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Subir Sachdev
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Quantum Phase Transitions

Second Edition

This is the first book to describe the physical properties of quantum materials near critical points with long-range many-body quantum entanglement. Readers are introduced to the basic theory of quantum phases, their phase transitions, and their observable properties.

This second edition begins with nine chapters, six of them new, suitable for an introductory course on quantum phase transitions, assuming no prior knowledge of quantum field theory. There are several new chapters covering important recent advances, such as the Fermi gas near unitarity, Dirac fermions, Fermi liquids and their phase transitions, quantum magnetism, and solvable models obtained from string theory. After introducing the basic theory, it moves on to a detailed description of the canonical quantum-critical phase diagram at nonzero temperatures. Finally, a variety of more complex models is explored. This book is ideal for graduate students and researchers in condensed matter physics and particle and string theory.

Subir Sachdev is Professor of Physics at Harvard University and holds a Distinguished Research Chair at the Perimeter Institute for Theoretical Physics. His research has focused on a variety of quantum materials, and especially on their quantum phase transitions.

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Second Edition

SUBIR SACHDEV
Harvard University

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To my parents and Menaka, Monisha, and Usha

Contents

From the Preface to the first edition

The past decade has seen a substantial rejuvenation of interest in the study of quantum phase transitions, driven by experiments on cuprate superconductors, heavy fermion materials, organic conductors, and related compounds. Although quantum phase transitions in simple spin systems, like the Ising model in a transverse field, were studied in the early 1970s, much of the subsequent theoretical work examined a particular example: the metal–insulator transition. While this is a subject of considerable experimental importance, the greatest theoretical progress was made for the case of the Anderson transition of noninteracting electrons, which is driven by localization of the electronic states in the presence of a random potential. The critical properties of this transition of noninteracting electrons constituted the primary basis upon which most condensed matter physicists have formed their intuition on the behavior of the systems near a quantum phase transition. However, it is clear that strong electronic interactions play a crucial role in the systems of current interest noted earlier, and simple paradigms for the behavior of such systems near quantum critical points are not widely known.

It is the purpose of this book to move interactions to center stage by describing and classifying the physical properties of the simplest interacting systems undergoing a quantum phase transition. The effects of disorder will be neglected for the most part but will be considered in the concluding chapters. Our focus will be on the dynamical properties of such systems at nonzero temperature, and it will become apparent that these differ substantially from the noninteracting case. We shall also be considering inelastic collision-dominated quantum dynamics and transport: our results will apply to clean physical systems whose inelastic scattering time is much shorter than their disorder-induced elastic scattering time. This is the converse of the usual theoretical situation in Anderson localization or mesoscopic system theory, where inelastic collision times are conventionally taken to be much larger than all other timescales.

One of the most interesting and significant regimes of the systems we shall study is one in which the inelastic scattering and phase coherence times are of order $\hbar/k_B T$, where T is the absolute temperature. The importance of such a regime was pointed out by Varma *et al.* [523, 524] by an analysis of transport and optical data on the cuprate superconductors. Neutron scattering measurements of Hayden *et al.* [210] and Keimer *et al.* [263] also supported such an interpretation in the low doping region. It was subsequently realized [86, 419, 440] that the inelastic rates are in fact a *universal number* times $k_B T/\hbar$, and they are a robust property of the high-temperature limit of renormalizable, interacting quantum field theories that are not asymptotically free at high energies. In the Wilsonian picture, such a field theory is defined by renormalization group flows away from a critical point describing a second-order quantum phase transition. It is not essential for this critical point to be in an experimentally accessible regime of the phase diagram: the quantum field theory it defines may still be an appropriate description of the physics over a substantial intermediate energy and temperature scale. Among the implications of such an interpretation of the experiments was the requirement that response functions should have prefactors of anomalous powers of T and a singular dependence on the wavevector; recent observations of Aeppli *et al.* [5], at somewhat higher dopings, appear to be consistent with this. These recent experiments also suggest that the appropriate quantum critical points involve competition between phases with or without conventional superconducting, spin-, or charge-density-wave order. There is no global theory yet for such quantum transitions, but we shall discuss numerous simpler models here that capture some of the basic features.

It is also appropriate to note here theoretical studies [25, 93, 94, 336, 514] on the relevance of finite temperature crossovers near quantum critical points of Fermi liquids [218] to the physics of heavy fermion compounds.

A separate motivation for the study of quantum phase transitions is simply the value in having another perspective on the physics of an interacting many-body system. A traditional analysis of such a system would begin from either a weak-coupling Hamiltonian, and then build in interactions among the nearly free excitations, or a strong-coupling limit, where the local interactions are well accounted for, but their coherent propagation through the system is not fully described. In contrast, a quantum critical point begins from an intermediate coupling regime, which straddles these limiting cases. One can then use the powerful technology of scaling to set up a systematic expansion of physical properties away from the special critical point. For many low-dimensional strongly correlated systems, I believe that such an approach holds the most promise for a comprehensive understanding. Many of the vexing open problems are related to phenomena at intermediate temperatures, and this is precisely the region over which the influence of a quantum critical point is dominant. Related motivations for the study of quantum phase transitions appear in a recent discourse by Laughlin [286].

The particular quantum phase transitions that are examined in this book are undoubtedly heavily influenced by my own research. However, I believe that my choices can also be justified on pedagogical grounds and lead to a logical development of the main physical concepts in the simplest possible contexts. Throughout, I have also attempted to provide experimental motivations for the models considered; this is mainly in the form of a guide to the literature, rather than in-depth discussion of the experimental issues. I have highlighted some especially interesting experiments in a recent popular introduction to quantum phase transitions [428]. An experimentally oriented introduction to the subject of quantum phase transitions can also be found in the excellent review article of Sondhi, Girvin, Carini, and Shahar [481]. Readers may also be interested in a recent introductory article [533], intended for a general science audience.

Acknowledgments

Chapter 21 was co-authored with T. Senthil and adapted from his 1997 Yale University Ph.D. thesis; I am grateful to him for agreeing to this arrangement.

Some portions of this book grew out of lectures and write-ups I prepared for schools and conferences in Trieste, Italy [418], Xiamen, China [419], Madrid, Spain [421], Geilo, Norway [424], and Seoul, Korea [427]. I am obliged to Professors Yu Lu, S. Lundqvist, G. Morandi, Hao Bai-Lin, German Sierra, Miguel Martin-Delgado, Arne Skjeltorp, David Sherrington, Jisoon Ihm, Yunkyu Bang, and Jaejun Yu for the opportunities to present these lectures. I also taught two graduate courses at Yale University and a mini-course at the Université Joseph Fourier, Grenoble, France on topics discussed in this book; I thank both institutions for arranging and supporting these courses. I am indebted to the participants and students at these lectures for stimulating discussions, valuable feedback, and their interest. Part of this book was written during a sojourn at the Laboratoire des Champs Magnétiques Intenses in Grenoble, and I thank Professors Claude Berthier and Benoy Chakraverty for their hospitality. My research has been supported by grants from the Division of Materials Research of the U.S. National Science Foundation.

I have been fortunate in having the benefit of interactions and collaborations with numerous colleagues and students who have generously shared insights that appear in many of these pages. I would particularly like to thank my collaborators Chiranjeeb Buragohain, Andrey Chubukov, Kedar Damle, Sankar Das Sarma, Antoine Georges, Ilya Gruzberg, Satya Majumdar, Reinhold Oppermann, Nick Read, R. Shankar, T. Senthil, Sasha Sokol, Matthias Troyer, Jinwu Ye, Peter Young, and Lian Zheng.

The evolution of the book owes a great deal to comments of readers of earlier versions, who unselfishly donated their time in working through unpolished drafts; naturally, they bear no responsibility for the remaining errors and obscurities. I am most grateful to Sudip Chakravarty, Andrey Chubukov, Kedar Damle, Ilya Gruzberg, Sankar Das Sarma, Bert Halperin, T. Senthil, R. Shankar, Oleg Starykh, Chandra Varma, Peter Young, Jan Zaanen, and two anonymous referees. The detailed comments provided by Steve Girvin and Wim van Saarloos were especially valuable. My thanks to them, and the others, accompany an admiration for their generous collegial spirit. I also acknowledge salutary encouragement from Jan Zaanen.

My wife, Usha, and my daughters, Monisha and Menaka, patiently tolerated my mental and physical absences during the writing (and rewritings) of this book. Ultimately, it was their cheerful support that made the project possible and worthwhile.

Preface to the second edition

Research on quantum phase transitions has undergone a vast expansion since the publication of the first edition, over a decade ago. Many new theoretical ideas have emerged, and the arena of experimental systems has grown rapidly. The cuprates have been firmly established to be d-wave superconductors, with a massless Dirac spectrum for their electronic excitations; the latter spectrum has also been observed in graphene and on the surface of topological insulators. Such fermions play a key role in a variety of quantum phase transitions. The observation of quantum oscillations in the presence of strong magnetic fields in the underdoped cuprates has highlighted the relevance of competing orders, and their quantum critical points. Optical lattices of ultracold atoms now offer a realization of the boson Hubbard model, and exhibit the superfluid–insulator transition. And ideas on quantum criticality and entanglement have had an interesting interplay with developments in quantum information science.

The second edition does not present a fully comprehensive survey of these ongoing developments. I believe the core topics of the first edition had a certain coherence, and they continue to be central to the more modern developments; I did not wish to dilute the global perspective they offer in understanding both condensed matter and ultracold atom experiments. However, wherever possible, I have discussed important advances, or directed the reader to review articles.

Also, in the last few years, a remarkable connection has developed between ideas on quantum criticality and the string theory of quantum black holes. I briefly survey the initial developments in Section 15.5. The subject has advanced rapidly since then, with interesting applications to quantum critical states of fermions at nonzero density: this recent work is not discussed here. In any case, this book should be useful background reading for this emerging and growing field of research.

The primary change in the second edition is pedagogical. I have had the benefit of teaching a course on quantum phase transitions several times since the first edition, both at Yale and at Harvard. I am also grateful for the opportunity to lecture at various summer and winter schools (Altenberg, Boulder, Cargese, Goa, Groningen, Jerusalem, Les Houches, Mahabaleshwar, Milos, Prague, Trieste, Windsor). The content of these lectures is now in the new Part II of the book. Chapters 3–8 are new, although they do extract some material from the earlier chapters of the first edition. Part II, titled “A first course,” is intended for a stand-alone course on the basic theory of quantum phase transitions, and for self-study. It should be accessible to students in both theory and experiment, after they have taken the core graduate courses on quantum mechanics and statistical mechanics. No prior knowledge of quantum field theory is assumed. Exercises are included at the ends of chapters, drawn from the problem sets of my courses.

After completing Part II, a course can choose from the more advanced topics in Parts III and IV. I recommend a basic survey of the nonzero temperature phase diagram from Chapters 10 and 11. This can be followed by a treatment of Fermi systems drawn from Chapters 17 and 18. Chapters 19 and 20 offer many possibilities for student presentations.

The chapters in the new Parts III and IV have been significantly updated from the first edition. Chapter 16 has a new section on the Fermi gas near unitarity: this was a simple and natural extension of the previous discussion on dilute quantum liquids. These results apply to ultracold atomic systems near a Feshbach resonance. Chapter 17, on Dirac fermions, is entirely new. I took this opportunity to introduce the basics of the theory of unconventional superconductivity induced by antiferromagnetism, as it applies to the cuprates and the pnictides. Dirac fermions also offer a gentle way of introducing non-trivial quantum phase transitions of Fermi systems. Chapter 18, on Fermi liquids and their phase transitions, has been almost completely re-written: this reflects advances in our understanding, and its relevance in many experimental contexts. Chapter 19, on quantum magnetism, has numerous updates to reflect our improved understanding of spin liquids, and a brief discussion of deconfined criticality. However, I have not attempted to cover the many modern developments in quantum magnetism: a more comprehensive starting point is offered by my Solvay lecture [430].

My web site, http://sachdev.physics.harvard.edu, will have updates and corrections.

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I am very grateful to all the students in my courses for their interest and valuable feedback. The notes of Suzanne Pittman and Jihye Seo were invaluable in writing Chapters 3–8. Gilad Ben-Shach, Thiparat Chotibut, Debanjan Chowdhury, Sean Hartnoll, Yejin Huh, Max Metlitski, and Eun Gook Moon provided very useful feedback on the initial drafts. The treatment of Fermi liquids in Chapter 18 is based on the ideas of Max Metlitski [333, 334].

I thank Simon Capelin, from Cambridge University Press, for guiding both editions over many years.

I thank the Perimeter Institute, Waterloo for hospitality while I was working on the second edition. Finally, I remain grateful to the National Science Foundation for continued support of my research.