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978-0-521-47236-4 - The Nonlinear Theory of Elastic Shells: Second Edition

A. Libai and J. G. Simmonds

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