling dictators and ubiquitous voting paradoxes by explaining what causes them. The explanations range from identifying the source of Arrow's discouraging result about a dictator and the cause of Sen's result, which asserts that there can be a fundamental conflict between societal needs and individual liberties, to all of those voting paradoxes that professionals in this field have learned to love. We should love them; they keep us employed.

### *What "Subtlety?"*

There does not seem to be anything subtle or complex about election rules. For some methods, such as the standard plurality vote where each person votes for a favorite candidate, simple counting is about the heaviest mathematics required to tally ballots. Even children in a kindergarten class can handle a "show-of-hands" – nothing complex about this.

As we have learned over the past two centuries, voting rules, including the plurality vote (also called "vote-for-one" or, in Europe, "first past the post"), are far from being simple or transparent. Instead, as frustrated voters in many countries may wonder in the aftermath of almost any election season, and as experts in this academic field have known since the work of Jean-Charles de Borda during the pre-revolutionary days of eighteenth-century France, so many things can go wrong with elections that we must worry whether election results accurately reflect the voters' beliefs. The complexity of voting rules, then, does not derive from the definitions or implementation of the rules but from their subtle discomfiting consequences.

Mind you, I am not talking about those widely discussed problems caused by malfeasance of election officials – actions explicitly directed

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toward “stealing” the election. Instead, I am referring to unexpected consequences caused by hidden properties of our standard and widely used election rules. Bluntly stated, even with the idealistic assumption that all procedural aspects of an election are honest and carefully followed, the choice of an election rule can seriously distort the outcome away from what arguably is the “true choice of the voters.” To illustrate with widely discussed contemporary examples, did George W. Bush’s victory over Albert Gore in the 2000 U.S. presidential election accurately reflect what the American, or even Floridian, voters wanted? I do not think so. Did the French electorate truly respect Jean-Marie Le Pen enough to justify advancing him to the runoffs in the 2002 French elections? I doubt it. These worrisome kinds of problems are discussed here.

## 1.2 And the Proud Father Is . . .

The field has a delightfully interesting history spiced with conflict thrown into the mix. Even though concerns about the choice of an appropriate voting rule can be traced back to the earliest of times, the academic pursuit started when Jean-Charles de Borda [6] worried in his June 1770 presentation whether the “wrong people” were being elected to the Paris Académie des Sciences.<sup>1</sup> With an insightful, explicit example, Borda proved that the academy’s election method allowed them to elect individuals who, in fact, the voters collectively viewed as being *inferior*. What a serious indictment!

What was this problematic election rule that allowed inferior conclusions? The culprit was the *plurality vote*, which is widely used across the world; it is the rule responsible for the questionable outcomes in the earlier mentioned American and French elections. Borda then introduced a voting rule (assign 2, 1, 0 points, respectively, to candidates who are positioned first, second, and third on a ballot) to resolve the difficulty. His “Borda Count” rule *does* solve the problem; at least, it works for examples of the type that he described.

By planting seeds of doubt about standard voting rules and by calling academic attention to this issue, Borda rightfully is the *Father of Social*

<sup>1</sup> The academy was abolished in 1793 during the Reign of Terror and later reestablished in the Institut de France.

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*Choice*; without question, his insights and presentation initiated this academic field. With that said, however, I am left with the feeling that when he delivered his 1770 address, Borda viewed voting theory as a side fling where his objective was not to start a new academic area, which he did, but to correct a troubling peculiarity afflicting the quality of newly chosen academy members. Perhaps recently elected classes of academy members failed to match expected standards, and this slip in quality provoked Borda into searching for the cause and a cure.

To place his contribution in a proper perspective, all of us, at one time or another, have worried about flaws within organizations in which we are members. To redress the problem, we explain to our colleagues what is wrong while suggesting ways to correct the deficiency. It might involve, for instance, inefficiencies in assigning students to classes or evaluating graduate students for advancement to Ph.D. candidacy. We point out the problem, argue for change, and then move on to what we view as our real work.

At least initially, this probably was the level of intensity that Borda assigned to the voting question.<sup>2</sup> He identified a serious defect in the academy's election rule, made his insightful change now called the Borda Count, explained why it was an improvement, and then moved on to his academic pursuits. After all, Borda, a mathematician and one of France's most notable experimental physicists, was more interested in the mathematics of astronomy and fluid dynamics and in his central role in creating

<sup>2</sup> Borda returned to this topic after issues and challenges were raised by Condorcet. But Borda's efforts were directed toward hydrodynamics, mathematical physics (the Borda harp remains of interest within partial differential equations), and the mathematics of astronomy (e.g., the research that resulted in his election to the French Académie des Sciences in 1753 at the age of 23 involved the behavior of projectiles). With his use of calculus and experimental methods, he helped to unify areas of mathematical physics. His "repeating circle" device, which had profound nautical applications because of its significant accuracy, was used to define the length of a meter! This distinguished man also played a central role in the establishment of the famous Bureau des Longitudes in Paris serving as its first president. As a side-note indicating the historical importance of this bureau, today the Prime Meridian passes through the original site of the Royal Observatory in Greenwich England. Before 1884, however, navigation was complicated by several choices – the French one was defined by a line embedded in the second floor of the Bureau des Longitudes and passing through the center of the clock on the wall of the Palais du Luxembourg that overlooks the outdoor pool.

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Author pointing to the French “Prime Meridian” in the astronomy institute founded by Borda.

the metric system: Some research issues he raised then are still studied today.

My interpretation is consistent with that of Duncan Black, who writes in his classic book [7] *The Theory of Committees and Elections*:

The initial step [to develop a theory for voting] was taken by Borda, who . . . [had] achieved distinction as a mathematician and had for the centre of his life the Academy of Sciences. It was no doubt elections to the Academy, membership of which was for him the most valuable of all privileges, and not the wider problems of politics that first directed his mind to the theory. In it he showed the same eye for the significant fact and for the simplifying assumption as in his other researches, and he broached the subject in a new way. His work has the robust good sense of the practical man.

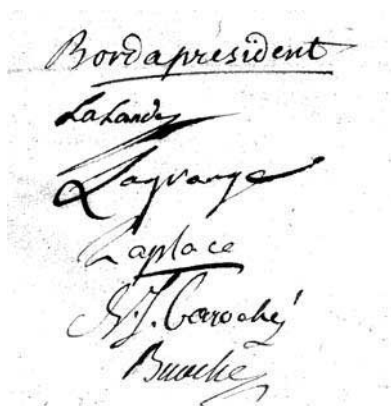
Although Borda’s contributions to voting remain central to this field, a rough measure of the relative insignificance that Borda might have placed on them compared to his many other mathematical and scientific contributions is that in Marcart’s [31] 636-page biography of Borda,

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The signatures of Borda, as president, and other notable mathematicians and astronomers on a document establishing the Bureau des Longitudes in Paris.

only 7 pages are devoted to Borda's interest in and contributions to voting theory.

Borda was a surprisingly well-rounded, productive researcher who was active in several scientific pursuits as well as a recognized naval officer. In recognition of his leadership of a fleet of six ships involved in the American Revolutionary War and winning notable victories, Borda was made a member of the American "Order of Cincinnatus." Borda's contributions include being a recognized military figure and a renowned experimental physicist, establishing the definition of the meter, creating the Bureau des Longitudes in Paris, discovering several mathematical results in hydrodynamics, initiating the study of voting rules, and on and on. His accomplishments were certainly a nice start for a curriculum vitae!

For voting theory, it was the Marquis de Condorcet [12] who nurtured and raised the child sired by Borda. Condorcet moved beyond addressing the voting problems facing the academy to initiate a general theory for voting rules. His masterful work is, and deserves to be, studied today. Without Condorcet, the development of a theory for voting most surely would have been delayed for at least another century. Combining Borda's fleeting affair with social choice – a fling with unexpected and lasting consequences – with Condorcet's more fatherly devotion, it is



Two streets in Paris; a concrete recognition of the founders of social choice.

appropriate to designate both gentlemen as the founding fathers of this area.

### 1.3 Enemies?

Condorcet was a major figure in French history whose many contributions were honored in 1989 when his remains<sup>3</sup> were moved into the Panthéon in Paris. His academic accomplishments, intellectual friendship with Thomas Jefferson, leadership role in the French Revolution, service to the French Academy, and deep contributions to social choice made him a major figure.

Let me offer a short, incomplete list of these contributions: Four score years before the American Civil War, Condorcet wrote persuasively against slavery and the slave trade. He took strong stands on issues being contested today, such as his opposition to the death penalty and his support for the equality of rights for women and the Jewish. During those days it was necessary to fight to protect Protestants, and he did so. As if this were not enough, he was the founder of the Condorcet jury theorem, a version of the central limit result, and the founder of a more general investigation of “social mathematics.” A particularly insightful message of his is that selecting the appropriate voting rule is just one step:

<sup>3</sup> As Maurice Salles reminded me, his “remains” were symbolically moved to the Panthéon; it is unknown where his actual remains are located or if they even exist.

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The rule is a tool that can be useless if not accompanied by a civilized and informed discourse. He was an incredible, courageous, perceptive man!

Perhaps of interest only for those in the area of social choice are the persistent assertions that Condorcet viewed Borda as an enemy. An enemy? Left unexplained, these comments resemble a meal without wine: Something of significance is missing. We want to know why, so it is worth exploring this issue.

From what I learned, this “enemy” notion was merely an irrelevant passing stage reflecting Condorcet’s insecurity and immaturity at a younger age exacerbated by political battles and a resource allocation problem – affairs that rarely bring out the best of anyone. Problems seemed to dissipate after Condorcet gained confidence in his leadership roles.<sup>4</sup>

As I write this, I have an amusing thought. Condorcet was elected to the Paris Académie des Sciences in 1769; in June of the next year, 1770, Borda explained to the academy how their voting rule could elect inferior choices. This timing forces one to wonder whether Borda was referring to the “inferiority” of Condorcet’s class of academy members. Did Condorcet wonder whether Borda was pointing to him? If so, this would explain Condorcet’s rumored enmity toward Borda. We may never know, but I wonder.

Let me start my commentary with an exemplar of mathematicians’ self-deprecating sense of humor – a joke widely circulated within our profession:

How can you tell which mathematician is an extrovert?

The answer:

The one who looks at *your* shoes when talking to you.

Converting this joke into a measure of social behavior, it appears that during his younger years and the early part of his career (when the “enemy” comments occurred) Condorcet was a world-class introvert striving to overcome a warped upbringing. To learn more about this, I highly

<sup>4</sup> Keith M. Baker, the author of *Condorcet* [5], suggested in an email exchange that this statement is consistent with his understanding of the situation.



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recommend Baker's fascinating book *Condorcet* [5], which is a principal reference for this man of great importance to French history. In Baker's book we learn (p. 3) how Condorcet's twice-widowed mother "sought to preserve herself from further loss by smothering the child in a mantle of piety. Dedicated for his protection to the virgin (so the tradition goes), he was kept in white dresses until the age of eight."

*White Dresses and Other Humiliations*

White dresses until the age of 8! This humiliation brings to mind the Shel Silverstein song, recorded in 1969 by Johnny Cash, entitled "A Boy Named Sue." As the daddy who left home explains years later, right after the son finally finds him and they have a knockdown physical brawl to exact revenge over his given name of Sue, the father knew he would not be around,

So I give ya that name and I said goodbye  
I knew you'd have to get tough or die  
And it's the name that helped to make you strong

It is easy to appreciate the consequences of Condorcet's white dress experience. Perhaps out of necessity, like the boy named Sue, he would develop street-brawling skills. In the physical sense of good ole fisticuffs, this did not occur, but in the sense of verbally and intellectually taking on opponents, this bare-knuckled trait served the feisty politician Condorcet quite well during the French revolutionary years. Even earlier, Mme Suard remarked (Baker [5], p. 5) that

Between the malice of his mind and the goodness of his heart, there was a contrast that I always found singularly striking. . . . His intolerance in matters of political opinion was incredible.

One must wonder whether Condorcet, who was carrying all of this emotional baggage, strived to dismiss lessons carefully learned at his mother's knee.

It got worse: Condorcet's Jesuit education did not reward him with happy years. Read Baker's description ([5], p. 3) of Condorcet's thoughts:

Among the Caribs [according to Condorcet], it was the customary practice to render newborn children completely stupid by flattening their heads between two boards. The mongols relieved themselves of the fear that a prince of the blood

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might foment trouble by the application of a narcotic potion producing imbecility by degrees. . . . Of all the known methods for reducing man to the intellectual level of the beast, however, Condorcet regarded as the surest that . . . [used by the monks]. A moral education fit to make debauched and hypocritical atheists or fanatically bigoted imbeciles; a philosophical education comprised of scholastic jargon and theological dreams; a closed educational environment calculated to foster and perpetuate the adolescent tendency to homosexuality; these were the principal aspects of his education at the hands of the Jesuits that Condorcet remembered at the age of thirty.

Wow! This description trumps all complaints I have ever heard about bad educational and scar-producing childhood experiences. It is easy to appreciate why Condorcet's later mentors strived to make him socially more acceptable. The year he was elected to the Paris Académie des Sciences (1769), Mlle de Lespinasse still labored

to repair the defects of Jesuit education by schooling [Condorcet] in the social graces. . . . Condorcet was admonished to leave off biting his nails and gnawing his lips in company; to refrain from folding himself in two while talking, . . . , to keep his ears free of chalk and his hair cut less close to his head; to leaven the madness of his long days of study with some cultivation of the science of love. ([5], p. 23)

Presumably Condorcet was also carefully coached to look at the other person's shoes during conversations.

### *Enemies and Money*

Take a talented, brilliant, quick-tempered young man with limited social graces, no history of tolerance, and a "malice of mind," mix in a fight over scarce resources, and what would you expect? It occurred, and it might explain Condorcet's "enemy" attitude toward Borda. Again, I encourage you to read Baker's book ([5], pp. 35–47). For a brief synopsis, the strategic machinations of d'Alembert and Condorcet with the goal of making Condorcet the academy leader required a significant pot of money. They discovered one – 12,000 livres designated to support experimental research. Could they divert this cache of cash for their purposes? Coconspirator Turgot used the power of his office of controller-general to redirect almost half of this money, 5,000 livres, to Condorcet.

Imagine the reaction when money dedicated for academic research projects suddenly is diverted to an administrator's pay. It happened;