In this exhaustive and definitive study, Christian Bidard develops a theory of prices of production. This theory, of classical inspiration, breaks down the symmetry between producers and consumers and gives more importance to reproduction than scarcity. In his analysis of multiple-product systems, Bidard focuses on the notion of an all-engaging system which elucidates the link with von Neumann’s theory; examines the notions of sector and vertical integration which make possible an elegant treatment of fixed capital; clarifies the status of the internal rate of return (IRR); and proposes a general theory of rent. In the discussion of capital theory and marginal equalities – or, more specifically, the treatment of exhaustible resources – Bidard compares and contrasts different readings of Sraffa’s work and revisits the question of the relationships between classical theory (Smith, Ricardo and Marx) and the general equilibrium theory (Walras, Arrow and Debreu).

CHRISTIAN BIDARD is Professor of Economics at the University of Paris X-Nanterre. He is the editor or author of several books on Sraffa’s theory – *Prix, reproduction, rareté* (1991), *La rente* (1987), *La production jointe* (1984) and its relationship with Keynesian theory. He is the author of over sixty papers in economic journals and books largely devoted to the theory of long-term competitive prices.
Prices, Reproduction, Scarcity

Christian Bidard
Contents

<table>
<thead>
<tr>
<th>Preface</th>
<th>page xiii</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acknowledgements</td>
<td>xv</td>
</tr>
</tbody>
</table>

Part I Single production

1 Principles 3

2 The corn model 5
   1 A simple economy 5
   2 Duality 5
   3 Expressions of price 7
   4 Choice of techniques 8
   5 Scope and limits of the model 12

3 A two-commodity economy 13
   1 A basic two-good economy 13
   2 Viability 14
   3 Accumulation and consumption 15
      3.1 Feasible rates of growth 15
      3.2 Consumption 16
   4 Profit and wage 17
      4.1 Profitability and prices 17
      4.2 Geometrical representation 19

4 A basic multisector economy 21
   1 A complex economy 21
   2 Properties of indecomposable matrices 22
   3 Economic properties 23
   4 Sensitivity analysis 26
   5 Linear models 26
      5.1 Sraffa and Leontief 26
      5.2 A taste of von Neumann’s theory 28
   6 Proofs 29

5 Non-basic economies 31
   1 Nested systems 31
   2 Relevant notions 31
### Contents

<table>
<thead>
<tr>
<th>Chapter</th>
<th>Title</th>
<th>Pages</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>Properties</td>
<td>33</td>
</tr>
<tr>
<td>4</td>
<td>The Austrian economy</td>
<td>34</td>
</tr>
<tr>
<td>5</td>
<td>Tax and tribute</td>
<td>36</td>
</tr>
<tr>
<td>6</td>
<td>Conclusion</td>
<td>36</td>
</tr>
<tr>
<td>7</td>
<td>Proof</td>
<td>37</td>
</tr>
<tr>
<td>6</td>
<td>Relative prices</td>
<td>39</td>
</tr>
<tr>
<td>1</td>
<td>Price and payment of wages</td>
<td>39</td>
</tr>
<tr>
<td>2</td>
<td>Price curves</td>
<td>40</td>
</tr>
<tr>
<td>2.1</td>
<td>Price space</td>
<td>40</td>
</tr>
<tr>
<td>2.2</td>
<td>Visualization</td>
<td>42</td>
</tr>
<tr>
<td>3</td>
<td>Distances between relative prices</td>
<td>44</td>
</tr>
<tr>
<td>3.1</td>
<td>Attractive cones</td>
<td>44</td>
</tr>
<tr>
<td>3.2</td>
<td>Hilbert distance</td>
<td>46</td>
</tr>
<tr>
<td>4</td>
<td>The golden sub-system</td>
<td>47</td>
</tr>
<tr>
<td>5</td>
<td>Wicksell price effects</td>
<td>48</td>
</tr>
<tr>
<td>6</td>
<td>References</td>
<td>48</td>
</tr>
<tr>
<td>7</td>
<td>Proofs</td>
<td>49</td>
</tr>
<tr>
<td>7</td>
<td>Ricardo’s theory of value</td>
<td>53</td>
</tr>
<tr>
<td>1</td>
<td>The theory of value as a stake</td>
<td>53</td>
</tr>
<tr>
<td>2</td>
<td>Difficulty of production and absolute price</td>
<td>55</td>
</tr>
<tr>
<td>3</td>
<td>The embodiments of value</td>
<td>57</td>
</tr>
<tr>
<td>4</td>
<td>A Ricardian standard of value</td>
<td>60</td>
</tr>
<tr>
<td>4.1</td>
<td>Corn as an absolute standard</td>
<td>60</td>
</tr>
<tr>
<td>4.2</td>
<td>Ricardo on measure</td>
<td>62</td>
</tr>
<tr>
<td>4.3</td>
<td>Multisector economies</td>
<td>64</td>
</tr>
<tr>
<td>5</td>
<td>Range of the construction</td>
<td>65</td>
</tr>
<tr>
<td>6</td>
<td>Notes on Sraffa’s interpretation</td>
<td>65</td>
</tr>
<tr>
<td>7</td>
<td>Mathematical discussion</td>
<td>67</td>
</tr>
<tr>
<td>8</td>
<td>The labour value</td>
<td>69</td>
</tr>
<tr>
<td>1</td>
<td>A measure of absolute value</td>
<td>69</td>
</tr>
<tr>
<td>2</td>
<td>The problem of transformation</td>
<td>70</td>
</tr>
<tr>
<td>2.1</td>
<td>From values to prices</td>
<td>70</td>
</tr>
<tr>
<td>2.2</td>
<td>The impossible transformation</td>
<td>72</td>
</tr>
<tr>
<td>3</td>
<td>The falling rate of profit</td>
<td>73</td>
</tr>
<tr>
<td>3.1</td>
<td>Statement of the law</td>
<td>73</td>
</tr>
<tr>
<td>3.2</td>
<td>A Marxian study</td>
<td>74</td>
</tr>
<tr>
<td>9</td>
<td>Choice of technique</td>
<td>77</td>
</tr>
<tr>
<td>1</td>
<td>The question</td>
<td>77</td>
</tr>
<tr>
<td>2</td>
<td>Dominant technique and market algorithm</td>
<td>77</td>
</tr>
<tr>
<td>3</td>
<td>Convergence and existence</td>
<td>79</td>
</tr>
<tr>
<td>4</td>
<td>Properties</td>
<td>82</td>
</tr>
<tr>
<td>5</td>
<td>Laws on prices</td>
<td>84</td>
</tr>
<tr>
<td>6</td>
<td>Sraffa on switches</td>
<td>86</td>
</tr>
<tr>
<td>7</td>
<td>Proofs</td>
<td>88</td>
</tr>
</tbody>
</table>
Contents

Part II Joint production

10 Joint production: a theoretical object 93
   1 A grain of sand 93
   2 The project 93

11 Paradoxes and tools 95
   1 Not amused 95
   2 Some catastrophes 95
   3 The price space 96
      3.1 A geometrical approach 96
      3.2 Some properties of the candidates 99
      3.3 Restoration of semi-positive prices 100
   4 The quantity space 101
   5 The Lippi problem 103
      5.1 Rigid demand, in particular 103
      5.2 The golden rule 105
   6 Conclusion 106
   7 References 107
   8 Proof 108

12 Engaging systems 110
   1 A desirable property 110
   2 Notion of an all-engaging system 111
   3 Bounds to regularity 114
   4 Geometrical interpretation 115
   5 Semi-engaging systems 116
   6 Reducibilities versus decomposability 117
   7 Conclusion 120
   8 References 121
   9 Proofs 122

13 From von Neumann to Sraffa 129
   1 A universal property 129
   2 The von Neumann theory 129
   3 Von Neumann and Sraffa 131
      3.1 An all-engagingness property 131
      3.2 Quantities and prices 132
   4 Geometrical illustration 133
   5 Conclusion 134
   6 Note on von Neumann 135

14 The notion of a sector 137
   1 From industries to sectors 137
   2 Sectors and final goods 138
   3 Adjustment with capital goods 139
   4 Sectors with capital goods 141
   5 Vertical integration 143
   6 A non-substitution result 146
   7 Conclusion 147
Contents

8 References 148
9 Proofs 148

15 Austrian and one-machine models 152
1 Two fixed-capital models 152
2 The Austrian project and truncation 154
  2.1 Investment criteria 154
  2.2 The truncation hypothesis 155
  2.3 The Ricardian law 159
3 The multisector one-machine model 160
  3.1 Centre and prices 160
  3.2 Effects of truncation 164
4 Fixed capital without truncation 165
  4.1 Austrian project without truncation 165
  4.2 One machine without truncation 167
5 Conclusion 168
6 Historical notes 169

16 Fixed-capital theory 172
1 One theory 172
2 A general fixed-capital model 172
3 Smoothness of vertical integration 176
4 Applications 179
5 Conclusion 180
6 References 180
7 Proofs 181

17 Rent theory 184
1 The difficulty of reproduction 184
2 Land and joint production 184
3 Two pure types of rent 187
  3.1 Extensive rent 187
  3.2 Intensive rent 189
4 Technical choice with rent 191
  4.1 Existence results 191
  4.2 Indeterminacy of distribution 193
5 A Ricardian model 193
6 References 196
7 Proofs 197

Part III Questions of method

18 An agenda 201

19 A Sraffian theory of choice of technique 202
  1 A theoretical divide 202
  2 An original methodology 203
  3 Market algorithm and consistency 205
  4 Candidates 206
  5 Pre-technologies 207
Contents

5.1 The insertion axiom 207
5.2 Existence and oddity 209
6 Technologies 210
6.1 Quasi-technology and convergence 210
6.2 Robustness, softness and uniqueness 211
7 Two applications 212
8 Conclusion 214
9 Switches and fakes 215
10 Proof 217

20 The structure of general equilibrium 220
1 Walras first 220
2 Static equilibrium 221
2.1 Utility, never 221
2.2 General equilibrium 222
3 Intertemporaleconomies 224
4 Efficiency and competition 226
4.1 The concept of efficiency 226
4.2 Where prices appear 226
5 Efficiency in an infinite horizon 228
5.1 The lost equivalence 228
5.2 Three efficiency criteria 230
5.3 Competition and inefficiency 232
6 The programme 233
7 Walras' theory of capitalization 233
7.1 Formalization 233
7.2 Discussion 235

21 The marginal equalities 237
1 The dispute over marginal equalities 237
2 The marginal productivity of labour 238
3 The demand for labour curve 240
3.1 Full-price adjustments 240
3.2 Quantity adjustments 241
4 The marginal productivity of capital 242
5 A numerical example 244
5.1 A WS calculation 244
5.2 Rule or fog 245
6 An Austrian forest 248
7 Capital theory 249
7.1 Time as capital 250
7.2 A miracle 251
8 Conclusion 252

22 Intertemporal models and prices of production 253
1 A comparative study 253
2 Reproduction in a Walrasian model 253
2.1 One-period model 253
2.2 Invariant relative prices 255
2.3 A general result 256
## Contents

5 A fruitful standpoint 305  
5.1 Which exogenous variables? 305  
5.2 The equilibrium paths 306  
5.3 The disequilibrium paths 307  
6 Omega 308

Mathematical appendix: elements of combinatorial geometry 309  
1 A combinatorial problem 309  
2 Candidates and techniques 309  
3 The naked Sperner lemma 313  
3.1 The simplicial model 313  
3.2 The combinatorial lemma 314  
3.3 Orientations 316  
4 Two applications 318  
4.1 The geometrical Sperner lemma 319  
4.2 Pre-technology and the Colour Theorem 320  
5 An algorithm for techniques 322  
5.1 An alternative approach 322  
5.2 Quasi-technology and existence 324  
5.3 Pseudo-technology and uniqueness 328  
5.4 Technology 330  
6 Application and comparisons 331  
7 Sources 333

References 334  
Author index 348  
Subject index 351
Preface

In 1968, when we undertook the study of political economy by reading Capital and Theory of Value, two books each inspiring in their own way, the ambition was to gain an understanding of the ways in which society works. We thus began with the essential issues. The present work is rooted in a profound perplexity in the face of the incompatibility between these constructs. The impossibility of linking labour value with capitalist competition, as opposed to the formal rigour of the general equilibrium theory, should have led us to abandon our quest, but the reading of Production of Commodities by Means of Commodities encouraged us to continue. In this book, a theory of prices of production is developed: the symmetry between producers and consumers is broken and more importance is given to reproduction than to scarcity, thereby reviving the accents of classical theory. Sraffa’s construct must be situated in the context of a radical programme that aims to establish the primacy of the classical way of thought. To reflect on this ambition seriously is to measure the scope of the critique and the long road still ahead. The project explains the structural analysis developed here in both its analytical and methodological aspects.

Our discourse keeps a varying, yet calculated distance from Sraffa’s. Part I, Single production, deals with familiar matters. Its innovations concern the study of relative prices and the interpretation of Ricardo’s theory of value. The economic properties referred to here are used as landmarks, but their real understanding requires the possibility of eluding them. This is the main interest of part II, Joint production. The study is centred on a few concepts: the notion of an all-engaging system elucidates the link with von Neumann’s theory; the notions of sector and vertical integration make possible an elegant treatment of fixed capital; and the theoretical status of the internal rate of return (IRR) is specified. Finally, a general theory of rent is proposed. The permanent principle is that of identification of the frame where such a property is true and its association with either a central concept or a small core of irreducible hypotheses.
Capital theory and marginal equalities are analysed in part III, Questions of method. The issue of the choice of technique illustrates our methodological and analytical effort. Two approaches to Sraffa's work are contrasted. Either it outlines an original theory which we develop in a resolutely 'fundamentalist' perspective and which endeavours to define principles while considering their limits, or it constitutes a variant of the general equilibrium model, which has to be explicitly recognized. Implicitly orienting itself in the second direction without acknowledging the change of theoretical status or accepting its consequences, the post-Sraffian analytical literature, whose contributions are remarkable in other respects, has rendered the identification of fundamental problems more difficult. If this interpretation were to be retained, the critique of economic theory would be significantly different from the one initially contemplated. To become conscious of this discordance is an intellectual prerequisite for economists interested in classical thought and in the pursuit of an ambitious programme: ‘We should regret that the truth only progresses slowly, but it shall surely triumph at last’ (Ricardo, letter to Morning Chronicle, 4 November 1810).

The book is an updated version of Prix, reproduction, rareté (Editions Dunod, 1991), which has been simplified and extended. It is our choice not to attempt to drown the reader in calculations. The most significant changes result from a stimulating and friendly cooperation with Guido Erreygers. I have shared the joys and the torments of research with many colleagues, but disclaiming their responsibility in the views and the opinions defended here is by no means conventional. My nearest and dearest have had the patience to bear a consuming intellectual passion – sustained thanks to Piero Sraffa.
Acknowledgements