

# Contents

---

	<b>Preface</b>	<b>xi</b>
	Notations, Cross-references, References	xiii
<b>1</b>	<b>Historical introduction</b>	<b>1</b>
	1.1. Definition of the zeta-function, 1.2. Euler's proof of the infinity of primes, 1.3. The values of $\zeta(2k)$ , 1.4. Riemann and the zeta-function, 1.5. The Hadamard Product Formula, 1.6. The explicit formula, 1.7. The Prime Number Theorem, 1.8. The Riemann Hypothesis and the Prime Number Theorem, 1.9. The approximate functional equation, 1.10. Disclaimer, 1.11. Some number-theoretic functions.	
	Exercises	10
<b>2</b>	<b>The Poisson Summation Formula and the functional equation</b>	<b>14</b>
	2.1. Summation formulae, 2.2. Poisson Summation Formula, 2.3. Mellin transform, 2.4. Cut-off functions, 2.5. Integral representation, 2.6. Local functional equation, 2.7. Analytic continuation and the functional equation, 2.8. Proofs, 2.9. Variant, 2.10. Further variant, 2.11. Estimates, 2.12. Growth in vertical strips, 2.13. First step towards the 'approximate functional equation', 2.14. Derivation of the Poisson Summation Formula from properties of the zeta-function, 2.15. Remarks.	
	Exercises	27
<b>3</b>	<b>The Hadamard Product Formula and 'explicit formulae' of prime number theory</b>	<b>33</b>
	3.1. Hadamard Product Formula, 3.2. Expansion of the logarithmic derivative, 3.3. Estimates for the logarithmic derivative, 3.4. Estimate on the zeros, 3.5. Proofs, 3.6. Explicit formula, 3.7. Proof, 3.8. Explicit formula, approximate version, 3.9. Explicit formula, classical version, 3.10. Proof, 3.11. Closing remarks.	
	Exercises	46
<b>4</b>	<b>The zeros of the zeta-function and the Prime Number Theorem</b>	<b>50</b>
	4.1. Introductory remarks, 4.2. Hadamard's zero-free region, 4.3. Prime Number Theorem, first version, 4.4. Proof of theorem 4.3, 4.5. Prime Number Theorem, second version, 4.6. The functions $N$ and $S$ , 4.7. The basic estimate of $S$ , 4.8. Further remarks, 4.9. Asymptotic estimate for $N$ , 4.10. Proof of Theorem 4.9, 4.11. The function $N(\sigma, T)$ , 4.12. The quadratic mean of the zeta-function, 4.13. Proof of Theorem 4.11, 4.14. Proof of Theorem 4.12, 4.15. General remarks on higher means.	
	Exercises	64
<b>5</b>	<b>The Riemann Hypothesis and the Lindelöf Hypothesis</b>	<b>67</b>
	5.1. Formulation of the Riemann Hypothesis, 5.2. Formulation of the Lindelöf Hypothesis, 5.3. Equivalent forms of the Lindelöf Hypothesis, 5.4. The Lindelöf Hypothesis and the zeros, 5.5. The Riemann Hypothesis implies the Lindelöf Hypothesis, 5.6. Proof of Theorem 5.4, 5.7. A further consequence of the Lindelöf	

Hypothesis, 5.8. The Riemann Hypothesis and prime numbers, 5.9. Proof of Theorem 5.8, 5.10. Reasons for believing the Riemann Hypothesis, 5.11. Zeta-functions of curves over finite fields,* 5.12. Places of a function-field,* 5.13. The zeta-function of a function-field,* 5.14. Analogue of the 'explicit formulae',* 5.15. Weil's criterion for the Riemann Hypothesis for the zeta-function of a function-field, 5.16. Weil's criterion for the Riemann Hypothesis,* 5.17. Weil's criterion and the Jacobian of the curve, *5.18. Significance of the previous section for the Riemann Hypothesis, 5.19. Concluding remarks on the Riemann Hypothesis.	87
Exercises	
<b>6 The approximate functional equation</b>	<b>91</b>
6.1. Formulation of the approximate functional equation, 6.2. A special case, 6.3. Estimates of certain integrals, 6.4. Proof of Theorem 6.1, 6.5. Integral means, 6.6. The estimate $\zeta(1/2 + it) = O( t ^{1/6} \log  t )$ , 6.7. Van der Corput's Summation Formula, 6.8. Weyl's Lemma, 6.9. Exponential sums, 6.10. More estimates on exponential sums, 6.11. Proof of Theorem 6.6.	
Exercises	108
<b>Appendices</b>	
<b>1 Fourier theory</b>	<b>111</b>
A1.1. The Riemann–Lebesgue Lemma, A1.2. The variation of a function, A1.3. A lemma, A1.4. The Dirichlet kernel, A1.5. Representation of a periodic function by a Fourier series, A1.6. A useful estimate, A1.7. The $L^2$ -theory, A1.8. Fourier integrals and the inversion formula, A1.9. Another useful estimate, A1.10. The Plancherel Theorem, A1.11. Convolution of periodic functions, A1.12. Convolution of integrable functions, A1.13. The Poisson Summation Formula.	
Exercises	119
<b>2 The Mellin transform</b>	<b>122</b>
A2.1. Definition of the Mellin transform, A2.2. The inversion formula, A2.3. Convolution, A2.4. A variant, A2.5. Perron's Formula.	
Exercises	124
<b>3 An estimate for certain integrals</b>	<b>127</b>
A3.1. An estimate for certain integrals, A3.2. Complements, A3.3. Further complements.	
Exercises	128
<b>4 The gamma-function</b>	<b>130</b>
A4.1. Definition of the gamma-function, A4.2. The beta-function, A4.3. The infinite product, A4.4. The logarithmic derivative, A4.5. A further integral formula, A4.6. Stirling's Theorem, A4.7. The functional equation, A4.8. Stirling's Theorem.	
Exercises	135
<b>5 Integral functions of finite order</b>	<b>138</b>
A5.1. Introductory remarks, A5.2. Jensen's Theorem, A5.3. Integral functions of finite order, A5.4. Integral functions of finite order without poles or zeros, A5.5. The general representation theorem, A5.6. A Maximum Principle for integral functions, A5.7. An extension of the previous section, A5.8. The Phragmén–Lindelöf Theorem, A5.9. A consequence of the Phragmén–Lindelöf Theorem, A5.10. Other convexity theorems.	
Exercises	145

Cambridge University Press

978-0-521-49905-7 - An Introduction to the Theory of the Riemann Zeta-Function

S. J. Patterson

Table of Contents

[More information](#)

<i>Contents</i>	ix
<b>6 Borel–Caratheodory Theorems</b>	<b>146</b>
A6.1. General remarks, A6.2. The Borel–Caratheodory Theorem, A6.3. An estimate of the argument.	
<b>7 Littlewood’s Theorem</b>	<b>148</b>
A7.1. Littlewood’s Theorem, A7.2.A theorem of Backlund.	
<b>Bibliography</b>	<b>152</b>
<b>Index</b>	<b>155</b>