Part one

Basic Option Theory

1 An Introduction to Options and Markets

1.1 Introduction

This book is about mathematical models for financial markets, the assets that are traded in them and, especially, financial derivative products such as options and futures. There are many kinds of financial market, but the most important ones for us are:

- **Stock markets**, such as those in New York, London and Tokyo;
- **Bond markets**, which deal in government and other bonds;
- **Currency markets** or **foreign exchange markets**, where currencies are bought and sold;
- **Commodity markets**, where physical assets such as oil, gold, copper, wheat or electricity are traded;
- **Futures** and **options** markets, on which the derivative products that are the subject of this book are traded.

The reader may not have encountered all of the financial terms in bold face in this list. Most will be explained in detail later in the book when we need them. However, we do assume that the *raison d'être* of the currency and commodity markets is clear, and we hope that readers are familiar with the idea behind **stocks** (also known as **shares** or **equities**). Roughly speaking, a company that needs to raise money, for example to build a new factory or develop a new product, can do so by selling shares in itself to investors. The company is then 'owned' by its shareholders; if the company makes a profit, part of this may be paid out to shareholders as a **dividend** of so much per share, and if the company is taken over or otherwise wound up, the proceeds (if any) are distributed to shareholders. Shares thus have a value that reflects the views of investors about the likely future dividend payments and capital growth of the company; this value is quantified by the price at which they are bought and sold on stock exchanges.[1]

We have, then, a collection of markets on which assets of various kinds are bought and sold. As markets have become more sophisticated, more complex contracts than simple buy/sell trades have been introduced. Known as **financial derivatives**, **derivative securities**, **derivative products**, **contingent claims** or just **derivatives**, they can give investors of all kinds a great range of opportunities to tailor their dealings to their investment needs. This book explains some of the financial theory and models that have been developed to analyse derivatives, a theory that is necessarily mathematical in character (the specialists now employed by all major financial institutions to work in this area are called 'rocket scientists'!), but which is at bottom a very elegant and clear combination of mathematical modelling and analysis. First, though, we need to become familiar with some of the necessary financial jargon, and to see how derivatives work. We begin with the example of an option, which is one of the commonest examples of a derivative security.

1.2 What is an Option?

The simplest financial option, a **European call option**, is a contract with the following conditions:

- At a prescribed time in the future, known as the **expiry date** or **expiration date**, the **holder** of the option *may*
- purchase a prescribed asset, known as the **underlying asset** or, briefly, the **underlying**, for a
- prescribed amount, known as the **exercise price** or **strike price**.

The word 'may' in this description implies that for the holder of the option, this contract is a *right* and not an *obligation*. The other party to the contract, who is known as the **writer**, does have a potential obligation: he *must* sell the asset if the holder chooses to buy it. Since the option confers on its holder a right with no obligation it has some value. Moreover, it must be paid for at the time of opening the contract. Conversely, the writer of the option must be compensated for the obligation he has assumed. Two of our main concerns throughout this book are:

[1] In practice, companies may have a much more complex structure for their equity, but in an introductory text we try not to get enmeshed in these details. The ideas we describe carry over with the appropriate modifications throughout.

- How much would one pay for this right, i.e. what is the value of an option?
- How can the writer minimise the risk associated with his obligation?

A Simple Example: A Call Option

How much is the following option now worth? Today's date is 22 August 1995.

- On 14 April 1996 the holder of the option *may*
- purchase one XYZ share for 250p.

In order to gain an intuitive feel for the price of this option let us imagine two possible situations that might occur on the expiry date, 14 April 1996, nearly eight months in the future.

If the XYZ share price is 270p on 14 April 1996, then the holder of the option would be able to purchase the asset for only 250p. This action, which is called **exercising** the option, yields an immediate profit of 20p. That is, he can buy the share for 250p and immediately sell it in the market for 270p:

$$270\text{p} - 250\text{p} = 20\text{p} \ \text{ profit.}$$

On the other hand, if the XYZ share price is only 230p on 14 April 1996 then it would not be sensible to exercise the option. Why buy something for 250p when it can be bought for 230p elsewhere?

If the XYZ share only takes the values 230p or 270p on 14 April 1996, with equal probability, then the expected profit to be made is

$$\tfrac{1}{2} \times 0 \ + \ \tfrac{1}{2} \times 20 = 10\text{p}.$$

Ignoring interest rates for the moment, it seems reasonable that the order of magnitude for the value of the option is 10p.

Of course, valuing an option is not as simple as this, but let us suppose that the holder did indeed pay 10p for this option. Now if the share price rises to 270p at expiry he has made a net profit calculated as follows:

profit on exercise	=	20p
cost of option	=	−10p
net profit	=	10p

This net profit of 10p is 100% of the up-front premium. The downside of this speculation is that if the share price is less than 250p at expiry he has lost all of the 10p invested in the option, giving a loss of 100%. If the investor had instead purchased the share for 250p on 22 August 1995, the corresponding profit or loss of 20p would have been only ±8% of the original investment. Option prices thus respond in an exaggerated way to changes in the underlying asset price. This effect is called **gearing**.

We can see from this simple example that the greater the share price on 14 April 1996, the greater the profit. Unfortunately, we do not know this share price in advance. However, it seems reasonable that the higher the share price is now (and this is something we *do* know) then the higher the price is likely to be in the future. Thus the value of a call option *today* depends on today's share price. Similarly, the dependence of the call option value on the exercise price is obvious: the lower the exercise price, the less that has to be paid on exercise, and so the higher the option value.

Implicit in this is that the option is to expire a significant time in the future. Just before the option is about to expire, there is little time for the asset price to change. In that case the price at expiry is known with a fair degree of certainty. We can conclude that the call option price must also be a function of the time to expiry.

Later we also see how the option price depends on a property of the 'randomness' of the asset price, the volatility. The larger the volatility, the more jagged is the graph of asset price against time. This clearly affects the distribution of asset prices at expiry, and hence the expected return from the option. The value of a call option should therefore depend on the volatility. Finally, the option price must depend on prevailing bank interest rates; the option is usually paid for up-front at the opening of the contract whereas the payoff, if any, does not come until later. The option price should reflect the income that would otherwise have been earned by investing the premium in the bank.

Put Options

The option to *buy* an asset discussed above is known as a **call** option. The right to *sell* an asset is known as a **put** option and has payoff properties which are opposite to those of a call. A put option allows its holder to sell the asset on a certain date for a prescribed amount. The writer is then obliged to buy the asset. Whereas the holder of a call option wants the asset price to rise – the higher the asset price at expiry the greater the profit – the holder of a put option wants the asset price to fall as low as possible. The value of a put option also increases with the exercise price, since with a higher exercise price more is received for the asset at expiry.

1.3 Reading the Financial Press

Armed with the jargon of calls, puts, expiry dates and so forth, we are in a position to read the options pages in the financial press. Our examples are taken from the *Financial Times* of Thursday 4 February 1993.

In Figure 1.1 is shown the traded[1] options section of the *Financial Times*. This table shows the prices of some of the options traded on the London International Financial Futures and Options Exchange (LIFFE). The table lists the last quoted prices on the previous day for a large number of options, both calls and puts, with a variety of exercise prices and expiry dates. Most of these examples are options on individual equities, but at the bottom of the third column we see options on the *FT-SE* index, which is a weighted arithmetic average of 100 equity shares quoted on the London Stock Exchange.

First, let us concentrate on the prices quoted for Rolls-Royce options, to be found in the third column labelled 'R. Royce'. Immediately beneath R. Royce is the number 134 in parentheses. This is the closing price, in pence, of Rolls-Royce shares on the previous day. To the right of R. Royce/(134) are the two numbers 130 and 140: these are two exercise prices, again in pence. Note that for equity options the *Financial Times* prints only those exercise prices each side of the closing price. Many other exercise prices exist (at intervals of 10p in this case) but are not printed in the *Financial Times* for want of space.

Now examine the six numbers to the right of the 130. The first three (11, 15, 19) are the prices of call options with different expiry dates, and the next three (9, 14, 17) are the prices of put options. The expiry date of each of these options can be found by looking at the top of its column. There we see that there are options on Rolls-Royce shares expiring in March, June and September, at a specified time on a specified date in each month, in this case at 18:00 on the third Wednesday of the month concerned (trading ceases slightly earlier). Option prices are quoted on an exchange only for a small number of expiry dates and only for exercise prices at discrete intervals (here ..., 130, 140, ...). For LIFFE-traded options on equities the expiry dates come in intervals of three months. When it is created, the longest dated option has a lifespan

[1] The word 'traded' here refers to an option that is traded on an exchange such as LIFFE or the CBOE (Chicago Board Options Exchange).

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LIFFE EQUITY OPTIONS

Option		Calls Apr	Calls Jul	Calls Oct	Puts Apr	Puts Jul	Puts Oct
Alld Lyons	550	48	60	68	10	24	30
(*585)	600	22	34	44	34	50	55
ASDA	57	9	12½	14	4	7	9
(*61)	67	4	8	9	8½	11½	14
Brit Airways	280	27	33	39	9½	20	24
(*296)	300	17	24	29	18	30	34
SmKl Bchm A	460	30	47	53	17	26	52
(*470)	500	13	27	36	42	48	54
Boots	500	23	32	43	21	33	37
(*501)	550	6½	15	23	56	67	70
B P	260	16½	23	29	11	17	21
(*265)	280	8	15	19	22	28	34
British Steel	70	13	16½	19	3½	6½	8½
(*79)	80	8	12	14	7½	11½	13
Bass	600	31	48	61	24	38	45
(*604)	650	12	27	39	58	70	73
C & Wire	700	40	60	70	25	43	50
(*710)	750	18	35	46	56	73	80
Courtaulds	550	40	52	61	17	30	37
(*568)	600	16	29	38	45	60	64
Com Union	600	35	49	57	21	32	40
(*623)	650	11	26	34	53	61	70
Fisons	220	22	31	39	18	30	35
(*222)	240	14	22	31	31	43	48
GKN	460	27	36	45	21	28	34
(*472)	500	8½	20	28	48	53	57
Grand Met	420	38	53	59	11	21	27
(*439)	460	16	31	36	30	41	47
I C I	1100	53	82	92	50	68	88
(*1132)	1150	31	62	72	80	100	117
Kingfisher	550	34	48	53	21	38	45
(*559)	600	12	25	33	52	68	73
Ladbroke	200	16	25	29	18	26	32
(*202)	220	8	16	22	32	38	44
Land Secur	460	45	49	53	5	17	21
(*493)	500	17	25	31	20	38	41
M & S	330	18	25	34	12	20	24
(*333)	360	6	12	20	32	38	41
Sainsbury	550	43	54	63	12	24	29
(*577)	600	16	28	38	38	50	56
Shell Trans	550	32	44	50	12	19	26
(*576)	600	6	19	25	44	47	53
Storehouse	200	18	26	34	8	17	18
(*204)	220	11	17	22	21	25	28
Trafalgar	90	11	14	18	8	9	13
(*93)	100	6½	11	14	12	17	18
Utd Biscuits	360	18	25	33	19	25	29
(*366)	390	6	14	20	41	45	49
Unilever	1100	72	90	110	16	35	42
(*1149)	1150	39	60	82	42	56	63

Option		Calls Feb	Calls May	Calls Aug	Puts Feb	Puts May	Puts Aug
Brit Aero	280	23	40	53	16	34	46
(*287)	300	14	33	47	27	49	60
BAA	750	41	61	71	6	16	31
(*786)	800	11	33	45	29	41	55
BAT Inds	950	45	58	74	8	37	46
(*982)	1000	16	33	50	30	64	73
BTR	550	22	30	38	5½	20	28
(*565)	600	3	10	19	38	53	55
Brit Telecom	420	10	23	29	7	15	24
(*420)	460	1	8	12½	40	41	50
Cadbury Sch	460	15	25	35	8	24	30
(*465)	500	3½	10	19	37	51	55
Eastern Elec	400	18	32	-	6	15	-
(*411)	430	5	-	-	24	-	-
Guinness	460	24	36	46	8	25	30
(*473)	500	6	19	28	33	48	56
GEC	300	9	19	23	7	13	21
(*301)	330	1	7	11	30	32	41
Hanson	260	7½	14½	18	5½	12	16½
(*262)	280	1	5½	9½	20	25	28
LASMO	160	8	18	22	9	18	23
(*161)	180	3	8½	15	24	33	36
Lucas Inds	140	15	23	26	4	11	16
(*151)	160	4	13	17	14	24	28
P & O	550	24	42	52	15	40	52
(*564)	600	5½	21	33	45	72	82
Pilkington	100	7	15	18	6	12	16
(*102)	110	4	10	14	12	17	22
Prudential	300	24	30	34	2½	11	15
(*321)	330	6	13	19	14	26	31
R T Z	650	34	47	60	9	30	40
(*672)	700	10	25	39	36	59	69
Scot & New	320	23	38	45	5	13	24
(*343)	360	4½	17	24	26	35	48
Tesco	240	22	27	30	2	8	12
(*257)	260	7	14	21	9	19	22
Thames Wtr	460	24	37	42	3½	12	22
(*479)	500	3½	16	20	25	32	45
Vodafone	390	18	34	43	8	21	28
(*398)	420	5½	19	29	27	36	45

Option		Calls Mar	Calls Jun	Calls Sep	Puts Mar	Puts Jun	Puts Sep
Abbey Nat	360	28	34	42	11	17	23
(*379)	390	11	19	27	27	33	38
Amstrad	20	5½	6½	7½	1½	1½	2½
(*24)	25	2½	3½	4½	3½	4	4½
Barclays	420	45	54	59	11	21	30
(*458)	460	20	33	38	33	41	51
Blue Circle	220	22	30	35	10	21	27
(*230)	240	12	19	27	22	33	39
British Gas	280	15½	19	23	6½	15	18
(*287)	300	6½	9½	14	18	27	30
Dixons	220	15	25	28	13	19	25
(*221)	240	8	17	21	26	31	38
Eurotunnel	420	38	55	70	20	35	45
(*435)	460	18	37	50	45	57	67
Glaxo	650	38	60	82	29	42	52
(*664)	700	17	38	58	60	72	80
Hillsdown	140	17	22	24	6	15	20
(*147)	160	6	12	16	18	27	31
Lonrho	70	9	12	15	5	8½	11
(*75)	80	4½	8	11½	11	14	17
HSBC 75p shs	550	45	57	70	17	34	46
(*576)	600	18	32	48	44	60	70
Natl Power	280	24	32	36	6	11	18
(*294)	300	11	21	25	15	20	27
Reuters	1400	69	108	135	47	78	100
(*1426)	1450	40	82	113	78	103	122
R Royce	130	11	15	19	9	14	17
(*134)	140	6	11	16	16	20	23
Scot Power	200	20	25	28	2	5	10½
(*216)	220	6	13	17	9	13½	22
Sears	100	9	11	15	5	8½	11
(*104)	110	4	7	9½	10	14	16
Forte	180	15	20	25	10	19	24
(*186)	200	7	13	17	21	32	36
Thorn EMI	800	61	81	90	10	22	39
(*843)	850	28	50	60	29	43	66
TSB	160	19	23	27	5	8½	12
(*176)	180	6½	13	17	15	18	22
Vaal Reefs	30	6	6	6½	3	4	4½
(*534)	35	2	4	5½	5½	6	8
Wellcome	850	50	75	100	28	48	63
(*866)	900	26	52	75	57	77	92

EURO FT-SE INDEX (*2872)

	2675	2725	2775	2825	2875	2925	2975	3025
CALLS								
Feb	205	155	108	68	36	16	5	2
Mar	215	170	129	92	61	38	22	11
Apr	-	188	-	115	-	61	-	28
Jun	-	217	-	152	-	100	-	60
Sep	-	252	-	188	-	129	-	-
PUTS								
Feb	1½	3½	7	15	34	65	104	151
Mar	9	12	21	34	53	81	114	153
Apr	-	26	-	51	-	97	-	163
Jun	-	50	-	80	-	125	-	185
Sep	-	-	-	120	-	-	-	-

FT-SE INDEX (*2872)

	2650	2700	2750	2800	2850	2900	2950	3000
CALLS								
Feb	233	183	135	89	50	24	9	3
Mar	243	197	153	113	79	51	31	17
Apr	254	210	170	131	99	73	51	36
May	266	226	186	151	121	93	72	52
Jun	-	235	-	164	-	107	-	67
Dec †	-	-	-	235	-	187	-	130
PUTS								
Feb	1½	2½	4½	10	23	47	85	131
Mar	8	12	17	29	46	69	100	138
Apr	16	22	31	44	63	86	116	151
May	26	35	45	61	80	105	133	166
Jun	-	40	-	70	-	113	-	175
Dec †	-	90	-	125	-	185	-	-

February 3 Total Contracts 31,257
Calls 21,861 Puts 9,396
FT-SE Index Calls 7,946 Puts 4,410
Euro FT-SE Calls 816 Puts 278
*Underlying security price † Long dated expiry mths
Premiums shown are based on closing offer prices

Figure 1.1 The traded options section of the *Financial Times* of 4 February 1993.

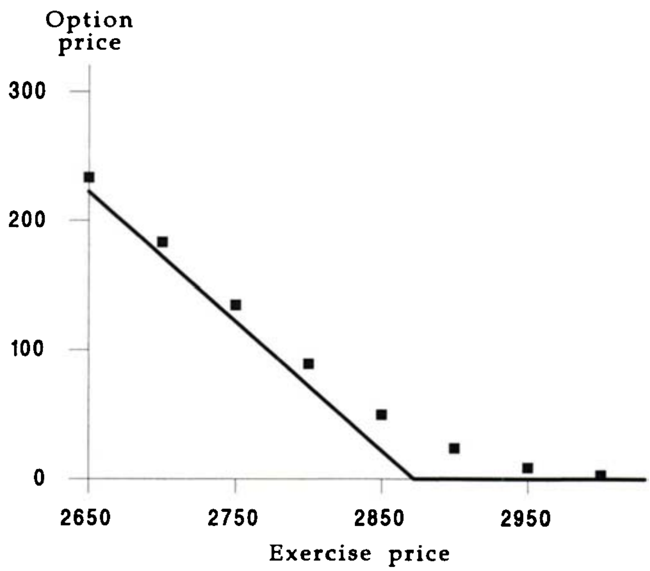

Figure 1.2 The *FT-SE* index call option values versus exercise price and the option values at expiry assuming that the index value is then 2872.

of nine months. Later in the year the December series of Rolls-Royce options will come into being.

Since a call option permits the holder to pay the exercise price to obtain the asset, we can see that call options with exercise price 140p are cheaper than those with exercise price 130p. This is because more must be paid for the share at exercise. The converse is true for puts: the holder of a 140p put can realise more by selling the share at exercise than the holder of a 130p put, and so the former is worth more.

Now let us look at the options on the *FT-SE* index. Towards the bottom of the third column we see prices for the *FT-SE* index call options. (Although the index is just a number, the contract is given a nominal price in pounds equal to 10 times the *FT-SE* value.) The exercise prices are quoted at intervals of 50 from 2650 to 3000 and expiry dates at monthly intervals. Since these options expire on the third Friday of the month, the February options have only about 10 days left. In Figure 1.2 we plot the value of the February call options against exercise price.

The closing value of the *FT-SE* index on 3 February 1993 was 2872. Suppose that the *FT-SE* index had exactly the same value at expiry as on 3 February 1993. Then the value of each call option contract at expiry would be the 'ramp function'

$$\begin{array}{ll} \pounds 10 \times (2872 - \text{exercise value}) & \text{for exercise value} \leq 2872 \\ 0 & \text{for exercise value} \geq 2872. \end{array}$$

In Figure 1.2 we also plot this ramp function. Notice that the data points are close to but above the ramp function. The difference between the two is due to the indeterminacy in the future index value: the index is unlikely to be at 2872 at the time of expiry of the February options. We return to the example of the *FT-SE* index call options in Chapter 3.

Finally, note that for each option type there is only one quoted price in this table. In reality the option could not be bought and sold for the same price since the market-maker has to make a living. Thus there are *two* prices for the option. The investor pays the ask (or offer) price and sells for the bid price, which is less than the ask price. The price quoted in the newspapers is usually a mid-price, the average of the bid and ask prices. The difference between the two prices is known as the bid-ask or bid-offer spread.

Technical Point: The trading of options.
Before 1973 all option contracts were what is now called 'over-the-counter' (OTC). That is, they were individually negotiated by a broker on behalf of two clients, one being the buyer and the other the seller. Trading on an official exchange began in 1973 on the Chicago Board Options Exchange (CBOE), with trading initially only in call options on some of the most heavily traded stocks. As increased competition followed the listing of options on an exchange, the cost of setting up an option contract decreased significantly.

Options are now traded on all of the world's major exchanges. They are no longer restricted to equity options but include options on indices, futures, government bonds, commodities, currencies etc. The OTC market still exists, and options are written by institutions to meet a client's needs. This is where exotic option contracts are created; they are very rarely quoted on an exchange.

When an option contract is initiated there must be two sides to the agreement. Consider a call option. On one side of the contract is the buyer, the party who has the right to exercise the option. On the other side is the party who must, if required, deliver the underlying asset. The latter is called the **writer** of the option.

Many options are registered and settled via a **clearing house**. This central body is also responsible for the collection of **margin** from the writers of options. This margin is a sum of money (or equivalent) which is held by the clearing house on behalf of the writer. It is a guarantee that he is able to meet his obligations should the asset price move against him.

The trade in the simplest call and put options (colloquially called **vanilla** options, because they are ubiquitous) is now so great that it can, in some markets, have a value in excess of that of the trade in the underlying. In some cases too the exchange-traded options are more liquid than the underlying asset. To give an idea of the size[3] of the derivatives (including futures) markets, there is an estimated $10,000 *billion* in derivatives investments worldwide in total (this is a gross figure; the net figure is much smaller). In late 1992, Citicorp alone had an estimated exposure equivalent to a notional contract value of $1426bn. As the number and type of derivative products have increased so there has been a corresponding growth in option pricing as a subject for academic and corporate research. This is especially true today as increasingly exotic types of options are created.

1.4 What are Options For?

Options have two primary uses: speculation and hedging. An investor who believes that a particular stock, XYZ again, say, is going to rise can purchase some shares in that company. If he is correct, he makes money, if he is wrong he loses money. This investor is speculating. As we have noted, if the share price rises from 250p to 270p he makes a profit of 20p or 8%. If it falls to 230p he makes a loss of 20p or 8%. Alternatively, suppose that he thinks that the share price is going to rise within the next couple of months and that he buys a call with exercise price 250p and expiry date in three months' time. We have seen in the earlier example that if such an option costs 10p then the profit or loss is magnified to 100%. Options can be a cheap way of exposing a portfolio to a large amount of risk.

If, on the other hand, the investor thinks that XYZ shares are going to fall he can, conversely, sell shares or buy puts. If he speculates by selling shares that he does not own (which in certain circumstances is perfectly legal in many markets) he is selling **short** and will profit from a fall in XYZ shares. (The opposite of a short position is a **long** position.) The

[3] These values are taken from a review of the derivatives market in the *Financial Times* of 8 December 1992.