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978-0-521-51328-9 - Fundamentals of Photonic Crystal Guiding
Maksim Skorobogatiy and Jianke Yang
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Fundamentals of Photonic Crystal Guiding

If you're looking to understand photonic crystals, this systematic, rigorous, and pedagogical introduction is a must. Here you'll find intuitive analytical and semi-analytical models applied to complex and practically relevant photonic crystal structures. You will also be shown how to use various analytical methods borrowed from quantum mechanics, such as perturbation theory, asymptotic analysis, and group theory, to investigate many of the limiting properties of photonic crystals, which are otherwise difficult to rationalize using only numerical simulations.

An introductory review of nonlinear guiding in photonic lattices is also presented, as are the fabrication and application of photonic crystals. In addition, end-of-chapter exercise problems with detailed analytical and numerical solutions allow you to monitor your understanding of the material presented. This accessible text is ideal for researchers and graduate students studying photonic crystals in departments of electrical engineering, physics, applied physics, and mathematics.

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M. Skorobogatiy dedicates this book to his family. He thanks his parents Alexander and Tetyana for never-ceasing support, encouragement, and participation in all his endeavors. He also thanks his wife Olga, his children, Alexander junior and Anastasia, and his parents for their unconditional love.

J. Yang dedicates this book to his family.

Contents

	<i>Preface</i>	<i>page</i> xi
	<i>Acknowledgements</i>	xii
1	Introduction	1
	1.1 Fabrication of photonic crystals	2
	1.2 Application of photonic crystals	4
	1.2.1 Photonic crystals as low-loss mirrors: photonic bandgap effects	4
	1.2.2 Photonic crystals for out-of-bandgap operation	10
	<i>References</i>	11
2	Hamiltonian formulation of Maxwell's equations (frequency consideration)	14
	2.1 Plane-wave solution for uniform dielectrics	16
	2.2 Methods of quantum mechanics in electromagnetism	18
	2.2.1 Orthogonality of eigenstates	19
	2.2.2 Variational principle	20
	2.2.3 Equivalence between the eigenstates of two commuting Hamiltonians	22
	2.2.4 Eigenstates of the operators of continuous and discrete translations and rotations	23
	2.3 Properties of the harmonic modes of Maxwell's equations	30
	2.3.1 Orthogonality of electromagnetic modes	32
	2.3.2 Eigenvalues and the variational principle	32
	2.3.3 Absence of the fundamental length scale in Maxwell's equations	34
	2.4 Symmetries of electromagnetic eigenmodes	35
	2.4.1 Time-reversal symmetry	35
	2.4.2 Definition of the operators of translation and rotation	35
	2.4.3 Continuous translational and rotational symmetries	38
	2.4.4 Band diagrams	43
	2.4.5 Discrete translational and rotational symmetries	44

viii	Contents	
	2.4.6 Discrete translational symmetry and discrete rotational symmetry	52
	2.4.7 Inversion symmetry, mirror symmetry, and other symmetries	53
	2.5 Problems	55
3	One-dimensional photonic crystals – multilayer stacks	59
	3.1 Transfer matrix technique	59
	3.1.1 Multilayer stack, TE polarization	59
	3.1.2 Multilayer stack, TM polarization	61
	3.1.3 Boundary conditions	62
	3.2 Reflection from a finite multilayer (dielectric mirror)	63
	3.3 Reflection from a semi-infinite multilayer (dielectric photonic crystal mirror)	64
	3.3.1 Omnidirectional reflectors I	68
	3.4 Guiding in a finite multilayer (planar dielectric waveguide)	69
	3.5 Guiding in the interior of an infinitely periodic multilayer	70
	3.5.1 Omnidirectional reflectors II	80
	3.6 Defect states in a perturbed periodic multilayer: planar photonic crystal waveguides	82
	3.7 Problems	86
4	Bandgap guidance in planar photonic crystal waveguides	93
	4.1 Design considerations of waveguides with infinitely periodic reflectors	93
	4.2 Fundamental TE mode of a waveguide with infinitely periodic reflector	96
	4.3 Infinitely periodic reflectors, field distribution in TM modes	98
	4.3.1 Case of the core dielectric constant $\epsilon_c < \epsilon_h \epsilon_l / (\epsilon_h + \epsilon_l)$	98
	4.3.2 Case of the core dielectric constant $\epsilon_l \geq \epsilon_c > \epsilon_h \epsilon_l / (\epsilon_h + \epsilon_l)$	101
	4.4 Perturbation theory for Maxwell’s equations, frequency formulation	103
	4.4.1 Accounting for the absorption losses of the waveguide materials: calculation of the modal lifetime and decay length	104
	4.5 Perturbative calculation of the modal radiation loss in a photonic bandgap waveguide with a finite reflector	106
	4.5.1 Physical approach	106
	4.5.2 Mathematical approach	108
5	Hamiltonian formulation of Maxwell’s equations for waveguides (propagation-constant consideration)	110
	5.1 Eigenstates of a waveguide in Hamiltonian formulation	110
	5.1.1 Orthogonality relation between the modes of a waveguide made of lossless dielectrics	111

	Contents	ix
5.1.2	Expressions for the modal phase velocity	114
5.1.3	Expressions for the modal group velocity	114
5.1.4	Orthogonality relation between the modes of a waveguide made of lossy dielectrics	115
5.2	Perturbation theory for uniform variations in a waveguide dielectric profile	116
5.2.1	Perturbation theory for the nondegenerate modes: example of material absorption	118
5.2.2	Perturbation theory for the degenerate modes coupled by perturbation: example of polarization-mode dispersion	120
5.2.3	Perturbations that change the positions of dielectric interfaces	123
5.3	Problems	126
	<i>References</i>	127
6	Two-dimensional photonic crystals	129
6.1	Two-dimensional photonic crystals with diminishingly small index contrast	129
6.2	Plane-wave expansion method	132
6.2.1	Calculation of the modal group velocity	134
6.2.2	Plane-wave method in 2D	134
6.2.3	Calculation of the group velocity in the case of 2D photonic crystals	135
6.2.4	Perturbative formulation for the photonic crystal lattices with small refractive index contrast	138
6.2.5	Photonic crystal lattices with high-refractive-index contrast	142
6.3	Comparison between various projected band diagrams	142
6.4	Dispersion relation at a band edge, density of states and Van Hove singularities	144
6.5	Refraction from photonic crystals	147
6.6	Defects in a 2D photonic crystal lattice	148
6.6.1	Line defects	148
6.6.2	Point defects	158
6.7	Problems	167
	<i>References</i>	171
7	Quasi-2D photonic crystals	172
7.1	Photonic crystal fibers	172
7.1.1	Plane-wave expansion method	172
7.1.2	Band diagram of modes of a photonic crystal fiber	176
7.2	Optically induced photonic lattices	177
7.2.1	Light propagation in low-index-contrast periodic photonic lattices	178
7.2.2	Defect modes in 2D photonic lattices with localized defects	181
7.2.3	Bandgap structure and diffraction relation for the modes of a uniform lattice	182

7.2.4	Bifurcations of the defect modes from Bloch band edges for localized weak defects	185
7.2.5	Dependence of the defect modes on the strength of localized defects	188
7.2.6	Defect modes in 2D photonic lattices with nonlocalized defects	192
7.3	Photonic-crystal slabs	195
7.3.1	Geometry of a photonic-crystal slab	195
7.3.2	Eigenmodes of a photonic-crystal slab	197
7.3.3	Analogy between the modes of a photonic-crystal slab and the modes of a corresponding 2D photonic crystal	200
7.3.4	Modes of a photonic-crystal slab waveguide	204
7.4	Problems	207
	<i>References</i>	208
8	Nonlinear effects and gap–soliton formation in periodic media	210
8.1	Solitons bifurcated from Bloch bands in 1D periodic media	211
8.1.1	Bloch bands and bandgaps	211
8.1.2	Envelope equations of Bloch modes	212
8.1.3	Locations of envelope solitons	215
8.1.4	Soliton families bifurcated from band edges	216
8.2	Solitons bifurcated from Bloch bands in 2D periodic media	218
8.2.1	Two-dimensional Bloch bands and bandgaps of linear periodic systems	219
8.2.2	Envelope equations of 2D Bloch modes	220
8.2.3	Families of solitons bifurcated from 2D band edges	223
8.3	Soliton families not bifurcated from Bloch bands	226
8.4	Problems	227
	<i>References</i>	228
	Problem solutions	230
	Chapter 2	230
	Chapter 3	236
	Chapter 5	244
	Chapter 6	246
	Chapter 7	257
	Chapter 8	260
	<i>Index</i>	263

Preface

The field of photonic crystals (aka periodic photonic structures) is experiencing an unprecedented growth due to the dramatic ways in which such structures can control, modify, and harvest the flow of light.

The idea of writing this book came to M. Skorobogatiy when he was developing an introductory course on photonic crystals at the Ecole Polytechnique de Montréal/University of Montréal. The field of photonic crystals, being heavily dependent on numerical simulations, is somewhat challenging to introduce without sacrificing the qualitative understanding of the underlying physics. On the other hand, exactly solvable models, where the relation between physics and quantitative results is most transparent, only exist for photonic crystals of trivial geometries. The challenge, therefore, was to develop a presentational approach that would maximally use intuitive analytical and semi-analytical models, while applying them to complex and practically relevant photonic crystal structures.

We would like to note that the main purpose of this book is not to present the latest advancements in the field of photonic crystals, but rather to give a systematic, logical, and pedagogical introduction to this vibrant field. The text is largely aimed at students and researchers who want to acquire a rigorous, while intuitive, mathematical introduction into the subject of guided modes in photonic crystals and photonic crystal waveguides. The text, therefore, favors analysis of analytically or semi-analytically solvable problems over pure numerical modeling. We believe that this is a more didactical approach when trying to introduce a novice into a new field. To further stimulate understanding of the book content, we suggest many exercise problems of physical relevance that can be solved analytically.

In the course of the book we extensively use the analogy between the Hamiltonian formulation of Maxwell's equations and the Hamiltonian formulation of quantum mechanics. We present both frequency and propagation-constant based Hamiltonian formulations of Maxwell's equations. The latter is particularly useful for analyzing photonic crystal-based linear and nonlinear waveguides and fibers. This approach allows us to use a well-developed machinery of quantum mechanical semi-analytical methods, such as perturbation theory, asymptotic analysis, and group theory, to investigate many of the limiting properties of photonic crystals, which are otherwise difficult to investigate based only on numerical simulations.

M. Skorobogatiy has contributed Chapters 2, 3, 4, 5, and 6 of this book, and J. Yang has contributed Chapter 8. Chapters 1 and 7 were co-authored by both authors.

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