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No-Cloning in Categorical Quantum Mechanics

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Abstract

The no-cloning theorem is a basic limitative result for quantum mechanics, with particular significance for quantum information. It says that there is no unitary operation that makes perfect copies of an unknown (pure) quantum state. We re-examine this foundational result from the perspective of the categorical formulation of quantum mechanics recently introduced by the author and Bob Coecke. We formulate and prove a novel version of the result, as an incompatibility between having a "natural" copying operation and the structural features required for modeling quantum entanglement coexisting in the same category. This formulation is strikingly similar to a well-known limitative result in categorical logic, Joyal's lemma, which shows that a "Boolean cartesian closed category" trivializes and hence provides a major roadblock to the computational interpretation of classical logic. This shows a heretofore unsuspected connection between limitative results in proof theory and no-go theorems in quantum mechanics. The argument is of a robust, topological character and uses the graphical calculus for monoidal categories to advantage.

1.1 Introduction

The no-cloning theorem (Dieks 1982; Wootters and Zurek 1982) is a basic limitative result for quantum mechanics, with particular significance for quantum information. It says that there is no unitary operation that makes perfect copies of an unknown (pure) quantum state. A stronger form of this result is the no-broadcasting theorem (Barnum et al. 1996), which applies to mixed states. There is also a no-deleting theorem (Pati and Braunstein 2000).

Recently, the author and Bob Coecke have introduced a categorical formulation of quantum mechanics (Abramsky and Coecke 2004, 2005, 2008), as a basis for a more structural, high-level approach to quantum information and computation. This has been elaborated by ourselves, our colleagues, and other workers in the field (Abramsky 2004, 2005, 2007; Abramsky and Duncan 2006; Coecke and Pavlovic

2007; Coecke and Duncan 2008; Selinger 2007; Vicary 2008) and has been shown to yield an effective and illuminating treatment of a wide range of topics in quantum information. Diagrammatic calculi for tensor categories (Joyal and Street 1991; Turaev 1994), suitably extended to incorporate the various additional structures that have been used to reflect fundamental features of quantum mechanics, play an important role, both as an intuitive and vivid visual presentation of the formalism and as an effective calculational device.

It is clear that such a novel reformulation of the mathematical formalism of quantum mechanics, a subject more or less set in stone since von Neumann's classic treatise (von Neumann 1932), has the potential to yield new insights into the foundations of quantum mechanics. In the present paper, we use it to open up a novel perspective on no-cloning. What we find, quite unexpectedly, is a link to some fundamental issues in logic, computation, and the foundations of mathematics. A striking feature of our results is that they are visibly in the same genre as a well-known result by Joyal in categorical logic (Lambek and Scott 1986) showing that a "Boolean cartesian closed category" trivializes, which provides a major roadblock to the computational interpretation of classical logic. In fact, they strengthen Joyal's result, insofar as the assumption of a full categorical product (diagonals *and* projections) in the presence of a classical duality is weakened. This shows a heretofore unsuspected connection between limitative results in proof theory and no-go theorems in quantum mechanics.

The further contents of the paper are as follows:

- In the next section, we briefly review the three-way link between logic, computation, and categories and recall Joyal's lemma.
- In Section 1.3, we review the categorical approach to quantum mechanics.
- Our main results are in Section 1.4, where we prove our limitative result, which shows the incompatibility of structural features corresponding to quantum entanglement (essentially, the existence of Bell states enabling teleportation) with the existence of a "natural" (in the categorical sense, corresponding essentially to *basis-independent*) copying operation. This result is mathematically robust, since it is proved in a very general context and has a topological content that is clearly revealed by a diagrammatic proof. At the same time it is delicately poised, since *non-natural*, basis-dependent copying operations do in fact play a key role in the categorical formulation of quantum notions of measurement. We discuss this context, and the conceptual reading of the results.
- We conclude with some discussion of extensions of the results, further directions, and open problems.

1.2 Categories, Logic, and Computational Content: Joyal's Lemma

Categorical logic (Lambek and Scott 1986) and the Curry-Howard correspondence in Proof Theory (Sørensen and Urzyczyn 2006) give us a beautiful three-way correspondence:

$$\begin{tikzcd}
\text{Logic} \arrow[rr, leftrightarrow] \arrow[dr, leftrightarrow] & & \text{Computation} \arrow[dl, leftrightarrow] \\
& \text{Categories} &
\end{tikzcd}$$

More particularly, we have as a paradigmatic example:

$$\begin{tikzcd}
\text{Intuitionistic Logic} \arrow[rr, leftrightarrow] \arrow[dr, leftrightarrow] & & \lambda\text{-calculus} \arrow[dl, leftrightarrow] \\
& \text{Cartesian Closed Categories} &
\end{tikzcd}$$

Here we are focusing on the fragment of intuitionistic logic containing conjunction and implication, and the simply typed λ-calculus with product types.

We shall assume familiarity with basic notions of category theory (Mac Lane 1998; Lawvere and Schanuel 1997). Recall that a cartesian closed category is a category with a terminal object, binary products, and exponentials. The basic cartesian closed adjunction is

$$\mathcal{C}(A \times B, C) \cong \mathcal{C}(A, B \Rightarrow C)\,.$$

More explicitly, a category $\mathcal{C}$ with finite products *has exponentials* if for all objects A and B of $\mathcal{C}$ there is a couniversal arrow from $- \times A$ to B, *i.e.*, an object $A \Rightarrow B$ of $\mathcal{C}$ and a morphism

$$\mathsf{ev}_{A,B} : (A \Rightarrow B) \times A \longrightarrow B$$

with the couniversal property: for every $g : C \times A \longrightarrow B$, there is a unique morphism $\Lambda(g) : C \longrightarrow A \Rightarrow B$ such that

$$\begin{tikzcd}
A \Rightarrow B & \\
C \arrow[u, dashed, "\Lambda(g)"] &
\end{tikzcd}
\qquad
\begin{tikzcd}
(A \Rightarrow B) \times A \arrow[r, "\mathsf{ev}_{A,B}"] & B \\
C \times A \arrow[u, dashed, "\Lambda(g) \times \mathsf{id}_A"] \arrow[ur, "g"'] &
\end{tikzcd}$$

The correspondence between the intuitionistic logic of conjunction and implication and cartesian closed categories is summarized in the following table:

Axiom	$\dfrac{}{\Gamma, A \vdash A}\ \mathsf{Id}$	$\dfrac{}{\pi_2 : \Gamma \times A \longrightarrow A}$
Conjunction	$\dfrac{\Gamma \vdash A \qquad \Gamma \vdash B}{\Gamma \vdash A \wedge B}\ \wedge\mathsf{I}$ $\dfrac{\Gamma \vdash A \wedge B}{\Gamma \vdash A}\ \wedge\mathsf{E}_1$ $\dfrac{\Gamma \vdash A \wedge B}{\Gamma \vdash B}\ \wedge\mathsf{E}_2$	$\dfrac{f : \Gamma \longrightarrow A \qquad g : \Gamma \longrightarrow B}{\langle f, g\rangle : \Gamma \longrightarrow A \times B}$ $\dfrac{f : \Gamma \longrightarrow A \times B}{\pi_1 \circ f : \Gamma \longrightarrow A}$ $\dfrac{f : \Gamma \longrightarrow A \times B}{\pi_2 \circ f : \Gamma \longrightarrow B}$
Implication	$\dfrac{\Gamma, A \vdash B}{\Gamma \vdash A \supset B}\ \supset\mathsf{I}$ $\dfrac{\Gamma \vdash A \supset B \quad \Gamma \vdash A}{\Gamma \vdash B}\ \supset\mathsf{E}$	$\dfrac{f : \Gamma \times A \longrightarrow B}{\Lambda(f) : \Gamma \longrightarrow (A \Rightarrow B)}$ $\dfrac{f : \Gamma \longrightarrow (A \Rightarrow B) \quad g : \Gamma \longrightarrow A}{\mathsf{ev}_{A,B} \circ \langle f, g\rangle : \Gamma \longrightarrow B}$

1.2.1 Joyal's Lemma

It is a very natural idea to seek to extend the preceding correspondence to the case of *classical logic*. Joyal's lemma shows that there is a fundamental impediment to doing so.[1]

The natural extension of the notion of cartesian closed category, which corresponds to the *intuitionistic logic* of conjunction and implication, to the classical case is to introduce a suitable notion of classical negation. We recall that it is customary in intuitionistic logic to *define* the negation by

$$\neg A := A \supset \bot$$

where $\bot$ is the *falsum*. The characteristic property of the falsum is that it implies every proposition. In categorical terms, this translates into the notion of an initial object. Note that for any fixed object B in a cartesian closed category, there is a well-defined contravariant functor

$$\mathcal{C} \longrightarrow \mathcal{C}^{\mathsf{op}} :: A \mapsto (A \Rightarrow B)\,.$$

This will always satisfy the properties corresponding to negation in minimal logic, and if $B = \bot$ is the initial object in $\mathcal{C}$, then it will satisfy the laws of intuitionistic negation. In particular, there is a canonical arrow

$$A \longrightarrow (A \Rightarrow \bot) \Rightarrow \bot$$

that is just the curried form of the evaluation morphism. This corresponds to the valid intuitionistic principle $A \supset \neg\neg A$. What else is needed in order to

[1] It is customary to refer to this result as Joyal's lemma, although, apparently, he never published it. The usual reference is to Lambek and Scott (1986), who attribute the result to Joyal, but follow the proof given by Freyd (1972). Our statement and proof are somewhat different from those by Lambek and Scott (1986).

obtain classical logic? As is well known, the missing principle is that of *proof by contradiction*: the converse implication $\neg\neg A \supset A$.

This leads us to the following notion. A *dualizing object* $\perp$ in a closed category is one for which the canonical arrow

$$A \longrightarrow (A \Rightarrow \perp) \Rightarrow \perp$$

is an isomorphism for all A.

We can now state Joyal's lemma:

Proposition 1.2.1 (Joyal's Lemma). *Any cartesian closed category with a dualizing object is a preorder (hence* trivial *as a semantics for proofs or computational processes).*

Proof. Note first that, if $\perp$ is dualizing, the induced negation functor $\mathcal{C} \longrightarrow \mathcal{C}^{\mathsf{op}}$ is a *contravariant equivalence* $\mathcal{C} \simeq \mathcal{C}^{\mathsf{op}}$. Since $(\top \Rightarrow A) \cong A$ where $\top$ is the terminal object, it follows that $\perp$ is the dual of $\top$, and hence initial. So it suffices to prove Joyal's lemma under the assumption that the dualizing object is initial.

We assume that $\perp$ is a dualizing initial object in a cartesian closed category $\mathcal{C}$. By cartesian closure, $\mathcal{C}(A \times \perp, B) \cong \mathcal{C}(\perp, A \Rightarrow B)$, which is a singleton by initiality of $\perp$. It follows that $A \times \perp$ is initial.[2]

Now

$$\mathcal{C}(A, B) \cong \mathcal{C}(B \Rightarrow \perp, A \Rightarrow \perp) \cong \mathcal{C}((B \Rightarrow \perp) \times A, \perp). \tag{1.1}$$

Given any $h, k : C \longrightarrow \perp$, note that

$$h = \pi_1 \circ \langle h, k\rangle, \qquad k = \pi_2 \circ \langle h, k\rangle.$$

But $\perp \times \perp \cong \perp$, hence by initiality $\pi_1 = \pi_2$, and so $h = k$, which by (1.1) implies that $f = g$ for $f, g : A \longrightarrow B$. □

1.2.2 Linearity and Classicality

However, we know from linear logic that there is no impediment to having a closed structure with a dualizing object, provided we weaken our assumption on the underlying context-building structure, from *cartesian* $\times$ to *monoidal* $\otimes$.

Then we get a wealth of examples of $*$-*autonomous categories* (Barr 1979), which stand to multiplicative linear logic as cartesian closed categories do to intuitionistic logic (Seely 1998).

Joyal's lemma can thus be stated in the following equivalent form.

Proposition 1.2.2. *A $*$-autonomous category in which the monoidal structure is cartesian is a preorder.*

[2] A slicker proof simply notes that $A \times (-)$ is a left adjoint by cartesian closure and hence preserves all colimits, in particular initial objects.

Essentially, a cartesian structure is a monoidal structure plus natural diagonals, and with the tensor unit a terminal object, *i.e.*, *plus cloning and deleting*!

1.3 Categorical Quantum Mechanics

In this section, we provide a brief review of the structures used in categorical quantum mechanics, their graphical representation, and how these structures are used in formalizing some key features of quantum mechanics. Further details can be found elsewhere (Abramsky and Coecke 2008; Abramsky 2005; Selinger 2007).

1.3.1 Symmetric Monoidal Categories

We recall that a *monoidal category* is a structure $(\mathcal{C}, \otimes, I, a, l, r)$ where:

- $\mathcal{C}$ is a category,
- $\otimes : \mathcal{C} \times \mathcal{C} \to \mathcal{C}$ is a functor (*tensor*),
- I is a distinguished object of $\mathcal{C}$ (*unit*),
- a, l, r are natural isomorphisms (*structural isos*) with components:

$$a_{A,B,C} : A \otimes (B \otimes C) \cong (A \otimes B) \otimes C$$
$$l_A : I \otimes A \cong A \qquad r_A : A \otimes I \cong A$$

  such that certain diagrams commute, which ensure *coherence* (Mac Lane 1998), described by the slogan:

$$\boxed{\text{All diagrams only involving } a, l \text{ and } r \text{ must commute.}}$$

Examples:

- Both products and coproducts give rise to monoidal structures – which are the common denominator between them. (But in addition, products have *diagonals* and *projections*, and coproducts have *codiagonals* and *injections*.)
- $(\mathbb{N}, \leqslant, +, 0)$ is a monoidal category.
- **Rel**, the category of sets and relations, with cartesian product (which is *not* the categorical product).
- $\mathbf{Vect}_k$ with the standard tensor product.

Let us examine the example of **Rel** in some detail. We take $\otimes$ to be the cartesian product, which is defined on relations $R : X \to X'$ and $S : Y \to Y'$ as follows:

$$\forall (x, y) \in X \times Y, (x', y') \in X' \times Y'. \ (x, y) R \otimes S (x', y') \iff x R x' \wedge y S y' .$$

It is not difficult to show that this is indeed a functor. Note that, in the case that R, S are *functions*, $R \otimes S$ is the same as $R \times S$ in **Set**. Moreover, we take each $a_{A,B,C}$ to be the associativity function for products (in **Set**), which is an iso in **Set** and hence also in **Rel**. Finally, we take I to be the one-element set, and l_A, r_A to be the projection functions: their relational converses are their inverses in **Rel**. The monoidal coherence diagrams commute simply because they commute in **Set**.

Tensors and products. As mentioned earlier, products are tensors with extra structure: natural diagonals and projections, corresponding to cloning and deleting operations. This fact is expressed more precisely as follows.

Proposition 1.3.1. *Let $\mathcal{C}$ be a monoidal category $(\mathcal{C}, \otimes, I, a, l, r)$. The tensor $\otimes$ induces a product structure iff there exist natural diagonals and projections, i.e., natural transformations*

$$\Delta_A : A \longrightarrow A \otimes A\,, \qquad p_{A,B} : A \otimes B \longrightarrow A\,, \qquad q_{A,B} : A \otimes B \longrightarrow B$$

such that the following diagrams commute.

$$\begin{tikzcd}
 & A \arrow[dl, "\mathrm{id}_A"'] \arrow[d, "\Delta_A"] \arrow[dr, "\mathrm{id}_A"] & \\
A & A \otimes A \arrow[l, "p_{A,A}"] \arrow[r, "q_{A,A}"'] & A
\end{tikzcd}
\qquad
\begin{tikzcd}
A \otimes B \arrow[r, "\Delta_{A,B}"] \arrow[dr, "\mathrm{id}_{A\otimes B}"'] & (A \otimes B) \otimes (A \otimes B) \arrow[d, "p_{A,B} \otimes q_{A,B}"] \\
 & A \otimes B
\end{tikzcd}$$

Symmetry. A *symmetric monoidal category* is a monoidal category $(\mathcal{C}, \otimes, I, a, l, r)$ with an additional natural isomorphism (*symmetry*),

$$\sigma_{A,B} : A \otimes B \cong B \otimes A$$

such that $\sigma_{B,A} = \sigma_{A,B}^{-1}$, and some additional coherence diagrams commute.

1.3.2 Scalars

Let $(\mathcal{C}, \otimes, I, l, a, l, r)$ be a monoidal category. We define a *scalar* in $\mathcal{C}$ to be a morphism $s : I \to I$, *i.e.*, an endomorphism of the tensor unit.

Example 1.3.2. *In* $\mathbf{FdVec}_{\mathbb{K}}$*, linear maps* $\mathbb{K} \to \mathbb{K}$ *are uniquely determined by the image of* 1 *and hence are in bijective correspondence with elements of* $\mathbb{K}$*; composition corresponds to multiplication of scalars. In* **Rel***, there are just two scalars, corresponding to the Boolean values* 0, 1.

The (multiplicative) monoid of scalars is then just the endomorphism monoid $\mathcal{C}(I, I)$. The first key point is the elementary but beautiful observation by Kelly and Laplaza (1980) that this monoid is always commutative.

Lemma 1.3.3. *$\mathcal{C}(I, I)$ is a commutative monoid.*

Proof.

$$\begin{tikzcd}
I \arrow[r, "r_I^{-1}"] & I \otimes I \arrow[r, equal] & I \otimes I \arrow[r, "l_I"] \arrow[d, "1 \otimes t"] & I \arrow[d, "t"] \\
I \arrow[u, "s"] \arrow[r, "r_I^{-1}"] \arrow[d, "t"'] & I \otimes I \arrow[u, "s \otimes 1"'] \arrow[r, "s \otimes t"] \arrow[d, "1 \otimes t"'] & I \otimes I \arrow[r, "l_I"] & I \\
I \arrow[r, "l_I^{-1}"'] & I \otimes I \arrow[r, equal] & I \otimes I \arrow[u, "s \otimes 1"'] \arrow[r, "r_I"'] & I \arrow[u, "s"']
\end{tikzcd}$$

using the coherence equation $l_I = r_I$. □

The second point is that a good notion of *scalar multiplication* exists at this level of generality. That is, each scalar $s : I \to I$ induces a natural transformation

$$s_A : A \xrightarrow{\simeq} I \otimes A \xrightarrow{s \otimes 1_A} I \otimes A \xrightarrow{\simeq} A$$

with the naturality square

$$\begin{tikzcd}
A \arrow[r, "s_A"] \arrow[d, "f"'] & A \arrow[d, "f"] \\
B \arrow[r, "s_B"'] & B
\end{tikzcd}$$

We write $s \bullet f$ for $f \circ s_A = s_B \circ f$. Note that

$$\begin{aligned}
1 \bullet f &= f \\
s \bullet (t \bullet f) &= (s \circ t) \bullet f \\
(s \bullet g) \circ (t \bullet f) &= (s \circ t) \bullet (g \circ f) \\
(s \bullet f) \otimes (t \bullet g) &= (s \circ t) \bullet (f \otimes g)
\end{aligned}$$

which exactly generalizes the multiplicative part of the usual properties of scalar multiplication. Thus scalars act globally on the whole category.

1.3.3 Compact Closed Categories

A category $\mathbf{C}$ is $*$-*autonomous* (Barr 1979) if it is symmetric monoidal and comes equipped with a full and faithful functor

$$(\ )^* : \mathbf{C}^{op} \to \mathbf{C}$$

such that a bijection

$$\mathbf{C}(A \otimes B, C^*) \simeq \mathbf{C}(A, (B \otimes C)^*)$$

exists which is natural in all variables. Hence a $*$-autonomous category is closed, with

$$A \multimap B := (A \otimes B^*)^* .$$

These $*$-autonomous categories provide a categorical semantics for the multiplicative fragment of linear logic (Seely 1998).

A *compact closed category* (Kelly and Laplaza 1980) is a $*$-autonomous category with a self-dual tensor, *i.e.*, with natural isomorphisms

$$u_{A,B} : (A \otimes B)^* \simeq A^* \otimes B^* \qquad u_I : I^* \simeq I .$$

It follows that

$$A \multimap B \simeq A^* \otimes B .$$

An alternative definition arises when one considers a symmetric monoidal category as a one-object bicategory. In this context, compact closure simply means that every object A, qua 1-cell of the bicategory, has a specified adjoint (Kelly and Laplaza 1980).

Definition 1.3.4 (Kelly-Laplaza). A *compact closed category* is a symmetric monoidal category in which to each object A a *dual object* A^*, a *unit*

$$\eta_A : \mathrm{I} \to A^* \otimes A$$

and a *counit*

$$\epsilon_A : A \otimes A^* \to \mathrm{I}$$

are assigned, in such a way that the diagram

$$\begin{tikzcd}
A \arrow[r, "r_A^{-1}"] \arrow[d, "1_A"'] & A \otimes \mathrm{I} \arrow[r, "1_A \otimes \eta_A"] & A \otimes (A^* \otimes A) \arrow[d, "a_{A,A^*,A}"] \\
A & \mathrm{I} \otimes A \arrow[l, "l_A"] & (A \otimes A^*) \otimes A \arrow[l, "\epsilon_A \otimes 1_A"]
\end{tikzcd}$$

and the dual one for A^* both commute.

Examples. The symmetric monoidal categories (**Rel**, $\times$) of sets, relations, and cartesian product and ($\mathbf{FdVec}_{\mathbb{K}}$, $\otimes$) of finite-dimensional vector spaces over a field $\mathbb{K}$, linear maps, and tensor product are both compact closed. In (**Rel**, $\times$), we simply set $X^* = X$. Taking a one-point set $\{*\}$ as the unit for $\times$, and writing $R^{\cup}$ for the converse of a relation R:

$$\eta_X = \epsilon_X^{\cup} = \{(*, (x, x)) \mid x \in X\} .$$

For $(\mathbf{FdVec}_{\mathbb{K}}, \otimes)$, we take V^* to be the dual space of linear functionals on V. The unit and counit in $(\mathbf{FdVec}_{\mathbb{K}}, \otimes)$ are

$$\eta_V : \mathbb{K} \to V^* \otimes V :: 1 \mapsto \sum_{i=1}^{n} \bar{e}_i \otimes e_i$$

and

$$\epsilon_V : V \otimes V^* \to \mathbb{K} :: e_i \otimes \bar{e}_j \mapsto \bar{e}_j(e_i)$$

where n is the dimension of V, $\{e_i\}_{i=1}^n$ is a basis of V and $\bar{e}_i$ is the linear functional in V^* determined by $\bar{e}_j(e_i) = \delta_{ij}$.

Definition 1.3.5. The *name* $\ulcorner f \urcorner$ and the *coname* $\llcorner f \lrcorner$ of a morphism $f : A \to B$ in a compact closed category are

$$\begin{tikzcd} A^* \otimes A \arrow[r, "1_{A^*} \otimes f"] & A^* \otimes B \\ \mathrm{I} \arrow[u, "\eta_A"] \arrow[ur, "\ulcorner f \urcorner"'] & \end{tikzcd} \qquad \begin{tikzcd} & \mathrm{I} \\ A \otimes B^* \arrow[ur, "\llcorner f \lrcorner"] \arrow[r, "f \otimes 1_{B^*}"'] & B \otimes B^* \arrow[u, "\epsilon_B"'] \end{tikzcd}$$

For $R \in \mathbf{Rel}(X, Y)$ we have

$$\ulcorner R \urcorner = \{(*, (x, y)) \mid x R y, x \in X, y \in Y\}$$

and

$$\llcorner R \lrcorner = \{((x, y), *) \mid x R y, x \in X, y \in Y\}$$

and for $f \in \mathbf{FdVec}_{\mathbb{K}}(V, W)$ with (m_{ij}) the matrix of f in bases $\{e_i^V\}_{i=1}^n$ and $\{e_j^W\}_{j=1}^m$ of V and W respectively

$$\ulcorner f \urcorner : \mathbb{K} \to V^* \otimes W :: 1 \mapsto \sum_{i,j=1}^{n,m} m_{ij} \cdot \bar{e}_i^V \otimes e_j^W$$

and

$$\llcorner f \lrcorner : V \otimes W^* \to \mathbb{K} :: e_i^V \otimes \bar{e}_j^W \mapsto m_{ij}.$$

Given $f : A \to B$ in any compact closed category $\mathbf{C}$ we can define $f^* : B^* \to A^*$ as

$$\begin{tikzcd} B^* \arrow[r, "l_{B^*}^{-1}"] \arrow[d, "f^*"'] & \mathrm{I} \otimes B^* \arrow[r, "\eta_A \otimes 1_{B^*}"] & A^* \otimes A \otimes B^* \arrow[d, "1_{A^*} \otimes f \otimes 1_{B^*}"] \\ A^* & A^* \otimes \mathrm{I} \arrow[l, "r_{A^*}"] & A^* \otimes B \otimes B^* \arrow[l, "1_{A^*} \otimes \epsilon_B"] \end{tikzcd}$$