AN INTRODUCTION TO MATHEMATICAL COSMOLOGY

This book provides a concise introduction to the mathematical aspects of the origin, structure and evolution of the universe. The book begins with a brief overview of observational and theoretical cosmology, along with a short introduction to general relativity. It then goes on to discuss Friedmann models, the Hubble constant and deceleration parameter, singularities, the early universe, inflation, quantum cosmology and the distant future of the universe. This new edition contains a rigorous derivation of the Robertson–Walker metric. It also discusses the limits to the parameter space through various theoretical and observational constraints, and presents a new inflationary solution for a sixth degree potential.

This book is suitable as a textbook for advanced undergraduates and beginning graduate students. It will also be of interest to cosmologists, astrophysicists, applied mathematicians and mathematical physicists.

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Second edition

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Contents

Preface to the first edition

Ever since I wrote my semi-popular book *The Ultimate Fate of the Universe* I have been meaning to write a technical version of it. There are of course many good books on cosmology and it seemed doubtful to me whether the inclusion of a chapter on the distant future of the universe would itself justify another book. However, in recent years there have been two interesting developments in cosmology, namely inflationary models and quantum cosmology, with their connection with particle physics and quantum mechanics, and I believe the time is ripe for a book containing these topics. Accordingly, this book has a chapter each on inflationary models, quantum cosmology and the distant future of the universe (as well as a chapter on singularities not usually contained in the standard texts).

This is essentially an introductory book. None of the topics dealt with have been treated exhaustively. However, I have tried to include enough introductory material and references so that the reader can pursue the topic of his interest further.

A knowledge of general relativity is helpful; I have included a brief exposition of it in Chapter 2 for those who are not familiar with it. This material is very standard; the form given here is taken essentially from my book *Rotating Fields in General Relativity*.

In the process of writing this book, I discovered two exact cosmological solutions, one connecting radiation and matter dominated eras and the other representing an inflationary model for a sixth degree potential. These have been included in Sections 4.5 and 9.4 respectively as I believe they are new and have some physical relevance.

I am grateful to J. V. Narlikar and M. J. Rees for providing some useful references. I am indebted to a Cambridge University Press reader for helpful comments; the portion on observational cosmology has I believe improved considerably as a result of these comments. I am grateful to

F. J. Dyson for his ideas included in the last chapter. I thank Maureen Storey of Cambridge University Press for her efficient and constructive subediting.

I am grateful to my wife Suraiya and daughters Nargis and Sadaf and my son-in-law Kamel for support and encouragement during the period this book was written. I have discussed plans for my books with Mrs Mary Wraith, who kindly typed the manuscript for my first book. For more than three decades she has been friend, philosopher and mentor for me and my wife and in recent years a very affectionate godmother ('Goddy') to my daughters. This book is fondly dedicated to this remarkable person.

Jamal Nazrul Islam
Chittagong, 1991

Preface to the second edition

The material in the earlier edition, to which there appears to have been a favourable response, has been kept intact as far as possible in this new edition except for minor changes. A number of new additions have been made. Some standard topics have been added to the introduction to general relativity, such as Killing vectors. Not all these topics are used later in the book, but some may be of use to the beginning student for mathematical aspects of cosmological studies. Observational aspects have been brought up to date in an extended chapter on the cosmological constant. As this is a book on mathematical cosmology, the treatment of observations is not definitive or exhaustive by any means, but hopefully it is adequate. To clarify the role of the cosmological constant, much discussed in recent years, an exact, somewhat unusual solution with cosmological constant is included. Whether the solution is new is not clear: it is meant to provide a 'comprehension exercise'. One reviewer of the earlier edition wondered why the Hubble constant and the deceleration parameter were chosen for a separate chapter. I believe these two parameters are among the most important in cosmology; adequate understanding of these helps to assess observations generally. Within the last year or two, through analyses of supernovae in distant galaxies, evidence seems to be emerging that the universe may be accelerating, or at least the deceleration may be not as much as was supposed earlier. If indeed the universe is accelerating, the nomenclature 'deceleration parameter' may be called into question. In any case, much more work has to be done, both observational and theoretical, to clarify the situation and it is probably better to retain the term, and refer to a possible acceleration as due to a 'negative deceleration parameter' (in case one has to revert back to 'deceleration'!). I believe it makes sense, in most if not all subjects, constantly to refer back to earlier work, observational, experimental or practical, as well as theoretical aspects, for

this helps to point to new directions and to assess new developments. Some of the material retained from the first edition could be viewed in this way.

A new exact inflationary solution for a sixth degree potential has been added to the chapter on the very early universe. The chapter on quantum cosmology is extended to include a discussion on functional differential equations, material which is not readily available. This topic is relevant for an understanding of the Wheeler–De Witt equation. Some additional topics and comments are considered in the Appendix at the end of the book. Needless to say, in the limited size and scope of the book an exhaustive treatment of any topic is not possible, but we hope enough ground has been covered for the serious student of cosmology to benefit from it.

As this book was going to press, Fred Hoyle passed away. Notwithstanding the controversies he was involved in, I believe Hoyle was one of the greatest contributors to cosmology in the twentieth century. The controversies, more often than not, led to important advances. Hoyle's prediction of a certain energy level of the carbon nucleus, revealed through his studies of nucleosynthesis, confirmed later in the laboratory, was an outstanding scientific achievement. A significant part of my knowledge of cosmology, for what it is worth, was acquired through my association with the then Institute of Theoretical Astronomy at Cambridge, of which the Founder-Director was Hoyle, who was kind enough to give me an appointment for some years. I shall always remember this with gratitude.

I am grateful to Clare Hall, Cambridge, for providing facilities where the manuscript and proofs were completed.

I am grateful for helpful comments by various CUP readers and referees, although it has not been possible to incorporate all their suggestions. I thank the various reviewers of the earlier edition for useful comments. I am grateful to Simon Mitton, Rufus Neal, Adam Black and Tamsin van Essen for cooperation and help at various stages in the preparation of this edition. I thank 'the three women in my life' (Suraiya, Sadaf and Nargis) and my son-in-law Kamel for support and encouragement.

Jamal Nazrul Islam
Chittagong, November 2000

IN MEMORIAM
Mary Wraith (1908–1995)
in affection, admiration and gratitude