
Contents

<i>Preface</i>	<i>page</i> xiii
Part one: Introduction	1
1 Mathematical modeling	3
1.1 What is a model?	3
1.2 The procedure of modeling	4
1.3 Choosing the model	8
1.4 Some examples	9
Exercises	13
Part two: Methods	17
2 Nondimensionalization	19
2.1 Introduction	19
2.2 Damped pendulum	21
2.3 Shear flow, heat transport, and convection	23
2.4 Using numerical estimates: an example from mathematical biology	25
2.5 Notes and references	30
Exercises	31
3 Asymptotics	35
3.1 Order notation	35
3.2 Asymptotic sequences and expansions	36
3.3 Convergence versus divergence	37
3.4 An algebraic example	38
3.5 Laplace's method	41
3.6 Notes and references	42
Exercises	43
4 Perturbation methods	45
4.1 Elementary boundary layer theory	45
4.2 Matched asymptotic expansions	47
4.3 Interior layers	48

viii	<i>Contents</i>	
4.4	A nonlinear example	50
4.5	Nonlinear oscillations	51
4.6	Partial differential equations	53
4.7	Notes and references	55
	Exercises	55
	Part three: Classical models	59
5	Heat transfer	61
5.1	The diffusion equation	61
5.2	Notes and references	69
	Exercises	70
6	Viscous flow	76
6.1	The Navier–Stokes equation	76
6.2	Notes and references	86
	Exercises	87
7	Solid mechanics	96
7.1	Stress and strain	96
7.2	Linear elasticity	97
7.3	Plasticity	104
7.4	Viscoelasticity	109
7.5	Notes and references	113
	Exercises	114
8	Electromagnetism	118
8.1	Fundamentals	118
8.2	Maxwell’s equations	120
8.3	Notes and references	127
	Exercises	128
	Part four: Continuum models	131
9	Enzyme kinetics	133
9.1	Pseudo-steady state hypothesis	134
9.2	Nondimensionalization	135
9.3	Singular perturbation theory	136
9.4	Enzyme-substrate-inhibitor system	138
9.5	Notes and references	143
	Exercises	143
10	The Belousov–Zhabotinskii reaction	145
10.1	Reaction mechanism	146
10.2	Relaxation oscillation analysis	149
10.3	Notes and references	155
	Exercises	157

<i>Contents</i>		ix
11	Spruce budworm infestations	161
11.1	Nondimensionalization and scale analysis	164
11.2	Ludwig–Jones–Holling (LJH) analysis	166
11.3	Summary	167
11.4	Finite saturation foliage health, revisited	168
11.5	Synopsis	173
11.6	Notes and references	175
	Exercises	176
12	Chemical reactors	179
12.1	Mathematical modeling	181
12.2	Thermal runaway	185
12.3	More realistic models: heat and mass transfer	190
12.4	The case $Le \gg 1$, $\varepsilon \ll 1$, and $\gamma = O(1)$	191
12.5	Nonporous pellet	192
12.6	Macroscopic modeling	195
12.7	Notes and references	198
	Exercises	199
13	Groundwater flow	201
13.1	Basic groundwater flow	204
13.2	Dam seepage	205
13.3	Dupuit approximation	206
13.4	Consolidation	211
13.5	Solute dispersivity	214
13.6	Heterogeneous porous media	216
13.7	Notes and references	219
	Exercises	222
14	Convection in a porous medium	227
14.1	Introduction	227
14.2	Linear stability	227
14.3	Nonlinear stability	229
14.4	Convection	232
14.5	A mathematical model	233
14.6	Nondimensionalization	235
14.7	Stability analysis	236
14.8	Nonlinear stability analysis	239
14.9	Boundary layer theory	243
14.10	Notes and references	248
	Exercises	250
15	River flow	254
15.1	The role of fluid mechanics	254
15.2	The mechanics of drainage basins	254

x	<i>Contents</i>	
15.3	Mathematical model	255
15.4	The flood hydrograph	256
15.5	Acceleration: stability and waves	258
15.6	Nonlinear waves	262
15.7	Sediment transport	263
15.8	Drainage networks	265
15.9	Notes and references	269
	Exercises	270
16	One-dimensional two-phase flow	273
16.1	Introduction	273
16.2	Flow regimes	273
16.3	A simple two-fluid model	275
16.4	Other models	275
16.5	Characteristics	276
16.6	More on averaging	276
16.7	A simple model for annular flow	280
16.8	Mathematical model of a thermosyphon	284
16.9	A reduced model	289
16.10	Notes and references	292
	Exercises	293
	Part five: Advanced models	299
17	Alloy solidification	301
17.1	Introduction	301
17.2	Modeling mushy layers	303
17.3	A reduced model	309
17.4	No convection, similarity solution	312
17.5	Convection	314
17.6	Modeling queries	324
17.7	Notes and references	327
18	Ice sheet dynamics	328
18.1	Basic equations and the shallow ice approximation	329
18.2	Isothermal flow	334
18.3	Steady, nonisothermal flow	338
18.4	Drainage, sliding and ice-till coupling	340
18.5	Notes and references	344
19	Chemosensory respiratory control	346
19.1	Respiratory physiology	346
19.2	The Grodins model	349
19.3	Reducing the model	353
19.4	Oscillations and chaos	356
19.5	Notes and references	359

<i>Contents</i>		xi
20	Frost heave in freezing soils	362
20.1	Introduction	362
20.2	Primary frost heave models	366
20.3	Secondary frost heave	373
20.4	Miller model of secondary frost heave	374
20.5	Simplifications	378
20.6	A reduced model	382
20.7	Notes and references	386
	<i>References</i>	387
	<i>Index</i>	399