1 Introduction

When Phillip Thompson began to write the first widely read textbook[1] on numerical weather prediction[2] (NWP), the subject was in its infancy, even though an earlier book, *Weather Prediction by Numerical Process* by L. F. Richardson (1922), presaged what was to come later in the century after the advent of electronic computers. The availability of computers increased greatly in the 1960s, and universities began to offer courses in atmospheric modeling, but most modelers had to also be model developers because the untested codes had many errors, the numerical schemes for solving the equations and the physical-process representations were not well tested and understood, lateral-boundary conditions for limited-area models produced noisy solutions, and codes for defining the initial conditions needed to be further developed. These early practitioners learned the basics of atmospheric modeling from each other, through journal articles, in seminars and conferences, and from early courses on the subject. During the last 30 years of the twentieth century, graduate-level courses in atmospheric modeling flourished at many universities. And because computer modeling of the atmosphere was increasingly becoming an important tool in research and operational weather prediction, these courses were typically filled. Nevertheless, atmospheric modeling was still somewhat of a specialty, and models were not very accessible beyond national centers and a few research universities. Smagorinsky (1983), Thompson (1983), Shuman (1989), Persson (2005), Lynch (2007), and Harper (2008) should be consulted for additional history on atmospheric modeling.

In contrast, most of today's modelers are model users only, not developers, and have available, at no cost, well-tested community, global and limited-area models with complete documentation, regular tutorials, and help desks. Some models are being touted as "turn-key" systems that can be run on desk-top computers, and they are accessible to anyone in the meteorological and nonmeteorological communities having little experience in atmospheric modeling and knowledge of the model limitations. There are, of course, still the developers working on the next-generation in modeling capabilities, but they are distinct from the much-more-numerous model users who simply want to employ the model as

[1] Thompson (1961)

[2] Historically the expression "numerical weather prediction" has been used to describe all activities involving the numerical simulation of atmospheric processes, whether or not the models were being used for research or operational forecasting. But, some reserve the use of this reference only for model applications to forecasting. In this book we will use the term "numerical weather prediction" to refer to all types of model uses.

a tool to address practical questions related to physical processes, policy, or operational prediction.

The range of time and space scales simulated by contemporary models is great. Regarding time scales, in some cases models are used as the basis of data-assimilation schemes where the objective is to simply define the current state of the atmosphere in a way that is consistent with the data and the model dynamics. Model-based "nowcasts" have time horizons of 1–2 hours. Deterministic predictions of weather (i.e., specific meteorological events) extend to weeks, while interseasonal predictions of weather trends are produced with coupled ocean–atmosphere models. On the longest end of the time spectrum, climate models are integrated for hundreds of years of simulated time. Resolved spatial scales are shrinking as well. Some models that span the globe have sufficient horizontal resolution to simulate mesoscale processes. Other models can simulate winds in urban street canyons and in the wakes of buildings, in some cases quickly enough to be useful for operational applications.

With the growing skill of atmospheric models, and the availability of cheap computing power, a variety of new applications has emerged for specialized and standard versions of the models. When coupled with air-quality models, they are applied to regional airsheds to help government and business develop strategies for managing regional air quality. They are used by governments and private industry for operational prediction of weather to which agriculture is sensitive, for purposes of estimating crop-disease spread, timing planting and harvesting operations, and scheduling irrigation. Militaries employ models for producing specialized forecasts of weather that affects the conduct of their operations on the land and sea, and in the air. Models are used for planning the emergency response to accidental or intentional releases of hazardous chemical, biological, or radiological material into the atmosphere. And they predict quantities such as wind-shear, turbulence, cloud ceiling, visibility, and aircraft icing that affect the safety and efficiency of commercial and private aviation. Atmospheric models are coupled with river-discharge models for prediction of floods. Wind-energy companies use models to "prospect" for the best places to locate farms of wind turbines. Energy companies use atmospheric models to predict cloud cover, temperature, and other quantities that influence the near-future demand for electricity for heating and cooling. And, there are dozens of other sectors of industry and government that have found that model-based weather forecasts improve the profitability and safety of their operations. In general, it has been found that better weather predictions lead to better decisions.

Global atmospheric models have been at the center of the climate-change challenge and controversy for decades, and our increasing confidence in their skill is mirrored in the worldwide call to reduce emissions of carbon dioxide and other greenhouse gases. Even though climate-change processes are of global proportions, there is evidence that the specific manifestations (precipitation and temperature changes) will vary greatly from region to region. Thus, high-resolution regional models are being embedded within the global models in order to provide specific guidance to local decision makers. The models can also be used to better understand and anticipate climate change that is unrelated to greenhouse-gas concentrations. For example, worldwide land-use degradation and modification, such as from deforestation and urbanization, are known to have significant effects on atmospheric processes. Thus, "what if" experiments are performed in which different scenarios are assumed for the landscape change, and the model is run for short or long periods to define the effects of the change on precipitation, for example. The results can be used as motivation for reversing those trends that have negative consequences.

A traditional use of global and regional models has been for basic research on atmospheric processes. Special field programs are very expensive to perform, and they only sample a small area of the atmosphere for a short period of time. Thus, it has been common practice in the research community to augment observations with model simulations. If the model reproduces the atmospheric conditions reasonably well at the observation locations and times, it is assumed that the model is also skillful elsewhere. Thus, the gridded four-dimensional (three space, and time) model data set is used as a surrogate for the real atmosphere, where the advantage, in addition to low cost, is that the availability of data on a regularly spaced grid, at high temporal frequency, makes it much easier to diagnose atmospheric structures and physical processes. However, it will be noted strongly in Chapter 10, about experimental designs in model-based research, that we should first thoroughly analyze all available observational data, and learn everything we can in that process, before running a model. Figure 1.1 emphasizes that observations and theory are as important as models, as research tools that we have at our disposal. And we should avoid the tendency to start running the model before we have learned all that we can from theory and observations. Indeed, it is the author's experience that using the model early in the process only prolongs the amount of time required to complete a research project, or a thesis.

Even though the historical trend has been to use specialized models for different scales and forecast durations, the cost of maintaining multiple modeling systems has

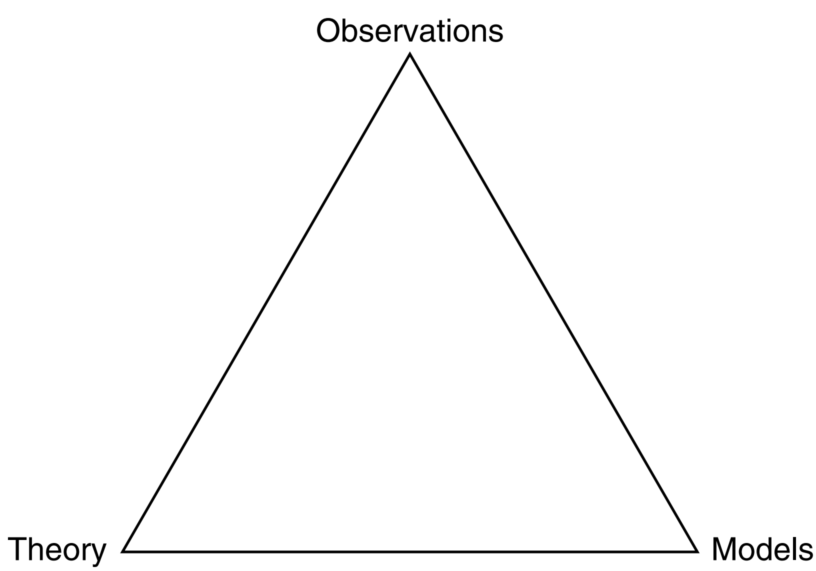

Fig. 1.1 Illustration of the equal importance of observations, theory, and models as tools in atmospheric research.

led to a trend toward a "unified" modeling approach by national meteorological services and other organizations. For example, instead of developing different models for mesoscale and global-scale applications, a single flexible system can be used for both. Similarly, weather-prediction and climate-simulation models used to be distinct, but there are efforts to merge the models used for these two purposes. Lastly, operational models have often not been used by the research community, which has meant that there has not been a straightforward path for operational implementation of improved numerical methods, physical-process parameterizations, initialization schemes, etc. But, there are now a number of examples where operational and research activities use the same models.

This book begins with a review of the governing equations that serve as the basis for atmospheric models (Chapter 2). It is assumed that the reader already has a good understanding of atmospheric dynamics, and the meaning of the various terms in these equations. One goal of the book is to educate the model user about the various components of the modeling process, and how the errors in those components affect the solution. Thus, the well-known sources of error will be described: the numerical approximations in the dynamical core (Chapter 3), the physical-process parameterizations (Chapters 4 and 5), the lateral-boundary conditions (Chapter 3), and the initial conditions (Chapter 6). The discussion of ensemble methods in Chapter 7 responds to the fact that most models, the operational ones at least, use this approach in order to provide valuable information to the model user about uncertainty in the forecast. The inherent predictability of the atmosphere has profound implications regarding the skill that we can expect from models, so this is discussed in Chapter 8. This is followed in Chapter 9 by the related topic of how we can best verify the skill of models. This is important for comparing different models, and for determining whether changes that we make in a single model have a positive or negative effect on the quality, and therefore the utility, of the output. Chapters 10 and 11 summarize common practices in designing research experiments with models, and the techniques for analyzing model output, respectively. Because models used for operational weather prediction often have different requirements and constraints than those used for research, some common differences are discussed in Chapter 12. The post processing of operational-model output to correct for biases and to make the forecast fields easier to interpret and support decision making is discussed in Chapter 13. As noted above, atmospheric models are sometimes coupled with other models that provide information about specialized processes, and these coupled applications are reviewed in Chapter 14. Even though computational fluid-dynamics models are normally applied on scales too small to be called weather, they nevertheless still simulate atmospheric processes, and are becoming more routinely used for a variety of purposes, so they are described in Chapter 15. Chapter 16 discusses how global and regional models are being used for simulation of current and future climates. Figure 1.2 summarizes the overall structural components of a modeling system, and the chapters that describe them.

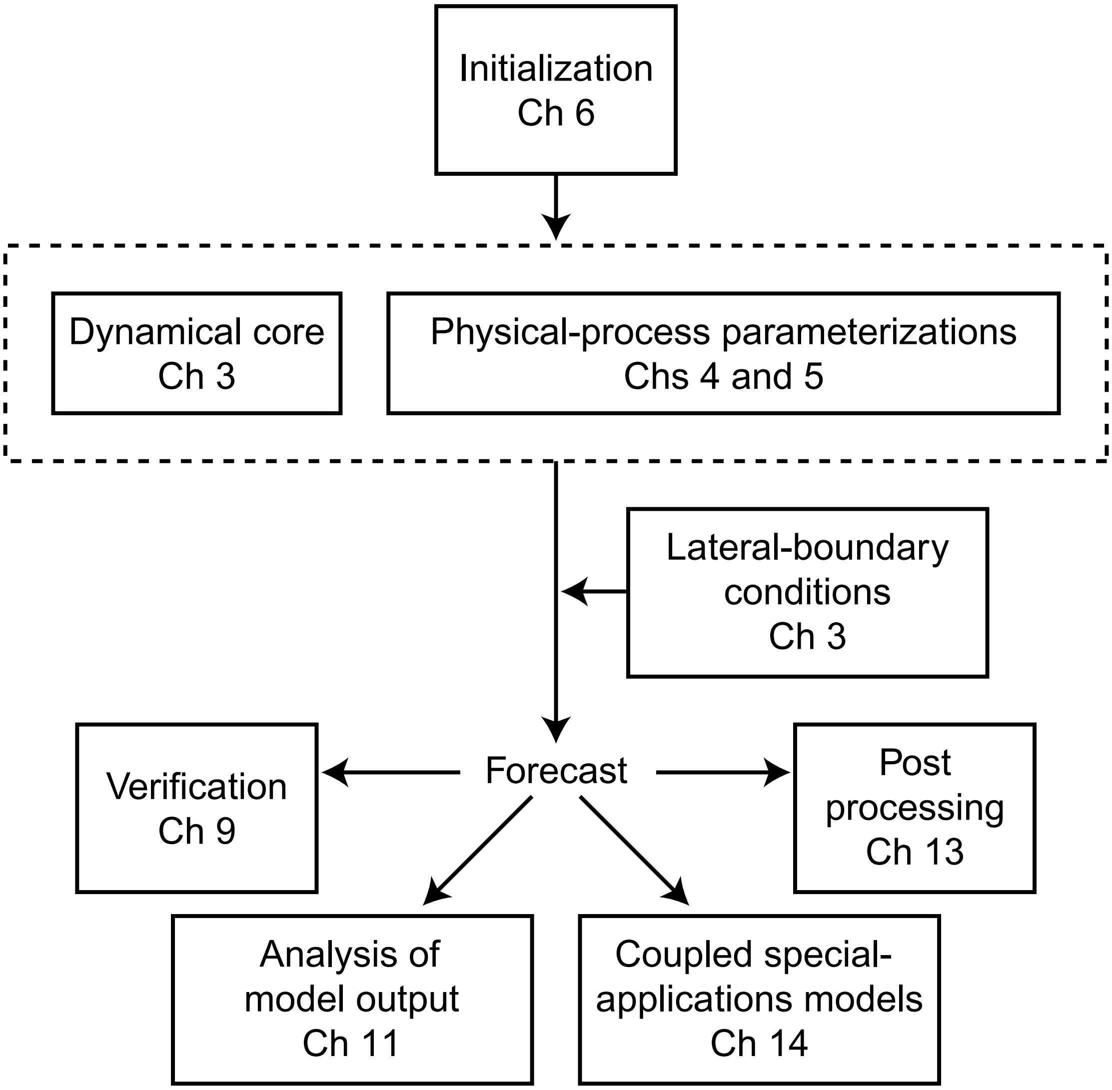

Fig. 1.2 Schematic of the overall structure of a modeling system, and the chapters that discuss the components. The dashed line encloses the two major components of the model code.

2 The governing systems of equations

2.1 The basic equations

This chapter describes the governing systems of equations that can serve as the basis for atmospheric models used for both operational and research applications. Even though most models employ similar sets of equations, the exact formulation can affect the accuracy of model forecasts and simulations,[1] and can even preclude the existence in the model solution of certain types of atmospheric waves. Because these equations cannot be solved analytically, they must be converted to a form that can be. The numerical methods typically used to accomplish this are described in Chapter 3.

The equations that serve as the basis for most numerical weather and climate prediction models are described in all first-year atmospheric-dynamics courses. The momentum equations for a spherical Earth (Eqs. 2.1–2.3) represent Newton's second law of motion, which states that the rate of change of momentum of a body is proportional to the resultant force acting on the body, and is in the same direction as the force. The thermodynamic energy equation (Eq. 2.4) accounts for various effects, both adiabatic and diabatic, on temperature. The continuity equation for total mass (Eq. 2.5) states that mass is neither gained nor destroyed, and Eq. 2.6 is analogous, but applies only to water vapor. The ideal gas law (Eq. 2.7) relates temperature, pressure, and density. The variables have their standard meteorological meaning. The independent variables u, v, and w are the Cartesian velocity components, p is pressure, ρ is density, T is temperature, q_v is specific humidity, Ω is the rotational frequency of Earth, ϕ is latitude, a is the radius of Earth, γ is the lapse rate of temperature, γ_d is the dry adiabatic lapse rate, c_p is the specific heat of air at constant pressure, g is the acceleration of gravity, H represents a gain or loss of heat, Q_v is the gain or loss of water vapor through phase changes, and Fr is a generic friction term in each coordinate direction.

$$\frac{\partial u}{\partial t} = -u\frac{\partial u}{\partial x} - v\frac{\partial u}{\partial y} - w\frac{\partial u}{\partial z} + \frac{uv\tan\phi}{a} - \frac{uw}{a} - \frac{1}{\rho}\frac{\partial p}{\partial x} - 2\Omega(w\cos\phi - v\sin\phi) + Fr_x \qquad (2.1)$$

$$\frac{\partial v}{\partial t} = -u\frac{\partial v}{\partial x} - v\frac{\partial v}{\partial y} - w\frac{\partial v}{\partial z} - \frac{u^2\tan\phi}{a} - \frac{uw}{a} - \frac{1}{\rho}\frac{\partial p}{\partial y} - 2\Omega u\sin\phi + Fr_y \qquad (2.2)$$

[1] In this text, the noun *simulation* refers to a model solution that is obtained for any purpose other than estimating the future state of the atmosphere (for example, for research). An estimate of the future state of the atmosphere is referred to as a *forecast*.

$$\frac{\partial w}{\partial t} = -u\frac{\partial w}{\partial x} - v\frac{\partial w}{\partial y} - w\frac{\partial w}{\partial z} - \frac{u^2+v^2}{a} - \frac{1}{\rho}\frac{\partial p}{\partial z} + 2\Omega u\cos\phi - g + Fr_z \tag{2.3}$$

$$\frac{\partial T}{\partial t} = -u\frac{\partial T}{\partial x} - v\frac{\partial T}{\partial y} + (\gamma - \gamma_d)w + \frac{1}{c_p}\frac{dH}{dt} \tag{2.4}$$

$$\frac{\partial \rho}{\partial t} = -u\frac{\partial \rho}{\partial x} - v\frac{\partial \rho}{\partial y} - w\frac{\partial \rho}{\partial z} - \rho\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) \tag{2.5}$$

$$\frac{\partial q_v}{\partial t} = -u\frac{\partial q_v}{\partial x} - v\frac{\partial q_v}{\partial y} - w\frac{\partial q_v}{\partial z} + Q_v \tag{2.6}$$

$$P = \rho RT \tag{2.7}$$

A complete model will also have continuity equations for cloud water, cloud ice, and the different types of precipitation (see Chapter 4). See Dutton (1976) and Holton (2004) for discussions of this set of prognostic,[2] coupled, nonlinear, nonhomogeneous partial differential equations. The equations are called the *primitive equations*, and models that are based on these equations are called *primitive-equation models*. This terminology is used to distinguish these models from ones that are based on differentiated versions of the equations, such as the vorticity equation. Virtually all contemporary research and operational models are based on some version of these primitive equations. Note that the terms in the equations related to diabatic effects (H), friction (Fr), and gains and losses of water through phase changes (Q_v) must be defined within the model. This particular example of the primitive equations has pressure as the vertical coordinate, but other options will be discussed in the next chapter.

2.2 Reynolds' equations: separating unresolved turbulence effects

The above equations apply to all scales of motion, even waves and turbulence that are too small to be represented by models designed for weather processes. Because this turbulence cannot be resolved explicitly in such models, the equations must be revised so that they apply only to larger nonturbulent motions. This can be accomplished by splitting all the dependent variables into mean and turbulent parts, or, analogously, spatially resolved and unresolved components, respectively. The mean is defined as an average over a grid cell, as described by Pielke (2002a). For example:

$$u = \bar{u} + u',$$

$$T = \bar{T} + T', \text{ and}$$

$$p = \bar{p} + p'.$$

[2] The word *prognostic* implies that an equation is predictive, in contrast to a *diagnostic* equation, which has no time derivative and simply relates the state of variables at the same time. For example, the ideal gas law is diagnostic.

These expressions are substituted into Eqs. 2.1–2.7, producing expansions such as the following one for the first term on the right side of Eq. 2.1:

$$u\frac{\partial u}{\partial x} = (\bar{u} + u')\frac{\partial}{\partial x}(\bar{u} + u') = \bar{u}\frac{\partial \bar{u}}{\partial x} + \bar{u}\frac{\partial u'}{\partial x} + u'\frac{\partial \bar{u}}{\partial x} + u'\frac{\partial u'}{\partial x}. \tag{2.8}$$

Because we want the equations to pertain to the mean motion, that is, the nonturbulent weather scales, we apply an averaging operator to all the terms. For the above term, we have

$$\overline{u\frac{\partial u}{\partial x}} = \overline{\bar{u}\frac{\partial \bar{u}}{\partial x}} + \overline{\bar{u}\frac{\partial u'}{\partial x}} + \overline{u'\frac{\partial \bar{u}}{\partial x}} + \overline{u'\frac{\partial u'}{\partial x}}. \tag{2.9}$$

Note that the last term on the right is a *covariance term*. Its value depends on whether the first quantity in the product covaries with the second. For example, if positive values of the first part tend to be paired with negative values of the second, the covariance, and the term, would be negative. If the two parts of the product are not physically correlated, the mean has a value of zero. We then simplify the equations using Reynolds' postulates (Reynolds 1895, Bernstein 1966). For variables a and b,

$$\overline{a'} = 0,$$

$$\bar{\bar{a}} = \bar{a} \text{ and } \overline{\bar{a}b} = \bar{\bar{a}}\bar{b} = \bar{a}\bar{b}\text{, and}$$

$$\overline{\bar{a}b'} = \bar{\bar{a}}\overline{b'} = \bar{a}\overline{b'} = 0\,.$$

Given these postulates, the terms in Eq. 2.9 become

$$\overline{u\frac{\partial u}{\partial x}} = \bar{u}\frac{\partial \bar{u}}{\partial x} + \bar{u}\cancelto{0}{\frac{\partial \overline{u'}}{\partial x}} + \cancelto{0}{\overline{u'}\frac{\partial \bar{u}}{\partial x}} + \overline{u'\frac{\partial u'}{\partial x}} = \bar{u}\frac{\partial \bar{u}}{\partial x} + \overline{u'\frac{\partial u'}{\partial x}}. \tag{2.10}$$

Before we show how to apply these methods to all the terms in Eqs. 2.1–2.7, let us rewrite Eq. 2.1 with a typical representation for the friction terms, Fr_x, without the Earth-curvature terms, and with only the dominant Coriolis term. In these equations, which explicitly represent turbulent motion, subgrid friction results only from viscous forces, which are a consequence of molecular motion.

$$\frac{\partial u}{\partial t} = -u\frac{\partial u}{\partial x} - v\frac{\partial u}{\partial y} - w\frac{\partial u}{\partial z} - \frac{1}{\rho}\frac{\partial p}{\partial x} + fv + \frac{1}{\rho}\left(\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z}\right). \tag{2.11}$$

Here, τ_{zx} is the force per unit area, or the momentum or shearing stress, exerted in the x direction by the fluid on one side of a constant-z plane with the fluid on the other side of the z plane, and τ_{xx} and τ_{yx} are the forces in the x direction across the other two coordinate planes. In hypothetical, inviscid fluids, there would be no "communication" between the flow on either side of a plane. But, in real fluids, the molecular motion, or molecular diffusion, across each of the coordinate surfaces will allow for the exchange of properties. A typical representation for the stress is

$$\tau_{zx} = \mu \frac{\partial u}{\partial z},$$

where μ is dynamic viscosity coefficient. This is called Newtonian friction, or Newton's law for the stress. Referring to the two (infinitesimally shallow) layers of fluid on either side of the z plane, if there is no shear in the fluid, viscosity produces no stress, or force per unit area, of one layer on the other. Substituting these expressions for the Newtonian friction into the terms for Fr_x in Eq. 2.11, we have

$$\frac{\partial u}{\partial t} \propto \frac{1}{\rho}\left(\mu\frac{\partial^2 u}{\partial x^2} + \mu\frac{\partial^2 u}{\partial y^2} + \mu\frac{\partial^2 u}{\partial z^2}\right) = \frac{\mu}{\rho}\nabla^2 u. \tag{2.12}$$

Now apply the averaging process to all the terms in Eq. 2.11. In particular, we represent each dependent variable by the sum of a resolved mean and an unresolved turbulent component, and then apply the averaging operator. Using Reynolds' postulates, and the assumption that $\rho' \ll \bar{\rho}$, we obtain

$$\frac{\partial \bar{u}}{\partial t} = -\bar{u}\frac{\partial \bar{u}}{\partial x} - \bar{v}\frac{\partial \bar{u}}{\partial y} - \bar{w}\frac{\partial \bar{u}}{\partial z} - \frac{1}{\bar{\rho}}\frac{\partial \bar{p}}{\partial x} + f\bar{v} - \overline{u'\frac{\partial u'}{\partial x}} - \overline{v'\frac{\partial u'}{\partial y}} - \overline{w'\frac{\partial u'}{\partial z}} + \frac{1}{\bar{\rho}}\left(\frac{\partial \bar{\tau}_{xx}}{\partial x} + \frac{\partial \bar{\tau}_{yx}}{\partial y} + \frac{\partial \bar{\tau}_{zx}}{\partial z}\right). \tag{2.13}$$

Stull (1988) uses a scale analysis to show that, for turbulence scales of motion, the following continuity equation applies:

$$\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} + \frac{\partial w'}{\partial z} = 0. \tag{2.14}$$

Multiply this by u', average it, and add it to Eq. 2.13 to put the turbulent advection terms into *flux form*:

$$\frac{\partial \bar{u}}{\partial t} = -\bar{u}\frac{\partial \bar{u}}{\partial x} - \bar{v}\frac{\partial \bar{u}}{\partial y} - \bar{w}\frac{\partial \bar{u}}{\partial z} - \frac{1}{\bar{\rho}}\frac{\partial \bar{p}}{\partial x} + f\bar{v} - \frac{\partial \overline{u'u'}}{\partial x} - \frac{\partial \overline{u'v'}}{\partial y} - \frac{\partial \overline{u'w'}}{\partial z} + \frac{1}{\bar{\rho}}\left(\frac{\partial \bar{\tau}_{xx}}{\partial x} + \frac{\partial \bar{\tau}_{yx}}{\partial y} + \frac{\partial \bar{\tau}_{zx}}{\partial z}\right). \tag{2.15}$$

By analogy with the molecular viscosity-related stresses, we define turbulent stresses (also, eddy stresses or Reynolds' stresses) as follows:

$$T_{xx} = -\bar{\rho}\overline{u'u'},$$
$$T_{yx} = -\bar{\rho}\overline{u'v'},$$
$$T_{zx} = -\bar{\rho}\,\overline{u'w'}.$$

Substituting these expressions into Eq. 2.15, and assuming that the spatial derivatives of the density are much smaller than those of the covariances, we have

$$\frac{\partial \bar{u}}{\partial t} = -\bar{u}\frac{\partial \bar{u}}{\partial x} - \bar{v}\frac{\partial \bar{u}}{\partial y} - \bar{w}\frac{\partial \bar{u}}{\partial z} - \frac{1}{\bar{\rho}}\frac{\partial \bar{p}}{\partial x} + f\bar{v}$$
$$+\frac{1}{\bar{\rho}}\left(\frac{\partial}{\partial x}(\tau_{xx} + T_{xx}) + \frac{\partial}{\partial y}(\tau_{yx} + T_{yx}) + \frac{\partial}{\partial z}(\tau_{zx} + T_{zx})\right). \tag{2.16}$$

This equation is the same as Eq. 2.11, except for the turbulent-stress terms and the mean-value symbols. The mean-value symbols are rarely used with the primitive equations, but it is still understood that the dependent variables represent only nonturbulent motions. And, the turbulent stresses are much larger than the viscous stresses, so the latter terms are usually not included. The turbulent-stress terms are sometimes represented symbolically as “*F*”, referring to friction. The representation of the turbulent stresses in terms of variables predicted by the model is the subject of turbulence parameterizations for the boundary layer, or for above the boundary layer, described in Chapter 4.

2.3 Approximations to the equations

There are a few reasons why we might desire to use approximate sets of equations as the basis for a model.

- Some approximate sets are more efficient to solve numerically than the complete equations. For example, the hydrostatic, *Boussinesq*, and *anelastic approximations* described below do not permit sound waves in the solutions, which, for reasons that will be explained in the next chapter, means that less computing resources are required to produce a simulation or forecast of a given length.
- The complete equations describe a physical system that is so complex that it is challenging to use them in a model for research, to better understand cause and effect relationships in the atmosphere. Thus, sometimes specific terms and equations (and the associated processes) are removed from the set of equations. For example, removing equations for water in all its phases, and the thermodynamic effect of phase changes, allows the study of processes in a simpler setting.
- Very simple forms of the equations are more amenable for pedagogical applications and for initial testing of new numerical algorithms. For example, the shallow-fluid equations, described below, are used as the basis for “toy models” in NWP classes (and in this text). But, they contain enough of the dynamics of the full set of equations that they can be profitably used to test new differencing schemes, which can later be evaluated in complete models.

The approximations described in the following subsections are commonly used in research and operational models.