This book provides a thorough examination of the use of elasticity in solving geotechnical engineering problems in a style that will be accessible to upper level students in civil engineering, geological engineering, and of the earth sciences. The first two chapters present a basic framework of the theory of elasticity and describe test procedures for determination of elastic parameters for soils. Chapters 3 and 4 present the fundamental solutions of Boussinesq, Kelvin, Cerrutti, and Mindlin, and use these to formulate solutions to problems of practical interest in geotechnical engineering. The book concludes with a sequence of appendixes designed to provide the interested student with details of the theory of elasticity that would be of considerable assistance to a deeper understanding of the main text.

# ELASTICITY AND GEOMECHANICS

# ELASTICITY AND GEOMECHANICS

R. O. DAVIS
*University of Canterbury*

A. P. S. SELVADURAI
*McGill University*

CAMBRIDGE UNIVERSITY PRESS

CAMBRIDGE UNIVERSITY PRESS
Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore,
São Paulo, Delhi, Dubai, Tokyo

Cambridge University Press
The Edinburgh Building, Cambridge CB2 8RU, UK

Published in the United States of America by Cambridge University Press, New York

www.cambridge.org
Information on this title: www.cambridge.org/9780521498272



First published 1996

*A catalogue record for this publication is available from the British Library*

*Library of Congress Cataloguing in Publication data*

Davis, R. O.
Elasticity and geomechanics / R. O. Davis, A. P. S. Selvadurai.
p. cm.
Includes bibliographical references.
ISBN 0-521-49506-7 (hc). – ISBN 0-521-49827-9 (pbk.)
1. Engineering geology – Mathematics. 2. Elasticity.
I. Selvadurai, A. P. S. II. Title.
TA705.D3 1996
624.1′51 – dc20 95-17507
CIP

ISBN 978-0-521-49506-6 Hardback
ISBN 978-0-521-49827-2 Paperback

Transferred to digital printing 2009

# Contents

# Preface

The linear theory of elasticity has enjoyed a fairly long and profitable history in the field of foundation engineering. Geotechnical engineers have turned to elasticity for answers to a variety of questions, and despite the sure knowledge that the answers are at best approximate, they will continue to do so for some time. The reasons for this lie in the essential simplicity of the relevant elastic solutions. The point load solutions of Boussinesq and others, as well as the distributed load solutions that derive from them, can provide answers with only a few lines of calculations. This can be a significant advantage when compared with the time and effort involved in obtaining numerical solutions that employ one of the multitude of existing plasticity models for soil. Furthermore, quite a lot of information concerning soil properties is required to run any of the various plasticity models. In many practical situations, the information may simply be unavailable, and the geotechnical engineer is left with few alternatives. In contrast, elasticity solutions will generally require only a value for the soil modulus and Poisson's ratio, and if the values given are known to be rough approximations, then at least the solution method is in keeping with the input data.

This book is about the use of elasticity in solving geotechnical engineering problems. It is directed toward upper level students in civil engineering or engineering geology. It was motivated when, in 1992, Patrick Selvadurai visited the University of Canterbury. There he found a course taught by Rob Davis, similar to a course Patrick himself taught at Carleton University. Both courses had evolved independently but were surprisingly similar in content, covering some basic applications of elasticity theory in geotechnical engineering.

The book grew from lecture notes and is intended solely as a teaching tool. It is not exhaustive in content but is sufficiently complete to provide a grounding in the linear theory of elasticity, together with an understanding of applications in foundation engineering. The presentation is deliberately informal and conversational in tone. From the standpoint of a student, it should not be

intimidating, but should hopefully put a variety of new ideas into an accessible format.

Four chapters form the main body of the book. Chapter 1 presents a framework of basic ideas from the linear theory of elasticity; deformation, strain and stress, equilibrium and compatibility, and the formulation of problems. Chapter 2 delves into Hooke's law and the elastic constants. The first half of this chapter describes the elastic constants and explains their relation to the material in Chapter 1, while the second half discusses how elastic constants may be determined in a geotechnical context. In Chapter 3 the point load problems of Boussinesq, Kelvin, Cerrutti, and Mindlin, as well as the line load problem of Flamant, are described. The solutions to these problems are presented but techniques for finding solutions are not discussed. The thrust of the book is not to provide another elasticity text. The point load problems are fundamental to geotechnical applications and while their solutions are examined, the corresponding solution methods are of only marginal interest. Chapter 4 uses the solutions from Chapter 3 to consider some basic problems in foundation engineering. The development progresses from consideration of a simple uniformly loaded region on the surface of a homogeneous elastic half-space to more challenging problems involving nonuniform loads, rigid foundations, and layered half-spaces. Some remarks concerning consolidation and in situ testing complete this chapter. Finally, a sequence of appendixes bring the book to a close. These are an important component of the book, designed to provide the interested student with details of elasticity theory that are peripheral to the main text. The compatibility conditions, development of the stress tensor, Saint-Venant's principle, uniqueness, the virtual work principle, and reciprocity relations are all considered in the appendixes.

The book draws together material from a range of sources. The description of the linear theory of elasticity is phrased in a physical rather than purely mathematical context. While mathematical formulations are not avoided, they are construed as the end result of physical thought processes. Examples are used to illustrate important concepts, and a set of problems is placed at the end of each of the four chapters. The book does not directly compete with any other text. While nearly all undergraduate soil mechanics texts contain some elastic analyses, none are devoted to elastic theory. Two other books, Poulos and Davis' *Elastic Solutions in Soil and Rock Mechanics* and Selvadurai's *Elastic Analysis of Soil-Foundation Interaction*, both involve geotechnical uses of elasticity; but the former is a catalog of solutions, while the latter is a treatise. Neither would easily serve as a text for undergraduate students.

Finally, we would like to express our sincere appreciation to two people whose assistance made preparation of the book a far more pleasant task than it might have been. First, all the text and equations were typed with remarkable care and accuracy by Mrs. Pat Roberts. Second, the figures were skillfully drawn by Mrs. Val Grey. To both these people we offer our thanks for a job well done.

R. O. Davis
Christchurch
July 1995

A. P. S. Selvadurai
Montreal
July 1995