

Atmospheric data analysis

Atmospheric Data Analysis is intended to fill a void in the atmospheric science literature and curricula. The book is self-contained, and includes topics important in several other fields outside atmospheric observation, including atmospheric dynamics and statistics. The emphasis is on the theoretical foundation of the subject and most of the development is analytic, but many practical considerations and examples are introduced. Dr. Daley devotes the early chapters to spatial analysis. Later chapters are primarily concerned with temporal aspects and the book closes with discussions of mesoscale and oceanic data assimilation and the challenges offered by future observing systems. There are numerous exercises at the end of each chapter to aid the student in comprehending the material.

Atmospheric Data Analysis can be used as a textbook for graduate and senior undergraduate students, and it will also serve as important reference material for practicing meteorologists, atmospheric scientists, and by extension it has applications to the fields of oceanography and seismology.

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Preface

The demand for atmospheric and environmental information is escalating rapidly, requiring the design and deployment of increasingly exotic and comprehensive observing systems. Atmospheric data analysis is the diagnosis of the complete, physically consistent four-dimensional atmospheric structure from this vast, asynchronous, heterogeneous data base.

Analyses of the atmospheric state are widely used in forecasting, in observational and phenomenological studies, and in formulating and verifying theoretical models of atmospheric behavior. They are also valuable for climatological studies, oceanography, and other disciplines of the terrestrial environment. In their most popular form, animated full-color atmospheric analyses are the mainstay of the television weather show. Although atmospheric analyses are widely used, the process of atmospheric analysis is not as widely understood as it should be. In large part, this is due to the paucity of pedagogical material in the field. This book, *Atmospheric Data Analysis*, evolved from a series of discussions with Tony Hollingsworth in England during a sabbatical year in 1983. Both of us proceeded to develop pedagogical material on atmospheric analysis, which formed the basis for a course at Colorado State University in 1984. Among the student critiques of the lecturer was one perceptive comment that a textbook would have substantially improved the course. I could not disagree and set to work immediately.

Atmospheric Data Analysis covers the complete field of atmospheric objective analysis or data assimilation. It is designed to be self-contained and thus includes some topics from atmospheric dynamics and statistics. The emphasis is on the theoretical foundations of the subject, and most of the developments are analytic. However, practical aspects and examples are introduced where appropriate. The book is intended for graduate or advanced undergraduate students with no prior knowledge of the subject. The level of mathematics is at the advanced undergraduate level, and background material on less familiar mathematical concepts is provided. Some prior knowledge of undergraduate atmospheric dynamics would be helpful but

is not absolutely necessary. Problems of varying levels of difficulty are included after each chapter.

The graduate or undergraduate curricula could use this book for a self-contained course or could include it in courses on numerical weather prediction, climate modeling, or atmospheric and climate dynamics. The material is also applicable in oceanography and related disciplines.

The emphasis in early chapters is on spatial analysis, and the treatment is largely mathematical or statistical. Gradually, more atmospheric physics, in the form of multivariate constraints, is introduced into the discussion, and there is an increasing emphasis on temporal aspects. The last chapter introduces the student to the most recent developments in the field.

This book could not have been written without the support and cooperation of a number of people on three continents. Tony Hollingsworth provided the initial inspiration, gave me many useful ideas and endless encouragement, and reviewed large segments of the text. Andrew Lorenc and David Williamson reviewed the final draft of the manuscript. Steve Cohn, Ron Errico, Akira Kasahara, John Lewis, Hersh Mitchell, Tom Schlatter, J. J. Stephens, and Clive Temperton each reviewed substantial portions of the manuscript and provided me with much constructive criticism. Other portions of the manuscript were reviewed by Andrew Bennett, John Derber, Tzvi Gal-Chen, Carl Hane, Peter Lynch, Kikuro Miyakoda, Phil Rasch, Yoshi Sasaki, Bob Seaman, Adrian Simmons, and Grace Wahba. Helpful ideas and encouragement came from Wayman Baker, T. N. Krishnamurti, Norm McFarlane, I. M. Navon, Wayne Schubert, Duane Stevens, Phillip Thompson, Joe Tribbia, and Warren Washington.

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Finally, I am particularly grateful for the patience and forbearance of my wife, Lucia, during the course of this project and for her help in proofreading the final text.

Partial list of symbols

<i>a</i>	earth radius, Fourier coefficient, arbitrary constant
<i>b</i>	Fourier coefficient, arbitrary constant
<i>c</i>	expansion coefficient, weight (Section 4.1), constant
<i>d</i>	residual, derivative
\underline{d}	observation increment vector, observation vector (Section 13.1)
<i>e</i>	2.71828
\underline{e}	eigenvector
<i>f</i>	arbitrary function, Coriolis parameter
<i>g</i>	spectrally transformed variable, gravitational constant
<i>h</i>	height, equivalent depth, arbitrary basis function
<i>i</i>	$\sqrt{-1}$, index
\underline{i}	unit vector
<i>j</i>	index
\underline{j}	unit vector
<i>k</i>	two-dimensional wavenumber, index
\underline{k}	unit vector
<i>l</i>	longitudinal velocity component, norm, index
<i>m</i>	<i>x</i> wavenumber, index
<i>n</i>	<i>y</i> wavenumber, normal direction, index
<i>p</i>	probability, element of \underline{P} , arbitrary function, index
<i>q</i>	specific humidity, element of \underline{Q} , arbitrary function, index
\underline{q}	column vector (Section 5.1)
<i>r</i>	absolute distance, random noise component
\underline{r}	position vector
<i>s</i>	complex frequency, arbitrary state variable, index
\underline{s}	column vector of dependent variables
<i>t</i>	transverse velocity component, time
<i>u</i>	eastward (<i>x</i>) or radial velocity component

v	northward (y) or tangential velocity component
\mathbf{v}	horizontal velocity vector
w	specified a priori weight, vertical velocity component
x	eastward space coordinate, normal mode amplitude
\times	vector product
y	northward space coordinate, slow mode amplitude
\underline{y}	eigenvector (Section 4.5)
z	vertical space coordinate, fast mode amplitude
\underline{z}	arbitrary column vector
A, \underline{A}	analyzed value (Section 4.2), analysis error covariance matrix
B, \underline{B}	background value (Section 4.2), background error covariance matrix
C	covariance (Section 5.1), interaction coefficient (Section 10.3), condensation (Section 13.6), curve on complex plane (Section 13.4)
\underline{C}	weight matrix (Section 3.5), covariance matrix (Appendix D)
C_p	specific heat at constant pressure
D	divergence, damping parameter
E	eastward gravity mode (Section 9.4)
\underline{E}	error, evaporation
\underline{E}	matrix of eigenvectors
F	slack function, arbitrary function
\mathbf{F}	friction force
\underline{F}	forward interpolation error covariance matrix
G	inertia–gravity mode, arbitrary function
\underline{G}	Gram matrix
H	forward interpolation operator (Section 13.1)
\underline{H}	matrix of basis functions evaluated on network (Section 2.3), forward interpolation matrix (Section 13.1)
I, \underline{I}	functional, identity matrix
J	functional, Bessel function, summation limit
K	Kelvin mode (Section 9.4)
K	shallow water parameter (Section 6.4), frictional coefficient, summation limit
\underline{K}	Kalman–Bucy gain matrix
L	Laplace transform, characteristic scale, summation limit
\underline{L}	linearized f -plane shallow water equations matrix
L_H, L_Z, L_R	horizontal and vertical characteristic scales, Rossby radius
L_B, L_O	background and observation error characteristic scales
M	ellipticity measure, summation index
\underline{M}	diagonal matrix (Section 3.5), linear model matrix (Section 13.2)
N	Brunt–Väisälä frequency, summation index
\underline{N}	spline matrix (Section 2.7), Jacobian matrix (Section 10.7)
O, \underline{O}	observed value, observation error covariance matrix
P, \underline{P}	pressure, prediction error covariance matrix

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Q	potential vorticity, diabatic heating, summation limit
\underline{Q}	spline matrix (Section 2.7), diagonal SCM matrix (Section 3.7), KB system error matrix (Section 13.3)
R	Rossby mode (Section 9.4)
R	radius of influence, spectral response, gas constant, symbolic right-hand side
R_0	Rossby number
\underline{R}	$\underline{Q} + \underline{F}$
S, \underline{S}	horizontal domain, second-derivative vector (Section 2.7)
\underline{S}	symmetric SCM matrix (Section 3.5), model matrix (Section 12.3)
T	transpose
T	true value (Section 4.2), temperature (Section 7.3)
\underline{T}	tri-diagonal matrix
U	constant zonal wind component
V	variance
W	westward gravity mode
W	a posteriori weight
\underline{W}	a posteriori weight matrix
\underline{Y}	vector of slow mode amplitudes
Z	vertical structure function
\underline{Z}	vector of fast mode amplitudes
α	spectral transform of a posteriori weights, inertia-gravity wave frequency, direction cosine, Rossby projection angle
$\underline{\alpha}$	a posteriori weight matrix
β	latitudinal derivative of Coriolis parameter, nondimensional data insertion frequency (Section 12.3)
γ	constant
Γ	static stability
δ	Dirac delta function, Kronecker delta, variational operator
∂	partial differential operator
$\nabla, \nabla \cdot$	gradient and divergence operators
$\nabla \times, \nabla^2$	curl and Laplacian operators
ε	error, small parameter (Section 7.3), spherical harmonic parameter (Appendix A)
ζ	relative vorticity vector
ζ	$\mathbf{k} \cdot \zeta$
κ	R/C_p
η	mean value, arbitrary function
$\theta, \underline{\theta}$	potential temperature, Hessian matrix
λ	eigenvalue, Lagrange multiplier, longitude
$\underline{\Lambda}$	diagonal matrix of eigenvalues
μ	sin(latitude), geostrophic coupling parameter, eigenvalue, x wave-number, index

ν	eigenvalue, divergent coupling parameter, y wavenumber, index
π	3.14159
Π, \mathbb{I}	vertical pressure scale, slow mode projection matrix
ρ	correlation, density, spectral radius
$\underline{\rho}, \underline{\rho}$	correlation vector, matrix
σ	standard deviation, frequency
Σ	summation
τ, τ	time scale, tangential unit vector
ϕ	tangential coordinate, latitude
Φ, \mathfrak{F}	geopotential, horizontally averaged geopotential
χ	velocity potential
ψ	streamfunction
ω	normalized a posteriori weight, vertical velocity (P coordinates), frequency
Ω	earth angular rotation vector
Ω	$\mathbf{k} \cdot \Omega$, forward interpolation operator
Ω	forward interpolation matrix