This book deals with the elastic stability of solids and structures, for which Warner Koiter was the world’s leading expert of his time. It begins with fundamental aspects of stability, relating the basic notions of dynamic stability to more traditional quasi-static approaches. The book is concerned not only with buckling, or linear instability, but most importantly with nonlinear postbuckling behavior and imperfection sensitivity. After laying out the general theory, Koiter applies the theory to a number of applications, with a chapter devoted to each. These include a variety of beam, plate, and shell structural problems and some basic continuum elasticity problems. Koiter’s classic results on the nonlinear buckling and imperfection sensitivity of cylindrical and spherical shells are included. The treatments of both the fundamental aspects and the applications are completely self-contained. This book was recorded as a detailed set of notes by Arnold van der Heijden from W. T. Koiter’s last set of lectures on stability theory at TU Delft.

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These lecture notes were made after Professor Koiter’s last official course at Delft’s University of Technology, in the academic year 1978–79. Although these notes were prepared in close collaboration with Professor Koiter, they are written in the author’s style. The author is therefore fully responsible for possible errors.

This course covers the entire field of elastic stability, although recent developments in the field of stiffened plates and shells are not included. Hopefully, these lecture notes reflect some of the atmosphere of Dr. Koiter’s unique lectures.

Delft, June 10, 2008

A. M. A. v. d. Heijden