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978-0-521-49789-3 - The Mathematics of Financial Derivatives: A Student Introduction

Paul Wilmott, Sam Howison and Jeff Dewynne

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